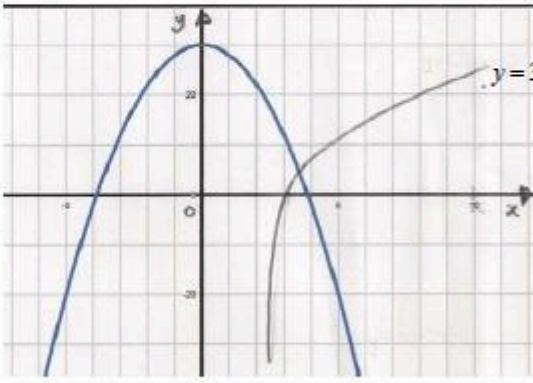


Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$f(3.5) = -4.8, f(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval [3.5, 4] $\Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 4 - \frac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x - 5)$	(2)	
(c)		M1	3.1a
	Attempts to sketch both $y = \ln(2x - 5)$ and $y = 30 - 2x^2$		
	States that $y = \ln(2x - 5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x - 5) = 30 - 2x$ has just one root $\Rightarrow f(x) = 0$ has just one root	A1	2.4
		(2)	
(6 marks)			

Notes:**(a)****M1:** Attempts $f(x)$ at both $x = 3.5$ and $x = 4$ with at least one correct to 1 significant figure**A1*:** $f(3.5)$ and $f(4)$ correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or $f(3.5) \times f(4) < 0$ or similar with $f(x)$ being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval'**(b)****M1:** Attempts $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ evidenced by $x_1 = 4 - \frac{3.099}{16.67}$ **A1:** Correct answer only $x_1 = 3.81$ **(c)****M1:** For a valid attempt at showing that there is only one root. This can be achieved by

- Sketching graphs of $y = \ln(2x - 5)$ and $y = 30 - 2x^2$ on the same axes
- Showing that $f(x) = \ln(2x - 5) + 2x^2 - 30$ has no turning points
- Sketching a graph of $f(x) = \ln(2x - 5) + 2x^2 - 30$

A1: Scored for correct conclusion

Q2.

Question	Scheme	Marks	AOs
(a)	$\ln x \rightarrow \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^3 + x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2 \times \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{dy}{dx} = 3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}} *$	A1*	2.1
		(4)	
(b)	$12x^2 + x - 16\sqrt{x} = 0 \Rightarrow 12x^{\frac{3}{2}} + x^{\frac{1}{2}} - 16 = 0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}} = 16 - \sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}} *$	A1*	2.1
		(3)	
(c)	$x_2 = \sqrt[3]{\left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	$x_2 = \text{awrt } 1.13894$	A1	1.1b
	$x = 1.15650$	A1	2.2a
		(3)	
(10 marks)			

Notes:

(a)

B1: Differentiates $\ln x \rightarrow \frac{1}{x}$ seen or implied

M1: Correct method to differentiate $\frac{4x^2 + x}{2\sqrt{x}}$:

Look for $\frac{4x^2 + x}{2\sqrt{x}} \rightarrow \dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}}$ being then differentiated to $Px^{\frac{1}{2}} + \dots$ or $\dots + Qx^{-\frac{1}{2}}$

Alternatively uses the quotient rule on $\frac{4x^2 + x}{2\sqrt{x}}$.

Condone slips but if rule is not quoted expect $\left(\frac{dy}{dx}\right) = \frac{2\sqrt{x}(Ax+B) - (4x^2+x)Cx^{\frac{1}{2}}}{(2\sqrt{x})^2} (A, B, C > 0)$

But a correct rule may be implied by their u, v, u', v' followed by applying $\frac{vu' - uv'}{v^2}$ etc.

Alternatively uses the product rule on $(4x^2 + x)(2\sqrt{x})^{-1}$

Condone slips but expect $\left(\frac{dy}{dx}\right) = Ax^{-\frac{1}{2}}(Bx+C) + D(4x^2+x)x^{-\frac{3}{2}} (A, B, C > 0)$

In general condone missing brackets for the M mark. If they quote $u = 4x^2 + x$ and $v = 2\sqrt{x}$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have v rather than v^2 in the denominator.

A1: Correct differentiation of $\frac{4x^2 + x}{2\sqrt{x}}$ although may not be simplified.

Examples: $\left(\frac{dy}{dx}\right) = \frac{2\sqrt{x}(8x+1) - (4x^2+x)x^{\frac{1}{2}}}{(2\sqrt{x})^2}, \frac{1}{2}x^{-\frac{1}{2}}(8x+1) - \frac{1}{4}(4x^2+x)x^{\frac{3}{2}}, 2 \times \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}}$

A1*: Obtains $\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$ via $3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x}$ or a correct application of the quotient or product rule and with sufficient working shown to reach the printed answer.
There must be no errors e.g. missing brackets.

(b)

M1: Sets $12x^2 + x - 16\sqrt{x} = 0$ and divides by $\sqrt{x}$ or equivalent e.g. divides by x and multiplies by $\sqrt{x}$

dM1: Makes the term in $x^{\frac{3}{2}}$ the subject of the formula

A1*: A correct and rigorous argument leading to the given solution.

Alternative - working backwards:

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}} \Rightarrow x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow 12x^{\frac{3}{2}} = 16 - \sqrt{x} \Rightarrow 12x^2 = 16\sqrt{x} - x \Rightarrow 12x^2 - 16\sqrt{x} + x = 0$$

M1: For raising to power of $3/2$ both sides. **dM1:** Multiplies through by $\sqrt{x}$. **A1:** Achieves printed answer and makes a minimal comment e.g. tick, #, QED, true etc.

(c)

M1: Attempts to use the iterative formula with $x_1 = 2$. This is implied by sight of $x_2 = \left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$ or awrt 1.14

A1: $x_2 =$ awrt 1.13894

A1: Deduces that $x = 1.15650$

Q3.

Question	Scheme	Marks	AOs	
(a) Way 1	$\{y = x^x \Rightarrow\} \ln y = x \ln x$	B1	1.1a	
	$\frac{1}{y} \frac{dy}{dx} = 1 + \ln x$	M1	1.1b	
		A1	2.1	
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \frac{x}{x} + \ln x = 0 \text{ or } 1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \dots$	M1	1.1b	
	$x = e^{-1} \text{ or awrt } 0.368$	A1	1.1b	
	Note: $k \neq 0$	(5)		
(a) Way 2	$\{y = x^x \Rightarrow\} y = e^{x \ln x}$	B1	1.1a	
	$\frac{dy}{dx} = \left(\frac{x}{x} + \ln x \right) e^{x \ln x}$	M1	1.1b	
		A1	2.1	
	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} \frac{x}{x} + \ln x = 0 \text{ or } 1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = \dots$	M1	1.1b	
	$x = e^{-1} \text{ or awrt } 0.368$	A1	1.1b	
	Note: $k \neq 0$	(5)		
(b) Way 1	Attempts both $1.5^{1.5} = 1.8\dots$ and $1.6^{1.6} = 2.1\dots$ and at least one result is correct to awrt 1 dp	M1	1.1b	
	$1.8\dots < 2$ and $2.1\dots > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1	
		(2)		
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b	
	$\{x_4 = 1.67313\dots \Rightarrow\} x_4 = 1.673$ (3 dp) cao	A1	1.1b	
		(2)		
(d)	Give 1 st B1 for any of - oscillates - periodic - non-convergent - divergent - fluctuates - goes up and down 1, 2, 1, 2, 1, 2 - alternates (condone)	Give B1 B1 for any of - periodic {sequence} with period 2 - oscillates between 1 and 2	B1	2.5
		Condone B1 B1 for any of - fluctuates between 1 and 2 - keep getting 1, 2 - alternates between 1 and 2 - goes up and down between 1 and 2 1, 2, 1, 2, 1, 2, ...	B1	2.5
			(2)	
(11 marks)				

