

Math Teaching Toolkit

San Francisco Unified School District Math Department



Acknowledgements

The SFUSD Math Teaching Toolkit was created in collaboration across departments within the district as well as with educational partners from outside.

SFUSD Multilingual Pathways Department

SFUSD Science Department

SFUSD iSTEM Initiative

SFUSD Educational Technology Department

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California Education Partners

Math in Common

Oakland Unified School District

The SFUSD Math Teaching Toolkit lives online at www.sfusdmath.org/toolkit.html.

This version of the toolkit has not been revised since June 2015.

Introduction to the 2015 Revision

Jim Ryan, Executive Director of STEM, SFUSD

"...teachers are able to sustain change when there are mechanisms in place at multiple levels of the system to support their efforts. This includes the presence of a supportive professional community of colleagues in the school that reinforces changes and provides continuing opportunities to learn."
Cynthia Coburn

As Dr. Coburn describes above, it is through the support of our peers that we sustain change over time. SFUSD is in the midst of a dramatic change in how we teach and understand mathematics. This shift in outlook and practice is one that must be sustained and continually revised over time. The Common Core State Standards for Mathematics (CCSS-M) have served to redefine what we expect students to know and be able to do; that also redefines our work as educators. As a district we learned a great deal in our first year of CCSS-M full implementation. Our hope is that these learnings are reflected in this Teaching Toolkit and how it interacts with the newly revised SFUSD Math Core Curriculum, as well as the associated professional development.

Constant reflection and revision is part of every teacher's life. It was your reflection and feedback that informed the revisions within this toolkit as well as the curriculum. The pedagogical strategies included in this toolkit should not be perceived as either exhaustive or limiting. Rather, these are instructional approaches that lend themselves to the structure and intentionality of the new curriculum and embedded tasks.

We understand that for deep and sustainable change in mathematics, teachers, students, and leaders must continue to grapple with what rich mathematics and instruction look like. It is our goal that teachers and site leaders work collaboratively towards a shared vision of good math instruction that develops *mathematically proficient students* as defined by the CCSS-M. It continues to be our hope that this toolkit provides a common instructional foundation for this collaboration.

Lizzy Hull Barnes, Program Administrator of Mathematics, SFUSD

During the 2014 - 2015 school year, math teachers in San Francisco took the brave step of supporting students to describe their thinking as they grappled with complex mathematical tasks. In our math classrooms, teachers and students together had public conversations about complex mathematical ideas, showing their thinking with both digital platforms and the old-school, tried and true, poster paper. All over our city, students presented their arguments about math and demonstrated that math is much more than simple answer-getting and speed.

Meanwhile, we heard many suggestions about how to improve the curriculum for teachers and students alike. Teams of teachers gathered during the summer to take this classroom feedback and work on the first complete revision of the SFUSD PK - 12 Core Curriculum. Revisions include (but are not limited to):

- Coherence of mathematics within and across units, including flow of instructional models, manipulatives, and materials;
- Lesson plans which clearly name the core mathematics;
- Additional resources such as regular homework and letters to families;
- Usability, including page numbers and more consistent structures;
- Increased access for all students, including guidance for language learners and students with special needs.

This toolkit is designed to be a practical source for strategies and ideas to support your mathematically powerful classroom. While the SFUSD Math Core Curriculum itself—the units built upon rich math tasks and formative assessment—is the “what,” this toolkit represents the “how.” Thank you for your good faith and professionalism as we continue to move together with a sense of what is possible for every student in our San Francisco Schools.

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The Common Core State Standards for Mathematical Practice

What is the SFUSD Math Teaching Toolkit?

When working with the Common Core State Standards for Mathematics, one central question is, “How can we help students keep the focus on personal and group sense-making, understanding, and reasoning as they become confident problem solvers?” In some ways, changing the content standards (what we learn) is the easy part. It is changing the mathematical practices (how we learn) where the challenge lies. The shift needs to be toward helping students to express their mathematical thinking, learn from their mistakes, experiment effectively, and pursue their mathematical interests to a deeper, more personal level.

The SFUSD Mathematics Department has created this Math Teaching Toolkit to support teachers and students as we continue our move away from a more directive style of teaching mathematics and toward a more inquiry-based style. It is designed to be a practical resource of teaching strategies and tools that allow all our students the opportunity to grapple with rich mathematical tasks in a discourse-rich classroom environment.

