Roll No…………………….. **Total No. of Printed Pages: [01]**

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B.Sc. (Hons.) Math, (Semester – 4th)

DIFFERENTIAL EQUATIONS-II

Subject Code: BMATS1421

Paper ID: [22131215]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Define the partial differential equation with examples.

b) Show that if a is a constant, then $u(x,t) = \sin(at).\cos(x)$ is a solution of $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$.

c) Solve the equation $a(p+q) = z$.

d) Write the condition of the pfaffian differential equation is integrable.

e) What is the condition of second order linear PDEs, if it is hyperbolic.

f) Write the two dimensional wave equation.

g) Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

h) Solve $x^2\left(\frac{\partial^2 z}{\partial x^2}\right) - y^2\left(\frac{\partial^2 z}{\partial y^2}\right) - y\left(\frac{\partial z}{\partial y}\right) + x\frac{\partial z}{\partial x}$.

i) Write the method of separation of variables.

j) Solve $2r + 5s + 2t = 0$.

Section – B **(8 marks each)**

Q2. Solve the $2p + 3q = 1$.

Q3. Find the characteristics of $y^2 r - x^2 t = 0$.

Q4. Classify the equation $u_{xx} + u_{yy} + u_{zz} + u_{yz} + u_{zy} = 0$.

Q5. Form the partial differential equation by eliminating arbitrary functions f and g from

$$Z = f(x^2 - y) + g(x^2 + y).$$

Q6. Solve $[D^2 + 3D(D') + 2(D')^2]z = x + y$.

Section – C **(10 marks each)**

Q7. Find the complete integral of $f = z^2 - pq\,xy = 0$ by Charpit's method.

Q8. Solve the one-dimension wave equation $\frac{\partial^2 y}{\partial x^2} = a^2 \frac{\partial^2 y}{\partial t^2}$ by the Monge' method.

Q9. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,$ where $u(x,0) = 6e^{-3x}$.