(11 marks)

Note	A common solution A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x \log x \Rightarrow \frac{1}{y} \frac{dy}{dx} = 1 + \log x$ $\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} 1 + \log x = 0 \Rightarrow x = 10^{-1}$ 1st B1 for $\log y = x \log x$ 1st M1 for $\log y \rightarrow \lambda \frac{1}{y} \frac{dy}{dx}; \lambda \neq 0$ or $x \log x \rightarrow 1 + \log x$ or $\frac{x}{x} + \log x$ 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = \dots; k \neq 0$
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Question	Scheme	Marks	AOs
(b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(b) Way 3	For $\ln y = x \ln x$, attempts both $1.5 \ln 1.5 = 0.608...$ and $1.6 \ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(b) Way 4	For $\log y = x \log x$, attempts both $1.5 \log 1.5 = 0.264...$ and $1.6 \log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question

(a)	Way 1
B1:	$\ln y = x \ln x$. Condone $\log_x y = x \log_x x$ or $\log_x y = x$
M1:	For either $\ln y \rightarrow \frac{1}{y} \frac{dy}{dx}$ or $x \ln x \rightarrow 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y} \frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y} \frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1 + \ln x)$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x \ln x \rightarrow x = 0.368$ with no intermediate working
(a)	Way 2
B1:	$y = e^{x \ln x}$
M1:	For either $y = e^{x \ln x} \Rightarrow \frac{dy}{dx} = f(\ln x) e^{x \ln x}$ or $x \ln x \rightarrow 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right) e^{x \ln x}$ or $\frac{dy}{dx} = (1 + \ln x) e^{x \ln x}$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\} \ln y = x \ln x \Rightarrow \frac{dy}{dx} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = e^{-1}$ or awrt 0.368

Notes for Question Continued	
(b)	Way 1
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} = \text{awrt } 1.8...$ and $1.6^{1.6} = \text{awrt } 2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	Way 2
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	Way 3
M1:	Attempts both $1.5 \ln 1.5 = 0.608...$ and $1.6 \ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5 \ln 1.5 = 0.608...$ and $1.6 \ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	Way 4
M1:	Attempts both $1.5 \log 1.5 = 0.264...$ and $1.6 \log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5 \log 1.5 = 0.264...$ and $1.6 \log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ cao (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 = \text{awrt } 1.673$
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for "Converges in a cob-web pattern" or "Converges up and down to α "

Q4.

Question	Scheme	Marks	AOs
(a)	$\dots xe^x + \dots e^x$	M1	1.1b
	$k(xe^x + e^x)$	A1	1.1b
	$\frac{d}{dx}(\sqrt{e^{3x}-2}) = \frac{1}{2} \times 3e^{3x}(e^{3x}-2)^{-\frac{1}{2}}$	B1	1.1b
	$(f'(x) =) \frac{(e^{3x}-2)^{\frac{1}{2}}(7xe^x + 7e^x) - \frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times 7xe^x}{e^{3x}-2}$	dM1	2.1
	$f'(x) = \frac{7e^x(e^{3x}(2-x) - 4x - 4)}{2(e^{3x}-2)^{\frac{3}{2}}}$	A1	1.1b
	(5)		
(b)	$e^{3x}(2-x) - 4x - 4 = 0 \Rightarrow x(\dots e^{3x} \pm \dots) = \dots e^{3x} \pm \dots$	M1	1.1b
	$\Rightarrow x = \frac{2e^{3x}-4}{e^{3x}+4} *$	A1*	2.1
	(2)		
(c)	Draws a vertical line $x=1$ up to the curve then across to the line $y=x$ then up to the curve finishing at the root (need to see a minimum of 2 vertical and horizontal lines tending to the root)	B1	2.1
	(1)		
(d)(i)	$x_2 = \frac{2e^3-4}{e^3+4} = 1.5017756\dots$	M1	1.1b
	$x_2 = \text{awrt } 1.502$	A1	1.1b
(ii)	$\beta = 1.968$	dB1	2.2b
	(3)		
(e)	$h(x) = \frac{2e^{3x}-4}{e^{3x}+4} - x$ $h(0.4315) = -0.000297\dots \quad h(0.4325) = 0.000947\dots$	M1	3.1a
	Both calculations correct and e.g. states: - There is a change of sign - e.g $f'(x)$ is continuous $\alpha = 0.432$ (to 3dp)	A1cao	2.4
	(2)		
	(13 marks)		

Notes

(a)

M1: Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $\dots xe^x \pm \dots e^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.

A1: $k(xe^x + e^x)$ (e.g. $7(xe^x + e^x)$) or equivalent which may be unsimplified (may be implied by further work)

B1: $\left(\frac{d}{dx}(\sqrt{e^{3x}-2})\right) = \frac{1}{2} \times 3e^{3x}(e^{3x}-2)^{-\frac{1}{2}}$ (simplified or unsimplified)

dM1: Attempts to use the quotient rule. It is dependent on the previous method mark.

Score for achieving an expression of the form

$$(f'(x) =) \frac{(e^{3x}-2)^{\frac{1}{2}}(7xe^x + 7e^x) - \frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{1}{2}} \times 7xe^x}{e^{3x}-2} \text{ or equivalent (do not be}$$

concerned by the constants for their "7" or their " $\frac{3}{2}$ " which may be both 1)

If it is clear that the quotient rule has been applied the wrong way round then score M0.

Alternatively, applies the product rule. Score for achieving an expression of the form

$$(f'(x) =) (e^{3x}-2)^{\frac{1}{2}}(7xe^x + 7e^x) - \frac{3}{2}e^{3x}(e^{3x}-2)^{-\frac{3}{2}} \times 7xe^x \text{ or equivalent (do not be}$$

concerned by the constants for their "7" or their " $\frac{3}{2}$ " which may be both 1)

Do not condone invisible brackets.

A1: $(f'(x) =) \frac{7e^x(e^{3x}(2-x) - 4x - 4)}{2(e^{3x}-2)^{\frac{3}{2}}}$ following a fully correct differentiated expression.

You may need to check to see if (a) is continued after other parts for evidence of this.

Condone the lack of $f'(x) =$ on the left hand side or allow the use of $\frac{dy}{dx}$ or y' instead.