We firmly believe that a powerful mathematics classroom involves shared sense-making within a community of learners. This is reflected in the Common Core State Standard for Mathematical Practice 1: *Make sense of problems and persevere in solving them*. Making our thinking public allows us to negotiate meaning with each other. This is reflected in the Standard for Mathematical Practice 3: *Construct viable arguments and critique the reasoning of others*. This standard states that “students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.” Student discourse is our window to understanding the mathematical strengths and challenges of the whole class as well as of individual students, thus reflecting Mathematical Practice 6: *Attend to precision*. It presents us a view of what our students know, misconceptions they may have, and how these might have developed. And yet, it is not simply a one-way street. In the process of trying to understand our students’ thinking, we gain perspective into our own mathematical understanding, thereby laying the foundation for a supportive learning community for teachers as well as students.

The SFUSD Math Core Curriculum promotes discourse in the teaching and learning of mathematics. Each unit of study within the Core Curriculum has four rich math tasks as well as lesson series that are premised on group work and meaningful student-to-student interactions. Our role as a teacher is described, in broad terms, as a facilitator who is listening carefully to students, framing appropriate questions, and mediating competing perspectives. However, this is not to be interpreted as “teachers stay out of the way and students will learn.” In fact, a teacher’s role is proactive and includes:

- ❖ Designing lessons that promote student learning through discourse;
- ❖ Beginning the lesson with a carefully presented launch;
- ❖ Guiding the lesson with facilitated group work and class discussion;
- ❖ Summarizing the lesson to bring out students’ insights while mitigating possible misconceptions.

The SFUSD Math Teaching Toolkit continues to include many of the key strategies that have been part of the development of the Math Core Curriculum from the start. Among these are the three Signature Strategies of Math Talks, Three Read Protocol, and Participation Quiz or Groupwork Feedback. There are new additions, such as math notebooks, math technology tools, questioning strategies, and re-engagement strategies.. Finally, this Math Teaching Toolkit refers to many other resources that can be found on the SFUSD Math Department website at www.sfusdmath.org. We hope you continue to find this toolkit a practical and useful resource as you create powerful mathematical learning for your students.

SFUSD Math Department Guiding Principles

All students can and should develop a belief that mathematics is sensible, worthwhile, and doable.

All students are capable of making sense of mathematics in ways that are creative, interactive, and relevant.

All students can and should engage in rigorous mathematics through rich, challenging tasks.

Students' academic success in mathematics must not be predictable on the basis of race, ethnicity, gender, socioeconomic status, language, religion, sexual orientation, cultural affiliation, or special needs.

More at <http://www.sfusdmath.org/our-vision.html>

Math Tasks in the SFUSD Mathematics Curriculum

SFUSD units are designed around four tasks. These tasks offer all students opportunities to engage in meaningful and rigorous mathematics that allow for the development of the Standards for Mathematical Practice. They give information about how students are learning the core concepts and skills of the unit.



All tasks are used for formative assessment—gathering information about what students know and are able to do—but they are not tests. The Entry, Apprentice, and Expert Tasks allow for student collaboration and individual accountability without being used to grade students individually. The Milestone Task can be used as an assessment for grading students individually (see section on using rubrics for letter grades in this Math Teaching Toolkit).

Overarching Principles	<p>Tasks support productive struggle. Tasks...</p> <ul style="list-style-type: none"> • are relevant and engaging. • have multiple entry points that allow for initial success. • have high cognitive demand. • allow for divergent ways of thinking. • are not scaffolded in ways that reduce cognitive demand. • are not timed; students should not be rushed. <p>Tasks build conceptual understanding. Tasks...</p> <ul style="list-style-type: none"> • allow students to make connections to prior learning. • allow students to answer with multiple representations. • embed multiple Standards for Mathematical Practice. • can provide a preview into the next level of learning. <p>Tasks allow students to show what they know and are able to do. Tasks...</p> <ul style="list-style-type: none"> • cover multiple standards that are central to the unit. • contain a balance of skills, concepts, and problem solving. • generate student work that a teacher can analyze to measure understanding and to inform instruction in the next lesson series.
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More at <http://www.sfusdmath.org/rich-math-tasks.html>