Alternative (a) attempt using the triple product rule

$$\begin{aligned} \text{e.g. } \frac{d}{dx} \left(7xe^x (e^{3x} - 2)^{-\frac{1}{2}} \right) &= 7e^x (e^{3x} - 2)^{-\frac{1}{2}} + 7xe^x (e^{3x} - 2)^{-\frac{1}{2}} + 7xe^x \times \left(-\frac{1}{2} \right) \times 3e^{3x} (e^{3x} - 2)^{-\frac{3}{2}} \\ &\Rightarrow \frac{(7e^x + 7xe^x)(e^{3x} - 2) + 7xe^x \times \left(-\frac{1}{2} \right) \times 3e^{3x}}{(e^{3x} - 2)^{\frac{3}{2}}} = \frac{7e^x \left(e^{3x} - 2 + xe^{3x} - 2x - \frac{3}{2}xe^{3x} \right)}{(e^{3x} - 2)^{\frac{3}{2}}} \Rightarrow \frac{7e^x (e^{3x}(2-x) - 4x - 4)}{2(e^{3x} - 2)^{\frac{3}{2}}} \end{aligned}$$

M1: Attempts the product rule on $xe^x \rightarrow \dots xe^x \pm \dots e^x$ which may be seen within the expression $\dots e^x (e^{3x} - 2)^{-\frac{1}{2}} \pm \dots xe^x (e^{3x} - 2)^{-\frac{1}{2}} + \dots$ simplified or unsimplified.

A1: $k(xe^x + e^x)$ which may be seen within the expression $k \left(e^x (e^{3x} - 2)^{-\frac{1}{2}} + xe^x (e^{3x} - 2)^{-\frac{1}{2}} \right) + \dots$ simplified or unsimplified.

B1: $\left(-\frac{1}{2} \right) \times 3e^{3x} (e^{3x} - 2)^{-\frac{3}{2}}$ which may be seen within the expression $\dots + k \left(xe^x \times \left(-\frac{1}{2} \right) \times 3e^{3x} (e^{3x} - 2)^{-\frac{3}{2}} \right)$ simplified or unsimplified.

dM1: A complete method using all three products (which may appear all on one line). Do not condone invisible brackets.

A1: As above in main scheme notes.

(b) **Note that if they do not have values $A = -4$, $B = -4$ in (a) (which may be seen later) then maximum score is M1A0***

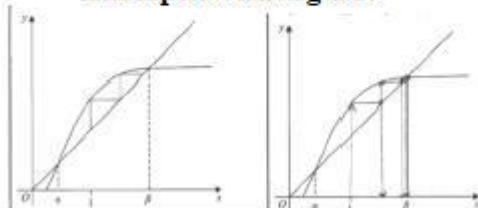
M1: Sets their $e^{3x}(2-x) - 4x - 4$ equal to zero, collects terms in x on one side of the equation and non x terms on the other and attempts to factorise the side with x as a common factor. Condone sign slips only for this mark. Allow A and B to be used instead of " -4 " and " -4 ".

A1*: Achieves the given answer with no errors including invisible brackets. If they do not reach the printed answer then it is A0. If they subsequently write $x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$ then isw

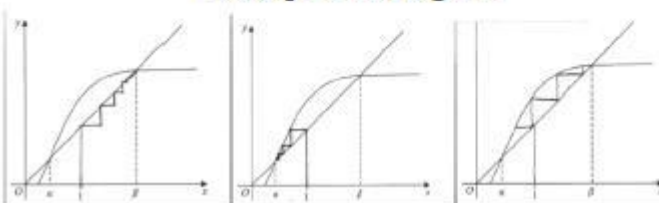
(c)

B1: Starting at $x_1 = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes) tending to β . Condone a lack of arrows on the lines but the sequence of lines should finish at the point of intersection where the root is. Condone the initial vertical line not starting from the x -axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of $x = 1$ this is B0. If they use both diagrams and do not indicate which one they want marking, then the "copy of Diagram 1" should be marked.

Examples scoring B1:



Examples scoring B0:



(d)(i)

M1: Substitutes 1 into the iterative formula. The values embedded in the formula is sufficient for this mark. May be implied by awrt 1.50

A1: awrt 1.502 isw

(d)(ii)

dB1: 1.968 cao (which can only be scored if M1 is scored in (d)(i))

SC: If (d)(i) is rounded to 1.50 then allow 1.97 in (d)(ii) to score M1A0dB1 for (d)

(e)

M1: Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root 0.4317388728...

If no function is stated then may be implied by their answers to e.g. $f'(0.4315)$, $f'(0.4325)$

You will need to check their calculation is correct.

Other possible functions include:

- $h(x) = x - \frac{2e^{3x} - 4}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974...$, $h(0.4325) = -0.0009479...$

- their $f'(x) = \pm \left(\frac{7e^x (e^{3x}(2-x) - 4x - 4)}{2(e^{3x} - 2)^{\frac{3}{2}}} \right)$

(If correct A and B then $f'(0.4315) = \mp 0.005789...$, $f'(0.4325) = \pm 0.01831...$)

- their $g(x) = \pm (e^{3x}(2-x) - 4x - 4)$

(If correct A and B then $g(0.4315) = \mp 0.002275...$, $g(0.4325) = \pm 0.007261...$)

A1: Requires

- Both calculations correct (rounded or truncated to 1sf)
- A statement that there is a change in sign and that their **function** is continuous (must refer to the function used for the substitution (which is not $f(x)$)
Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g.
 $f'(0.4315) \times f'(0.4325) < 0$, "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"
- A minimal conclusion e.g. "hence $\alpha = 0.432$ ", "so rounds to 0.432". Do not allow "hence root"

Q5.

Question	Scheme	Marks	AOs
(a)	$f'(x) = 2x + \frac{4x - 4}{2x^2 - 4x + 5}$	M1 A1	1.1b 1.1b
	$2x + \frac{4x - 4}{2x^2 - 4x + 5} = 0 \Rightarrow 2x(2x^2 - 4x + 5) + 4x - 4 = 0$	dM1	1.1b
	$2x^3 - 4x^2 + 7x - 2 = 0^*$	A1*	2.1
		(4)	
(b)	(i) $x_2 = \frac{1}{7}(2 + 4(0.3)^2 - 2(0.3)^3)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$h(x) = 2x^3 - 4x^2 + 7x - 2$ $h(0.3415) = 0.00366... \quad h(0.3405) = -0.00130...$	M1	3.1a
	States: - there is a change of sign $f'(x)$ is continuous $\alpha = 0.341$ to 3dp	A1	2.4
		(2)	
	(9 marks)		
Notes			

(a)

M1: Differentiates $\ln(2x^2 - 4x + 5)$ to obtain $\frac{g(x)}{2x^2 - 4x + 5}$ where $g(x)$ could be 1

A1: For $f'(x) = 2x + \frac{4x - 4}{2x^2 - 4x + 5}$

dM1: Sets their $f'(x) = ax + \frac{g(x)}{2x^2 - 4x + 5} = 0$ and uses "correct" algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax(2x^2 - 4x + 5) \pm g(x) = 0$ o.e., condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3 - 4x^2 + 7x - 2 = 0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1 = 0.3$. If no method is shown award for $x_2 = \text{awrt } 0.33$

A1: $x_2 = \text{awrt } 0.3294$ Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: $x_4 = \text{awrt } 0.3398$ Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute $x = 0.3415$ and $x = 0.3405$ into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3 - 4x^2 + 7x - 2$, $x - \frac{1}{7}(4x^2 - 2x^3 + 2)$ and $f'(x)$ as this has been

found in part (a) with $f'(0.3405) = -0.00067\dots$, $f'(0.3415) = (+) 0.0018$

There must be sufficient evidence for the function, which would be for example, a statement such as $h(x) = 2x^3 - 4x^2 + 7x - 2$ or sight of embedded values that imply the function, not just a value or values

even if both are correct. Condone $h(x)$ being mislabelled as f

$h(0.3415) = 2 \times 0.3415^3 - 4 \times 0.3415^2 + 7 \times 0.3415 - 2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. $\checkmark$, proven, $\alpha = 0.341$, root

Q6.