Entry Task	<p>Question</p> <ul style="list-style-type: none"> • What do you already know? <p>Purpose</p> <ul style="list-style-type: none"> • To gather information about what students already know and to help them access their prior knowledge. <p>Guidelines</p> <ul style="list-style-type: none"> • Only requires one class period, or less than an entire class period. • Does not require pre-teaching of concepts and skills. • Might preview the mathematics to be explored in the unit. • Allows for student collaboration and individual accountability. • A rubric can be used to assess and give feedback, but should not be used for grading students.
Apprentice Task	<p>Question</p> <ul style="list-style-type: none"> • What sense are you making of what you are learning? <p>Purpose</p> <ul style="list-style-type: none"> • To gather information about students' ability to express their thinking and justify their results as they apply the new concepts and skills they are learning. <p>Guidelines</p> <ul style="list-style-type: none"> • Should "ramp up" to allow students to demonstrate their level of understanding while making new connections. • Student work includes explanations or justifications. • Assesses current understanding through analysis of student approaches, struggles, and successes. • Allows for student collaboration and individual accountability. • A rubric can be used to assess and give feedback, but should not be used for grading students.
Expert Task	<p>Question</p> <ul style="list-style-type: none"> • How can you apply what you have learned so far to a new situation? <p>Purpose</p> <ul style="list-style-type: none"> • To gather information about students' ability to transfer the new concepts and skills they are learning to novel, real-world, creative, or complex contexts. <p>Guidelines</p> <ul style="list-style-type: none"> • Should be open, complex, and take a long time to solve. • Allows for student collaboration and individual accountability. • A rubric can be used to assess and give feedback, but should not be used for grading students.
Milestone Task	<p>Question</p> <ul style="list-style-type: none"> • Did you learn what was expected of you from this unit? <p>Purpose</p> <ul style="list-style-type: none"> • To assess students' understanding of the concepts and skills in the unit, including their ability to express their thinking and justify their results. <p>Guidelines</p> <ul style="list-style-type: none"> • Should be completed individually. • Used as one component of the SFUSD Interim Assessments in predetermined units. • A rubric should be used to assess, give feedback, and grade students individually.

The Three Signature Strategies of the SFUSD Mathematics Core Curriculum



The SFUSD Mathematics Core Curriculum emphasizes student discourse and group work in its lessons and math tasks. The three Signature Strategies are designed to support teachers and students as they participate in powerful and collaborative mathematics classrooms.

SFUSD Signature Strategy #1: Math Talks

What is this strategy?

A Math Talk is a pedagogical tool for building math thinking and academic discourse in a student-centered, teacher-led way. Math Talks should last for 10–15 minutes. They can be centered on any math topic. Math Talks work best, however, when the content is generally familiar to students up to their Zone of Proximal Development. They should not be used to introduce math content, but when a topic is new, a Math Talk can be an opportunity for informal assessment of student familiarity and background.



Why would I use this strategy?

Math Talks serve to further understanding of math content while addressing Standard for Mathematical Practice #3: *Construct viable arguments and critique the reasoning of others*. They give students the opportunity to develop flexibility and fluency with mental visualization and computation. They offer opportunities to revisit math topics as well as deepen understanding by sharing multiple ways of thinking about a concept or skill.

When do I use this strategy?

This strategy can be used at any time, but is often done at the beginning of a math class. Because it does not need to be focused on the lesson's content, the content of the Math Talk can vary according to the needs of students. Math Talks generally happen 2 or 3 times a week for 10–15 minutes each. In middle and high school, the Math Talk can replace the Do Now.

How do I use this strategy?

Teachers deliberately set up a safe environment where each child's thinking is valued.

Students practice making their thinking explicit.

Everyone practices understanding each other's thinking.

1. Teacher presents the problem.

A problem is presented to the whole class or a small group. Computation problems are always presented horizontally (e.g., $43 + 35 =$), to encourage mental strategies over reliance on algorithms.

2. Students think about the problem.

Students are given time (1–2 minutes) to silently, mentally (no pencils or paper) think about the problem and try to find an answer. They signal quietly to the teacher (e.g., with a thumb up against their chest) when they have an answer.




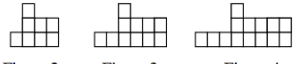
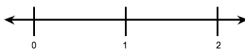
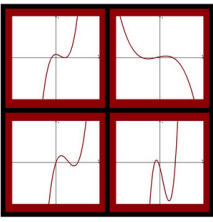
3. Students share their answers.