Question	Scheme	Marks	AOs
(a)	25	B1	3.4
		(1)	
(b)	Attempts to differentiate using the product rule $\frac{dv}{dt} = \ln(t+1) \times -0.4 + \frac{(10-0.4t)}{t+1}$	M1 A1	3.1b 1.1b
	Sets their $\frac{dv}{dt} = 0 \Rightarrow \frac{(10-0.4t)}{(t+1)} = 0.4 \ln(t+1)$ and then makes progress towards making "t" the subject (See notes for this)	dM1	1.1b
	$t = \frac{25 - \ln(t+1)}{1 + \ln(t+1)}$ $t = \frac{26}{1 + \ln(t+1)} - 1 \quad *$	A1*	2.1
		(4)	
(c)	(i) Attempts $t_2 = \frac{26}{1 + \ln 8} - 1$	M1	1.1b
	awrt 7.298	A1	1.1b
	(ii) awrt 7.33 seconds	A1	3.2a
		(3)	
(8 marks)			
Notes:			

(a)

B1: 25 but condone 25 seconds. If another value is given (apart from 0) it is B0

(b)

M1: Attempts to use the product rule in an attempt to differentiate $v = (10 - 0.4t) \ln(t+1)$

Look for $(10 - 0.4t) \times \frac{1}{(t+1)} \pm k \ln(t+1)$, where k is a constant, condoning slips.

If you see direct evidence of an incorrect rule used e.g. $vu' - uv'$ it is M0

You will see attempts from $v = 10 \ln(t+1) - 0.4t \ln(t+1)$ which can be similarly marked.

In this case look for $\frac{a}{t+1} \pm \frac{bt}{t+1} \pm c \ln(t+1)$

A1: Correct differentiation. Condone a missing left hand or it seen as v' , $\frac{dy}{dx}$ or even $= 0$

$\left(\frac{dv}{dt}\right) = \ln(t+1) \times -0.4 + \frac{(10-0.4t)}{t+1}$ or equivalent such as $\left(\frac{dv}{dt}\right) = \frac{10}{t+1} - \frac{0.4t}{(t+1)} - 0.4 \ln(t+1)$

dM1: Score for setting their $dV/dt = 0$ (which must be in an appropriate form) and proceeding to an equation where the variable t occurs only once – ignoring $\ln(t+1)$.

See two examples of how this can be achieved below. It is dependent upon the previous M.

Look for the following steps

- An allowable derivative set (or implied) = 0 E.g. $\ln(t+1) \times 0.4 = \frac{(10-0.4t)}{t+1}$
- Cross multiplication (or division) and rearrangement to form an equation where the variable t only occurs once.

E.g.1.

$$\begin{aligned}\ln(t+1) \times 0.4 &= \frac{(10-0.4t)}{t+1} \\ \Rightarrow \ln(t+1) &= \frac{25-t}{t+1} \\ \Rightarrow \ln(t+1) &= -1 + \frac{26}{t+1}\end{aligned}$$

E.g.2

$$\begin{aligned}\ln(t+1) \times 0.4 &= \frac{(10-0.4t)}{t+1} \\ \Rightarrow 0.4t \ln(t+1) + 0.4 \ln(t+1) &= 10 - 0.4t \\ \Rightarrow 0.4t(1 + \ln(t+1)) &= 10 - 0.4 \ln(t+1)\end{aligned}$$

A1*: Correctly proceeds to the given answer of $t = \frac{26}{1 + \ln(t+1)} - 1$ showing all key steps.

The key steps must include

- use of $\frac{dv}{dt}$ or v' which must be correct
- a correct line preceding the given answer, usually $t = \frac{25 - \ln(t+1)}{1 + \ln(t+1)}$ or $\frac{26}{t+1} - 1 = \ln(t+1)$

(c) (i)

M1: Attempts to use the iteration formula at least once.

Usually to find $t_2 = \frac{26}{1 + \ln 8} - 1$ which may be implied by awrt 7.44

A1: awrt 7.298. This alone will score both marks as iteration is implied. ISW after sight of this value. As t_3 is the only value that rounds to 7.298 just score the rhs, it does not need to be labelled t_3

(c)(ii)

A1: Uses repeated iteration until value established as awrt 7.33 seconds. Allow awrt 7.33 s

Requires units. It also requires some evidence of iteration which will be usually be awarded from the award of the M

Q7.

Question	Scheme	Marks	AOs
	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\{f(x) = 2x^3 + x^2 - 1 \Rightarrow f'(x) = 6x^2 + 2x\}$	B1	1.1b
	$\left\{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow \left\{x_{n+1}\right\} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}\right.$	M1	1.1b
	$= \frac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n} *$	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow x_2 = \frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)} \text{ or } x_2 = 1 - \frac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}\}$	M1	1.1b
	$\Rightarrow x_2 = \frac{3}{4}, x_3 = \frac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson method cannot be used with $x_1 = 0$ which refer or <i>allude</i> to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
(6 marks)			

Notes for Question	
(a)	
B1:	States that $f'(x) = 6x^2 + 2x$ or states that $f'(x_n) = 6x_n^2 + 2x_n$ (Condone $\frac{dy}{dx} = 6x^2 + 2x$)
M1:	Substitutes $f(x_n) = 2x_n^3 + x_n^2 - 1$ and their $f'(x_n)$ into $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $f'(x) = 6x^2 + 2x$ is applied as $f'(x_n)$ (or $f'(x)$) in the NR formula $\{x_{n+1}\} = x_n - \frac{f(x_n)}{f'(x_n)}$
Note:	Allow M1A1 for $\bullet \quad x_{n+1} = x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x} = \frac{x(6x^2 + 2x) - (2x^3 + x^2 - 1)}{6x^2 + 2x} \Rightarrow x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$
Note	Condone $x = x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x}$ for M1
Note	Condone $x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$ or $x - \frac{2x^3 + x^2 - 1}{6x^2 + 2x}$ (i.e. no $x_{n+1} = \dots$) for M1
Note:	Give M0 for $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ followed by $x_{n+1} = 2x_n^3 + x_n^2 - 1 - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*
(b)	
M1:	An attempt to use the given or their formula once. Can be implied by $\frac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or 0.75 o.e.
Note:	Allow one slip in substituting $x_1 = 1$
A1:	$x_2 = \frac{3}{4}$ and $x_3 = \frac{2}{3}$
Note:	Condone $x_2 = \frac{3}{4}$ and $x_3 = \text{awrt } 0.667$ for A1
Note:	Condone $\frac{3}{4}, \frac{2}{3}$ listed in a correct order ignoring subscripts
(c)	
B1:	See scheme
Note:	Give B0 for the following isolated reasons: e.g. - You cannot divide by 0 - The fraction (or the NR formula) is undefined at $x = 0$ - At $x = 0$, $f'(x_1) = 0$ x_1 cannot be 0 $6x^2 + 2x$ cannot be 0 - the denominator is 0 which cannot happen - if $x_1 = 0$, $6x^2 + 2x = 0$

Q8.