A few students volunteer to share their answers and the teacher records them on the board. Without judgment, the teacher records answers where all students can see. The teacher continues to take answers until all students' answers have been shared. Teacher can also ask the students to do a Turn-and-Talk with a partner before sharing answers.

4. Students share their thinking.

Students share how they got their answers with a partner or with the larger group. Any student can provide an explanation to any answer on the board. Equity sticks can be used to ensure every student has an equal opportunity to share. The teacher records the student's name and thinking using words, numbers, and symbols. It is important to capture student thinking without writing down every word. The teacher and other students ask questions that help students express themselves, understand each other, and clarify their thinking to make sense of the problem and its solution(s). The expectation of multiple solution pathways is emphasized.

Sample Math Talks By Level

K-2	3-5	6-8	High School
<p>Dot Talks</p> 	<p>How many dots do you see?</p>  <p>How did you see them?</p>	<p>Think of as many equivalent expressions as you can for</p> <p>$2x + 5x + 3 + 6$</p>	<p>Are any of the following equivalent to 1? Why or why not?</p> <p>$\frac{(t+3)}{(t+2)}$ $\frac{(t+3)}{(t-3)}$ $\frac{(t-3)}{(3-t)}$</p>
<p>How many triangles do you see?</p> 	<p>15×18</p>	<p>Always, Sometimes, or Never:</p> <p>$4p$ is greater than $9 + p$</p>	<p>How many tiles in figure 10?</p>  <p>Figure 2 Figure 3 Figure 4</p>
<p>Number Strings</p> <p>$12 + 12$ $12 + 13$ $13 + 13$ $13 + 14$</p>	<p>Place $\frac{1}{2}$ and $1\frac{1}{2}$ on this number line</p> 	<p>Estimate what 32% of 647 is.</p>	<p>Which one doesn't belong?</p> 

Share the “Why” with Students

Give the students the rationale behind the Math Talk. Let them know that they have great thinking that we can't see and this gives them a chance to share what's going on in their brains. This also gives everyone a chance to learn from each other lots of different ways we can think about a problem.

While Math Talks provide an important space for students to share different ways of thinking, teachers will often have a goal in mind and help bring out the important, grade-level mathematical understandings that arise during the Math Talk. The Math Talk Planning Tool on the SFUSD website (<http://www.sfusdmath.org/math-talks-resources.html>) can help teachers anticipate student responses and connect them to each other and to the target mathematics.

Initial Implementation

As you begin to implement Math Talks in your classroom, you will want to keep them simple. Your goal might be to have 2 or 3 students share their thinking, which you capture and record without much comment or questioning.

- Provide a safe environment.
- Start with easier problems so that students can learn the routine and to encourage wide participation.
- Present calculation problems horizontally.
- Provide quiet think time and a silent signal.
- Accept, respect, and consider all answers.
- Capture student thinking as faithfully as you can.
- Write the student's name so that you can refer to _____'s strategy.
- Develop your poker face. Respond neutrally to students' comments.

Developing Questions

As your and your students' familiarity with Math Talks grows, you can begin to ask more probing questions that help students to clarify their thinking and explain the steps they went through.

- Where did you get this number?
- How did you get this?
- Why did you do this operation?
- Do you mean this?
- Is this how you thought of it?
- So you are saying that...?

Adding Layers

- Ask students if they thought of the problem in the same or a different way. (This can be done verbally or with a signal. For example, students can pat their heads if they thought of it the same way.)
- Have students do a Turn-and-Talk. Use this strategy when many students want to talk and may not have a turn individually; when you want to generate more answers or discussion from students; or when students need time and practice articulating their math ideas and strategies before sharing with the whole group.
- Begin to ask questions that connect students thinking to each other:
 - What questions do you have for _____?
 - Who can paraphrase what _____ is saying?
 - Who can explain what _____ is thinking?
 - Do you agree or disagree with what they said? Can you explain why?
- Point out similarities and differences between different strategies.
- Ask students to point out similarities and differences between different strategies.

Further steps with Math Talks

As you become increasingly comfortable with using Math Talks, you will find yourself adjusting them and incorporating them into your pedagogical repertoire.

- Design new Math Talks based on issues that arise during math instruction.
- Design math instruction based on confusions that arise during Math Talks.
- Create class strategy posters that summarize the different strategies that your class is using in Math Talks.
- Simplify Math Talks when students have difficulty. Using smaller numbers can help students access a strategy that they can then apply to larger numbers.
- Offer more than one problem during a Math Talk and allow students to choose the one they want to solve. For example, 13×12 and 15×17 both get at multi-digit multiplication, but one uses numbers that may be easier for students to keep in their heads as they solve the problem mentally.

Further Resources

The SFUSD Math Department website has many more ideas and resources for Math Talks, including: how to design and plan for a Math Talk, how teachers use them, and ways to deepen and extend Math Talks.

<http://www.sfusdmath.org/math-talks.html>.

SFUSD Signature Strategy #2: Three Read Protocol

What is this strategy?

The Three Read Protocol is one way to do a close read of a complex math word problem or task. This strategy includes reading a math scenario three times with a different goal each time. The first read is to understand the context. The second read is to understand the mathematics. The third read is to elicit inquiry questions based on the scenario.



Why would I use this strategy?

The Three Read Protocol is designed to engage students in sense-making of language-rich math problems or tasks. It deepens student understanding by surfacing linguistic as well as mathematical clues. It focuses attention on the importance of understanding problems rather than rapidly trying to solve them. It allows for the use of authentic, instead of overly simplified, text. This strategy also allows for natural differentiation within a class of diverse learners.

When do I use this strategy?

This strategy can be used for math tasks that include complex language structures or language that lends itself to a variety of interpretations. While this is a particularly useful strategy for English Language Learners, all students can benefit from the deeper understanding of word problem structures and open-ended questioning.

How do I use this strategy?

The Three Read Protocol uses the “**problem stem**” of a word problem. This is essentially the word problem without the question at the end. The purpose of presenting the problem stem alone is to have students focus on the contextual and mathematical information before dealing with any question that is involved. This gives students the freedom to create their own questions for a given scenario, which is an excellent skill to develop both in math and in reading. It is important that the teacher choose the problem carefully and anticipate potential linguistic and mathematical roadblocks the students may encounter.

1. **First Read: Teacher reads the problem stem orally.**

The teacher may have visuals to accompany the oral read of the problem stem. Students listen to the story, then turn to a partner and share what they remember of it. Memorizing it is not necessary.

Key Question: What is this situation about?

After the Turn-and-Talk, the teacher asks students to volunteer information they remember from the story. Teachers and students ask clarifying questions about the vocabulary as needed.

2. **Second Read: Class does choral read or partner read of the problem stem.**

The teacher projects the problem stem so the whole class can see it. The teacher leads the class either in a choral read of the problem or has partners read the problem orally to each other. Choral read is preferable because it allows all students to participate without excessive pressure, but a partner read can work fine if that is a better fit. The teacher explains that math stories usually have information about quantities (numbers) and the units that are being counted.

Key Question: What are the quantities in the situation?

An example is 25 cats, where “25” is the quantity and “cat” is the unit. Sometimes the quantities are implied. For example, “some cats” implies a quantity but we do not know what it is. There can also be implied units. An example is “I have one at home.” The implied unit in this case depends on the context of the story. Bottom line: The discussion of quantities and units can be important for focusing student attention, but how deeply the teacher delves into the explicit and implicit information depends on the math and language objectives.

3. Third Read: Partner or choral read the problem stem orally one more time.

The teacher asks students to do one more read of the “story” and asks them to think, “What is missing to make this a good math problem?” Students volunteer their answers to that question. Responses will likely vary because many students assume there is a question without actually reading one. Without correcting student responses, the teacher probes until the class decides that a question is missing. The teacher asks, “Is there only one question that we can ask of this story?” Students responses may vary, but there are usually many different questions that can be asked of almost any scenario.

Key Question: What mathematical questions can we ask about the situation?

The teacher asks partners to determine at least two questions that can be asked using the problem stem. Students share their questions. The teacher writes a couple of the questions and clarifies language as appropriate. After each question, the teacher asks the class, “Can this question be answered with the information from this story?” and the class discusses why or why not.

4. Students work in collaborative groups on the problem.

Students work in groups to solve a question based on the problem stem. The teacher may assign a specific question for all groups to answer, or groups may choose a question from the list asked by the class. If groups are asked to choose their own questions, it is important that the teacher circulate and clarify expectations for the work. This can be an opportunity to differentiate the math work because the range of possible questions to a problem stem is broad.