Question	Scheme	Marks	AOs
(a)	$(f'(x) =) 4\cos\left(\frac{1}{2}x\right) - 3$	M1 A1	1.1b 1.1b
	Sets $f'(x) = 4\cos\left(\frac{1}{2}x\right) - 3 = 0 \Rightarrow x =$	dM1	3.1a
	$x = 14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $f(4) > 0$, $f(5) < 0$ and the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1 = 5 - \frac{8\sin 2.5 - 15 + 9}{"4\cos 2.5 - 3"}$ (NB $f(5) = -1.212\dots$ and $f'(5) = -6.204\dots$)	M1	1.1b
	$x_1 = \text{awrt } 4.80$	A1	1.1b
		(2)	
(7 marks)			
Notes:			

(a)

M1: Differentiates to obtain $k \cos\left(\frac{1}{2}x\right) \pm \alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $f'(x) = 4 \cos\left(\frac{1}{2}x\right) - 3$. Allow unsimplified e.g. $f'(x) = \frac{1}{2} \times 8 \cos\left(\frac{1}{2}x\right) - 3x^0$

There is no need for $f'(x) = \dots$ or $\frac{dy}{dx} = \dots$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for x .

Look for

- $f'(x) = a \cos\left(\frac{1}{2}x\right) + b = 0, a, b \neq 0$
- Correct method of finding a valid solution to $a \cos\left(\frac{1}{2}x\right) + b = 0$

Allow for $a \cos\left(\frac{1}{2}x\right) + b = 0 \Rightarrow \cos\left(\frac{1}{2}x\right) = \pm k \Rightarrow x = 2 \cos^{-1}(\pm k)$ where $|k| < 1$

If this working is not shown then you may need to check their value(s).

For example $4 \cos\left(\frac{1}{2}x\right) - 3 = 0 \Rightarrow x = 1.4\dots$ or $11.1\dots$ (or $82.8\dots$ or $637\dots$ or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x = 14.0$ and not just 14 or unrounded e.g. $14.011\dots$

Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the y coordinate.

Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)

Accept equivalent statements for $f(4) > 0, f(5) < 0$ e.g. $f(4) \times f(5) < 0$, "there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"

(c)

M1: Attempts $x_1 = 5 - \frac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a “changed” function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees $f(5) = -5.65\dots$ and $f'(5) = 0.996\dots$ and gives $x_1 = 10.67\dots$

There must be clear evidence that $5 - \frac{f(5)}{f'(5)}$ is being attempted.

$$\text{so e.g. } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80 \text{ scores M0 as does e.g. } x_1 = x - \frac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 =$ awrt 4.80 not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$$5 - \frac{f(5)}{f'(5)} = \text{awrt } 4.80 \text{ following a correct derivative scores M1A1}$$

$$5 - \frac{f(5)}{f'(5)} \neq \text{awrt } 4.80 \text{ with no evidence that } 5 - \frac{f(5)}{f'(5)} \text{ was attempted scores M0}$$

Q9.

Question	Scheme	Marks	AOs
(a)	Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x) - x)$	M1	2.1
	$f(3) = (2\ln(5) - x) = (+)0.22$ and $f(4) = (2\ln(4) - 4) = -1.23$ <u>Change of sign</u> and function <u>continuous</u> in interval $[3, 4] \Rightarrow$ <u>Root</u> *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1 = 4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula can be used to find an approximation for α because the cobweb spirals inwards for the cobweb diagram	A1	2.2a
		(2)	
(4 marks)			

Notes:

(a)

M1: Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x) - x)$ or alternatively compares $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

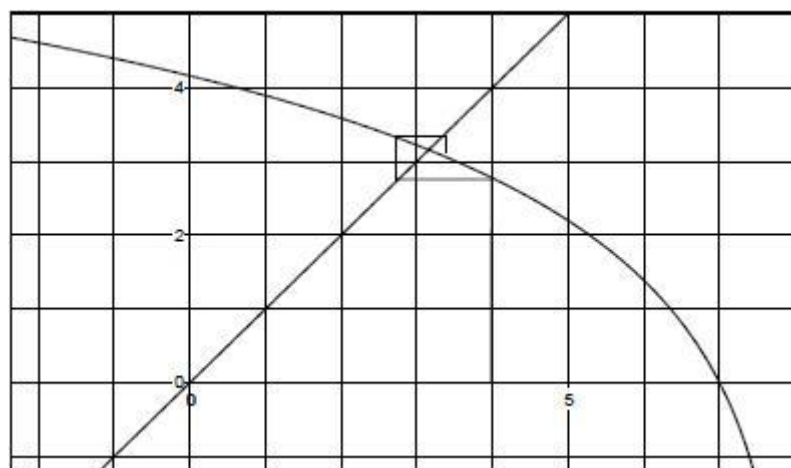
A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)

When comparing terms, allow reasons to be $2\ln 8 = 3.21 > 3$, $2\ln 4 = 2.77 < 4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be "it spirals inwards", "it gets closer to the root", "it converges"



Q10.

Question Number	Scheme	Marks
(a)	$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe. Puts $f'(x) = 0$ to give $x = -1$ and $x = 0$ or one coordinate Obtains $(0, -16)$ and $(-1, 25e^{-2} - 16)$	M1A1 dM1A1 CSO A1 (5)
(b)	Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm \frac{4}{5}e^{-x}$	B1* (1)
(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt } 0.485$ $\Rightarrow x_2 = \text{awrt } 0.492, x_3 = \text{awrt } 0.489$	M1A1 A1 (3)
(d)	$\alpha = 0.49$ $f(0.485) = -0.487, f(0.495) = (+)0.485$, sign change and deduction	B1 B1 (2)
(11 marks)		

Notes for Question

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu' + uv'$. If the rule is quoted it must be correct.

It can be implied by their $u = \dots, v = \dots, u' = \dots, v' = \dots$ followed by their $vu' + uv'$

If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x} + Bxe^{2x}$

A1 $f'(x) = 50x^2e^{2x} + 50xe^{2x}$.

Allow un simplified forms such as $f'(x) = 25x^2 \times 2e^{2x} + 50x \times e^{2x}$

dM1 Sets $f'(x) = 0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x

This is dependent upon the first M1 being scored.

A1 Both $x = -1$ and $x = 0$ or one complete coordinate. Accept $(0, -16)$ and $(-1, 25e^{-2} - 16)$ or $(-1, \text{awrt } -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.

$x = -1, 0$ $y = \frac{25}{e^2} - 16, -16$ or linked together by coordinate pairs $(0, -16)$ and $(-1, 25e^{-2} - 16)$ but the 'pairs' must be correct and exact.

Notes for Question Continued

(b)

B1 This is a show that question and all elements must be seen

Candidates must 1) State that $f(x)=0$ or writes $25x^2e^{2x} - 16 = 0$ or $25x^2e^{2x} = 16$

2) Show at least one intermediate (correct) line with either

$$x^2 \text{ or } x \text{ the subject. Eg } x^2 = \frac{16}{25}e^{-2x}, \quad x = \sqrt{\frac{16}{25}e^{-2x}} \text{ oe}$$

$$\text{or square rooting } 25x^2e^{2x} = 16 \Rightarrow 5xe^x = \pm 4$$

$$\text{or factorising by DOTS to give } (5xe^x + 4)(5xe^x - 4) = 0$$

$$3) \text{ Show the given answer } x = \pm \frac{4}{5}e^{-x}.$$

Condone the minus sign just appearing on the final line.

A 'reverse' proof is acceptable as long as there is a statement that $f(x)=0$

(c)

M1 Substitutes $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \dots$

This can be implied by $x_1 = \frac{4}{5}e^{-0.5}$, or awrt 0.49

A1 $x_1 = \text{awrt } 0.485 \text{ 3dp. Mark as the first value given. Don't be concerned by the subscript.}$

A1 $x_2 = \text{awrt } 0.492, x_3 = \text{awrt } 0.489 \text{ 3dp. Mark as the second and third values given.}$

(d)

B1 States $\alpha = 0.49$

B1 Justifies by

either calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp,

$$f(0.485) = -0.5, f(0.495) = (+)0.5 \text{ rounded}$$

$$f(0.485) = -0.4, f(0.495) = (+)0.4 \text{ truncated}$$

giving a reason – accept change of sign, $>0 <0$ or $f(0.485) \times f(0.495) < 0$

and giving a minimal conclusion. Eg. Accept hence root or $\alpha = 0.49$

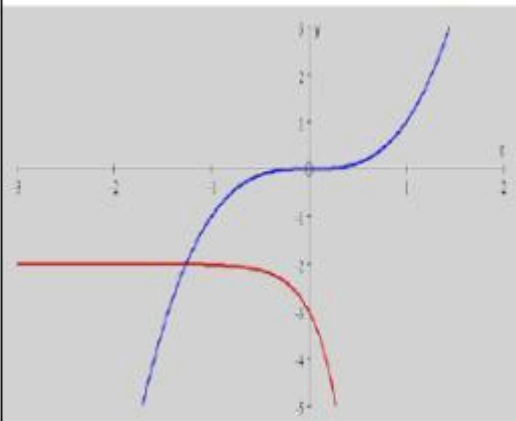
A smaller interval containing the root may be used, eg $f(0.49)$ and

$f(0.495)$. Root = 0.49007

or by stating that the iteration is oscillating

or by calculating by continued iteration to at least the value of $x_4 = \text{awrt } 0.491$ and stating (or seeing each value round to) 0.49

Q11.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$ Puts $\frac{dy}{dx} = 0$ to give $x^3 = -2 - e^{4x}$	M1, A1 A1 * (3)
(b)	 $y = x^3$ Shape of $y = -2 - e^{4x}$ $y = -2 - e^{4x}$ cuts y axis at (0,-3) $y = -2 - e^{4x}$ has asymptote at $y = -2$	B1 B1 B1 B1 (4)
(c)	Only one crossing point	B1 (1)
(d)	-1.26376, -1.26126 Accept answers which round to these answers to 5dp	M1 A1 (2)
(e)	$\alpha = -1.26$ and so turning point is at (-1.26, -2.55)	M1 A1cao (2)
		12 marks

(a)

M1 Two (of the four) terms differentiated correctly

A1 All correct $\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$

A1* States or sets $\frac{dy}{dx} = 0$, and proceeds correctly to achieve printed answer $x^3 = -2 - e^{4x}$.

(b)

B1 Correct shape and position for $y = x^3$. It must appear to go through the origin.

It must only appear in Quadrants 1 and 3 and have a gradient that is always ≥ 0 . The gradient should appear large at either end. Tolerate slips of the pen. See practice and qualification for acceptable curves.

B1 Correct shape for $y = -2 - e^{4x}$, its position is not important for this mark. The gradient must be approximately zero at the left hand end and increase negatively as you move from left to right along the curve. See practice and qualification for acceptable curves.

B1 Score for $y = -2 - e^{4x}$ cutting or meeting the y axis at $(0, -3)$. Its shape is not important.

Accept for the intention of $(0, -3)$, -3 being marked on the y - axis as well as $(-3, 0)$

Do not accept 3 being marked on the negative y axis.

B1 Score for $y = -2 - e^{4x}$ having an asymptote stated as $y = -2$. This is dependent upon the curve appearing to have an asymptote there. Do not accept the asymptote marked as '-2' or indeed $x = -2$. See practice and qualification for acceptable solutions.

(c)

B1 Score for a statement to the effect that the graphs cross at one point. Accept minimal statements such as 'one intersection'. Do not award if their diagram shows more than one intersection. They must have a diagram (which may be incorrect)

(d)

M1 Awarded for applying the iteration formula once. Possible ways in which this can be scored are the sight

of $\sqrt[3]{-2 - e^{-4}}$, $(-2 - e^{4 \times -1})^{\frac{1}{3}}$ or awrt -1.264

A1 Both values correct awrt -1.26376, -1.26126 5dps. The subscripts are unimportant for this mark. Score as the first and second values seen.

(e)

M1 Score for EITHER rounding their value in part (c) to 2 dp OR finding turning point of C by substituting a value of x generated from part (d) into $y = e^{4x} + x^4 + 8x + 5$ in order to find the y value. You may accept the appearance of a y value as evidence of finding the turning point (as long as an x value appears to be generated from part (d) and the correct equation is used.)

A1 $(-1.26, -2.55)$ and correct solution only. It is a deduction and you cannot accept the appearance of a correct answer for two marks.

Q12.

Question Number	Scheme	Marks
(a)	$2^{x+1} - 3 = 17 - x \Rightarrow 2^{x+1} = 20 - x$ $(x+1) \ln 2 = \ln(20 - x) \Rightarrow x = \dots$ $x = \frac{\ln(20 - x)}{\ln 2} - 1$	M1 dM1 A1* (3)
(b)	Sub $x_0 = 3$ into $x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \Rightarrow x_1 = 3.087$ (awrt) $x_2 = 3.080, x_3 = 3.081$ (awrt)	M1A1 A1 (3)
(c)	$A = (3.1, 13.9)$ cao	M1,A1 (2) (8 marks)
(a)Alt	$2^{x+1} - 3 = 17 - x \Rightarrow 2^x = \frac{20 - x}{2}$ $x \ln 2 = \ln \frac{20 - x}{2} \Rightarrow x = \dots$ $x = \frac{\ln(20 - x)}{\ln 2} - 1$	M1 dM1 A1* (3)
(a) backwards	$x = \frac{\ln(20 - x)}{\ln 2} - 1 \Rightarrow (x+1) \ln 2 = \ln(20 - x)$ $\Rightarrow 2^{x+1} = 20 - x$ Hence $y = 2^{x+1} - 3$ meets $y = 17 - x$	M1 dM1 A1* (3)

(a)

M1 Setting equations in x equal to each other and proceeding to make 2^{x+1} the subject

dM1 Take $\ln$'s or logs of both sides, use the power law and proceed to $x = \dots$

A1* This is a given answer and all aspects must be correct including $\ln$ or $\log_e$ rather than $\log_{10}$

Bracketing on both $(x+1)$ and $\ln(20-x)$ must be correct.

$$\text{Eg } x+1 \ln 2 = \ln(20-x) \Rightarrow x = \frac{\ln(20-x)}{\ln 2} - 1 \text{ is A0*}$$

Special case: Students who start from the point $2^{x+1} = 20-x$ can score M1 dM1A0*