In summary:

	What the teacher does	Key Question	What the students do
Preparation	<ul style="list-style-type: none">• Identifies appropriate problem stem• Anticipates linguistic and mathematical challenges• Creates visuals to support understanding		
1st Read	<ul style="list-style-type: none">• Shows visuals• Orally reads the “story” (problem stem)	What is this situation about?	<ul style="list-style-type: none">• Sit with a partner• Listen to the “story”• Turn to partners to discuss the “story” in their own words• Say what they remember of the story
2nd Read	<ul style="list-style-type: none">• Shows problem stem (for example, on overhead projector or poster)• Leads class in choral or partner read• Leads discussion of quantities and units	What are the quantities in the situation?	<ul style="list-style-type: none">• Read chorally with the class or with partners• Volunteer quantities and units they identify
3rd Read	<ul style="list-style-type: none">• Asks partners to read with specific goal• Leads discussion of potential questions• Clarifies language as needed	What mathematical questions can we ask about the situation?	<ul style="list-style-type: none">• Read one more time with partners• Brainstorm with partners several questions that could be asked using the problem stem• Volunteer questions

Considerations for use of the Three Read Protocol

1. Is the problem stem sufficiently interesting as a story?

It does not need to be long, but it should have some narrative structure.

2. Does the problem stem have quantities, both explicit and implicit?

Ideally it has easily identifiable explicit quantities, but may have implicit ones as well for a richer discussion and potentially more interesting math investigations.

3. Does the problem stem have extraneous explicit quantities or a variety of implicit quantities?

This strategy can model how to understand math problems with this characteristic and teach students how to discern salient information.

4. Is the language of the problem stem likely to create obstacles for English Language Learners or is the context of the problem likely to be unfamiliar to students from diverse backgrounds?

Surfacing the language structures and contextual clues within the problem stems allows students to focus on the mathematical structures as well as evaluate the reasonableness of their work.

Sample Problem Stems

Animal Shelter Problem

Tasha wants a pet. She goes to the animal shelter to ask how much it will cost to adopt and care for a dog. The vet at the shelter tells her that big dogs have an adoption fee of \$200, a vaccination fee of \$300, and that they eat about 35 pounds of food per month. Small dogs have an adoption fee of \$300, a vaccination fee of \$450, and they eat about 18 pounds of food per month. The vet says that dog food costs about \$3 per pound.

Judy's Berries

Judy loves to eat berries for breakfast, lunch, and dinner. She sees that Clear Lake School is having a fundraiser to raise money for a new playground. The students are selling fruit baskets to raise the money. Strawberries sell for \$3 per basket. Blueberries sell for \$4 per basket. Raspberries sell for \$5 per basket. Judy has \$20 to spend on berries.

Joining a Gym

Carlos wants to join a gym. The gym offers three membership options. The first one is called "Pay as you go" and costs \$6 each time you work out. The second one is called "Regular deal" and costs \$50 per month and \$2 each time you work out. The third one is called "All-in-one price!" and costs \$100 per month for unlimited use of the gym.

Squirrels and Their Acorns

Austin likes to watch squirrels find and store acorns for the winter. Brown Squirrels can carry two acorns at a time. Gray Squirrels can carry three acorns at a time, and Black Squirrels can carry five acorns at a time. There is a pile of 24 acorns.

Further Resources

SFUSD Mathematics Department web site: <http://www.sfusdmath.org/3-read-protocol.html>

SFUSD Signature Strategy #3: Participation Quiz - Secondary

What is this strategy?

A Participation Quiz¹ is a strategy to help establish or reinforce norms for group work in a cooperative environment. While students work together in their group on a math task, the teacher takes public notes—on a document camera, whiteboard, chart paper, or overhead projector—about the quality of their group work (social moves) and the quality of their mathematical discussions (math moves). The teacher then takes notes on how students work together, their use of classroom norms, or the specific language they use to communicate their mathematical ideas.



Why would I use this strategy?

Publicly taking notes on students' interactions allows the teacher to communicate the behaviors they value and wish to encourage, as well as mitigating perceived status differences between students—that is, highlighting strengths of students who are may not be perceived to be strong in math. Some teachers assign each group a grade at the end of a Participation Quiz. Other teachers prefer to focus on the feedback rather than giving it a score. This protocol might be named differently, for example, "Groupwork Feedback," to reflect the teacher's objective.

When do I use this strategy?

This strategy can be used whenever students are working in collaborative groups. It is especially useful when establishing classroom and groupwork norms, and when doing tasks that do not require much teacher assistance.

How do I use this strategy?