(b)

M1 Sub $x_0 = 3$ into $x_{n+1} = \frac{\ln(20-x_n)}{\ln 2} - 1$ to find $x_1 = \dots$

Accept as evidence $x_1 = \frac{\ln(20-3)}{\ln 2} - 1$, awrt $x_1 = 3.1$

Allow $x_0 = 3$ into the miscopied iterative equation $x_1 = \frac{\ln(20-3)}{\ln 2}$ to find $x_1 = \dots$

Note that the answer to this, 4.087, on its own without sight of $\frac{\ln(20-3)}{\ln 2}$ is M0

A1 awrt 3 dp $x_1 = 3.087$

A1 awrt $x_2 = 3.080$, $x_3 = 3.081$. Tolerate 3.08 for 3.080

Note that the subscripts are not important, just mark in the order seen

(c) Note that this appears as B1B1 on e pen. It is marked M1A1

M1 For sight of 3.1

Alternatively it can be scored for substituting their value of x or a rounded value of x from (b) into either $2^{x+1} - 3$ or $17 - x$ to find the y coordinate.

A1 (3.1, 13.9)

Q13.

Question	Scheme	Marks
(a)	(i) 21 (ii) $4e^{2x} - 25 = 0 \Rightarrow e^{2x} = \frac{25}{4} \Rightarrow 2x = \ln\left(\frac{25}{4}\right) \Rightarrow x = \frac{1}{2}\ln\left(\frac{25}{4}\right) \Rightarrow x = \ln\left(\frac{5}{2}\right)$ (iii) 25	B1 M1A1, A1 B1 (5)
(b)	$4e^{2x} - 25 = 2x + 43 \Rightarrow e^{2x} = \frac{1}{2}x + 17$ $\Rightarrow 2x = \ln\left(\frac{1}{2}x + 17\right) \Rightarrow x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$	M1 A1* (2)
(c)	$x_1 = \frac{1}{2}\ln\left(\frac{1}{2} \times 1.4 + 17\right) = \text{awrt } 1.44$ awrt $x_1 = 1.4368, x_2 = 1.4373$	M1 A1 (2)
(d)	Defines a suitable interval 1.4365 and 1.4375 ...and substitutes into a suitable function Eg $4e^{2x} - 2x - 68$, obtains correct values with both a reason and conclusion	M1 A1 (2) (11 marks)

In part (a) accept points marked on the graph. If they appear on the graph and in the text, the text takes precedence. If they don't mark (a) as (i) (ii) and (iii) mark in the order given. If you feel unsure then please use the review system and your team leader will advise.

(a) (i)

B1 Sight of 21. Accept (0, 21)

Do not accept just $|4 - 25|$ or (21, 0)

(a) (ii)

M1 Sets $4e^{2x} - 25 = 0$ and proceeds via $e^{2x} = \frac{25}{4}$ or $e^x = \frac{5}{2}$ to $x = ..$

Alternatively sets $4e^{2x} - 25 = 0$ and proceeds via $(2e^x - 5)(2e^x + 5) = 0$ to $e^x = ..$

A1 $\frac{1}{2}\ln\left(\frac{25}{4}\right)$ or awrt 0.92

A1 cao $\ln\left(\frac{5}{2}\right)$ or $\ln 5 - \ln 2$. Accept $\left(\ln\left(\frac{5}{2}\right), 0\right)$

(a) (iii)

B1 $k = 25$ Accept also 25 or $y = 25$

Do not accept just $|-25|$ or $x = 25$ or $y = \pm 25$

(b)

M1 Sets $4e^{2x} - 25 = 2x + 43$ and makes e^{2x} the subject. Look for $e^{2x} = \frac{1}{4}(2x + 43 + 25)$ condoning sign slips. Condone $|4e^{2x} - 25| = 2x + 43$ and makes $|e^{2x}|$ the subject. Condone for both marks a solution with $x = a / \alpha$

An acceptable alternative is to proceed to $2e^{2x} = x + 34 \Rightarrow \ln 2 + 2x = \ln(x + 34)$ using $\ln$ laws

A1* Proceeds correctly without errors to the correct solution. This is a given answer and the bracketing must be correct throughout. The solution must have come from $4e^{2x} - 25 = 2x + 43$ with the modulus having been taken correctly.

Allow $e^{2x} = \frac{1}{4}(2x + 43 + 25)$ going to $x = \frac{1}{2} \ln\left(\frac{1}{2}x + 17\right)$ without explanation

Allow $\frac{1}{2} \ln\left(\frac{1}{2}x + 17\right)$ appearing as $\frac{1}{2} \log_e\left(\frac{1}{2}x + 17\right)$ but not as $\frac{1}{2} \log\left(\frac{1}{2}x + 17\right)$

If a candidate attempts the solution backwards they must proceed from

$$x = \frac{1}{2} \ln\left(\frac{1}{2}x + 17\right) \Rightarrow e^{2x} = \frac{1}{2}x + 17 \Rightarrow 4e^{2x} - 25 = 2x + 43 \text{ for the M1}$$

For the A1 it must be tied up with a minimal statement that this is $g(x) = 2x + 43$

(c)

M1 Subs 1.4 into the iterative formula in an attempt to find x_1

$$\text{Score for } x_1 = \frac{1}{2} \ln\left(\frac{1}{2} \times 1.4 + 17\right) \quad x_1 = \frac{1}{2} \ln(17.7) \text{ or awrt } 1.44$$

A1 awrt $x_1 = 1.4368$, $x_2 = 1.4373$ Subscripts are not important, mark in the order given please.

(d)

M1 For a suitable interval. Accept 1.4365 and 1.4375 (or any two values of a smaller range spanning the root=1.4373) Continued iteration is M0

A1 Substitutes both values into a suitable function, which must be defined or implied by their working calculates both values correctly to 1 sig fig (rounded or truncated)

$$\text{Suitable functions could be } \pm(4e^{2x} - 2x - 68), \pm\left(x - \frac{1}{2} \ln\left(\frac{1}{2}x + 17\right)\right), \pm\left(2x - \ln\left(\frac{1}{2}x + 17\right)\right).$$

$$\text{Using } 4e^{2x} - 2x - 68 \quad f(1.4365) = -0.1, f(1.4375) = +0.02 \text{ or } +0.03$$

$$\text{Using } 2e^{2x} - x - 34 \quad f(1.4365) = -0.05/-0.06, f(1.4375) = +0.01$$

$$\text{Using } x - \frac{1}{2} \ln\left(\frac{1}{2}x + 17\right) \quad f(1.4365) = -0.0007 \text{ or } -0.0008, f(1.4375) = +0.0001 \text{ or } +0.0002$$

$$\text{Using } 2x - \ln\left(\frac{1}{2}x + 17\right) \quad f(1.4365) = -0.001 \text{ or } -0.002, f(1.4375) = +0.0003 \text{ or } +0.0004$$

and states a reason (eg change of sign)

and a gives a minimal conclusion (eg root or tick)

$$\text{It is valid to compare the two functions. Eg } g(1.4365) = 45.7(6) < 2 \times 1.4365 + 43 = 45.8(73)$$

$$g(1.4375) = 45.90 > 2 \times 1.4375 + 43 = 45.8(75)$$

but the conclusion should be $g(x) = 2x + 43$ in between, hence root.

Similarly candidates can compare the functions x and $\frac{1}{2} \ln\left(\frac{1}{2}x + 17\right)$

Q14.

Question Number	Scheme	Notes	Marks
	$f(x) = x^3 - \frac{5}{2x^{\frac{3}{2}}} + 2x - 3$		
(a)	$f(1.1) = -1.6359604,$ $f(1.5) = 2.0141723$	Attempts to evaluate both $f(1.1)$ and $f(1.5)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	Sign change (and $f(x)$ is continuous) therefore a root / α is between $x = 1.1$ and $x = 1.5$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. $-1.63.. < 0 < 2.014..$) and conclusion.	A1
			(2)
(b)	$f(x) = x^3 - \frac{5}{2}x^{-\frac{3}{2}} + 2x - 3$ $\Rightarrow f'(x) = 3x^2 + \frac{15}{4}x^{-\frac{5}{2}} + 2$	M1: $x^n \rightarrow x^{n-1}$ for at least one term	M1A1
		A1: Correct derivative oe	
			(2)
(c)	$f'(1.1) = 3(1.1)^2 + \frac{15}{4}(1.1)^{-\frac{5}{2}} + 2 (= 8.585)$	Attempt to find $f'(1.1)$. Accept $f'(1.1)$ seen and their value.	M1
	$\alpha_2 = 1.1 - \left(\frac{"-1.6359604"}{"8.585"} \right)$	Correct application of N-R	M1
	$\alpha_2 = 1.291$	cao	A1
			(3)
			Total 7

Q15.

Question Number	Scheme	Notes	Marks
	$f(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$		
	$f(x) = x^2 + \frac{3}{4}x^{-\frac{1}{2}} - 3x - 7$		
	$f'(x) = 2x - \frac{3}{8}x^{-\frac{3}{2}} - 3 \{+ 0\}$	M1: $x^n \rightarrow x^{n-1}$ on at least one term A1: Correct differentiation.	M1A1
	$f(4) = -2.625 = -\frac{21}{8} = -2\frac{5}{8}$ or $4^2 + \frac{3}{4\sqrt{4}} - 3 \times 4 - 7$	$f(4) = -2.625$ A correct <u>evaluation</u> of $f(4)$ or a correct <u>numerical expression</u> for $f(4)$. This can be implied by a correct answer below but in all other cases, <u>$f(4)$ must be seen explicitly evaluated</u> or as an <u>expression</u>.	B1
	$f'(4) = 4.953125 = \frac{317}{64} = 4\frac{61}{64}$	Attempt to insert $x = 4$ into their $f'(x)$. Not dependent on the first M but must be what they think is $f'(x)$.	M1
	$\alpha_2 = 4 - \left(\frac{"-2.625"}{"4.953125"} \right)$	Correct application of Newton-Raphson using their values.	M1
	$= 4.529968454... \quad \left(= \frac{1436}{317} = 4\frac{188}{317} \right)$		
	$= 4.53 \text{ (2 dp)}$	4.53 cso	A1 cao
	Note that the kind of errors that are being made in differentiating are sometimes giving 4.53 but the final mark is cso and the final A1 should not be awarded in these cases.		
	Ignore any further iterations		
	A correct derivative followed by $\alpha_2 = 4 - \frac{f(4)}{f'(4)} = 4.53$ can score full marks.		
			[6]
			6 marks

Q16.

Question Number	Scheme	Marks
(a)	$f(1.6) = \dots$ $f(1.7) = \dots$ (Evaluate both) $0.08\dots$ (or 0.09), $-0.3\dots$ One +ve, one -ve or sign change, $\therefore$ root	M1 A1 (2)
(b)	$f'(x) = -4\sin x - e^{-x}$ $1.6 - \frac{f(1.6)}{f'(1.6)}$ $= 1.6 - \frac{4\cos 1.6 + e^{-1.6}}{(-4\sin 1.6 - e^{-1.6})} \quad \left(= 1.6 - \frac{0.085\dots}{-4.2\dots} \right)$	B1 M1 A1 A1 (4)
		(6 marks)

Q17.

Question Number	Scheme	Marks
(a)	At P $x = -2 \Rightarrow y = 3$ $\frac{dy}{dx} = \frac{4}{2x+5} - \frac{3}{2}$ $\left. \frac{dy}{dx} \right _{x=-2} = \frac{5}{2} \Rightarrow \text{Equation of normal is } y - '3' = -\frac{2}{5}(x - (-2))$ $\Rightarrow 2x + 5y = 11$	B1 M1, A1 M1 A1 (5)
(b)	Combines $5y + 2x = 11$ and $y = 2 \ln(2x+5) - \frac{3x}{2}$ to form equation in x $5 \left(2 \ln(2x+5) - \frac{3x}{2} \right) + 2x = 11$ $\Rightarrow x = \frac{20}{11} \ln(2x+5) - 2$	M1 dM1 A1* (3)
(c)	Substitutes $x_1 = 2 \Rightarrow x_2 = \frac{20}{11} \ln 9 - 2$ Awrtr $x_2 = 1.9950$ and $x_3 = 1.9929$.	M1 A1 (2) (10 marks)

- (a)
- B1 $y = 3$ at point P . This may be seen embedded within their equation which may be a tangent
- M1 Differentiates $\ln(2x+5) \rightarrow \frac{A}{2x+5}$ or equivalent. You may see $\ln(2x+5)^2 \rightarrow \frac{A(2x+5)}{(2x+5)^2}$
- A1 $\frac{dy}{dx} = \frac{4}{2x+5} - \frac{3}{2}$ oe. It need not be simplified.
- M1 For using a correct method of finding the equation of the normal using their numerical value of $-\frac{dx}{dy}\bigg|_{x=-2}$ as the gradient. Allow for $(y-3) = -\frac{dx}{dy}\bigg|_{x=-2} (x+2)$, oe.
- At least one bracket must be correct for their $(-2, 3)$
- If the form $y = mx + c$ is used it is scored for proceeding as far as $c = ..$
- A1 $\pm k(5y+2x=11)$ It must be in the form $ax+by=c$ as stated in the question
- Score this mark once it is seen. Do not withhold it if they proceed to another form, $y = mx + c$ for example
- If a candidate uses a graphical calculator to find the gradient they can score a maximum of B1 M0 A0 M1 A1
- (b)
- M1 For combining 'their' **linear** $5y+2x=11$ with $y = 2\ln(2x+5) - \frac{3x}{2}$ to form equation in just x ,
condoning slips on the rearrangement of their $5y+2x=11$. Eg $2\ln(2x+5) - \frac{3x}{2} = \frac{11 \pm 2x}{5}$ is OK
- dM1 Collects the two terms in x and proceeds to $ax = b\ln(2x+5) + c$ Allow numerical slips
- A1* This is a given answer. All aspects must be correct including bracketing
- (c)
- M1 Score for substituting $x_1 = 2 \Rightarrow x_2 = \frac{20}{11} \ln(2 \times 2 + 5) - 2$ or exact equivalent
- This may implied by $x_2 = 1.99...$
- A1 Both values correct. Allow awrt $x_2 = 1.9950$ and $x_3 = 1.9929$ but condone $x_2 = 1.995$
- Ignore subscripts. Mark on the first and second values given.