1. Choose a worthy task.

The teacher chooses a task that is accessible, challenging, important, and requires students to read and talk together. If a task is too hard, the teacher may spend more time answering group questions than observing, and if a task is too routine students will naturally do these individually since little collaboration will be required. The task should also lend itself to a variety of math abilities.

2. Decide on a focus.

The teacher decides which group norms or Standards for Mathematical Practice he or she wants students to focus on. This decision depends on the context of the classroom. Early in the year, the teacher may focus on establishing norms, such as getting a quick start (reading problem promptly and making sure group understands), working together (heads leaning in and working in the middle of the group), and asking the group questions before asking the teacher. Later in the year, the teacher may focus on refining a norm that a particular class is struggling with, such as making statements with reasons, or the teacher may choose to highlight strengths of specific students that have low status (students who are not generally seen as strong in math).

3. Communicate the focus to students.

The teacher lets his or her class know that a Participation Quiz will be happening during the group work portion of the lesson. The teacher is clear about what he or she is looking for and uses language that students understand. The teacher explains that he or she will publically record a snapshot of the students working together in their groups.

¹ This description is based on *Smarter Together—Collaboration and Equity in the Elementary Math Classroom* by Featherstone, Crespo, Jilk, et. al. and the *Instructional Toolkit for Mathematics* produced by Oakland Unified School District.

For example, the teacher can say that he or she is looking for:

- Quick Start
- “Because” statements (addresses the norm: “We make statements with reasons”).
- Students “leaning in” (addresses the norm: “We work together on the same problem”).
- Group questions only.

As students work, the teacher publically records statements about how groups are working and making sense of the math together. This can be done on a document camera, whiteboard, chart paper, or overhead projector. The recording sheet is split into as many spaces as there are groups (see example diagram below). Some groups may not notice this public documentation, while other times they pay attention and change their behaviors to meet the norms.

4. Debrief the notes taken.

The teacher takes time before the end of class or in the middle of the task to debrief. Time is given for students to read comments. The teacher highlights key evidence that supports the groupwork norms. The focus on groupwork norms to start the class and then end the lesson can be a powerful way to reinforce the kinds of cooperative behaviors that teachers want to establish.

When students are used to seeing this structure, teachers can use these public notes as a “quiz” to assess students and groups on their groupwork skills. Generally, the focus should be on positive behaviors, although over time honest critiques of behavior may be included as well.

Example Participation Quiz Feedback Form

Group 1 QS (Quick Start) “Miguel, can you re-read the problem?” “I said this was $3x + 2$ because...”	Group 2 QS “So, I think this means...” Heads leaning in “What did you mean by...” “So, x means... do you get it?”
Group 3 QS “I’m not sure what to do. Can you...” Using a table of points “I think the pattern is $+3$; see, look at...” Using “because” statements	Group 4 Reading problem “Melissa, explain to me how you got the equation.”
Group 5 QS “Can you repeat your idea?” “Let’s make a graph.” “Something’s not right. What do you think?” Explaining graph and equation.	Group 6 QS “I don’t understand the where the 2 shows up on the graph.” “Oh...I see the adding 3 in the table and graph; now...that makes more sense.” Students leaning in on work in the middle

Further Resources

SFUSD Mathematics Department web site:

<http://www.sfusdmath.org/participation-quiz--group-feedback.html>

Smarter Together-Collaboration and Equity in the Elementary Math Classroom Featherstone, Crespo, Jilk, et. al.

SFUSD Signature Strategy #3: Groupwork Feedback - Elementary

What is this strategy?

Groupwork feedback is a strategy to publicly recognize the class norms and math focus of students as they work in groups. It is based on the Participation Quiz strategy, but modified to better fit the Elementary context. Group work is loosely defined to include partners, trios, or larger group sizes. The teacher takes public notes about the quality of the group work and the quality of the mathematical discussions. This feedback should focus on the specific nature of groupwork interactions as well as target mathematics of the lesson rather than general positive reinforcement.



Why would I use this strategy?

While math class norms are important, it is not enough to simply post them and discuss them in the beginning of the year. Public groupwork feedback allows the teacher to communicate the behaviors and norms he or she values and wishes to encourage. With regular use, it reminds students of the classroom norms and allows them to deepen their practice and incorporate new approaches towards meeting them. It is also a way to recognize the efforts of students who may be shy or struggling in class. By highlighting the many strengths that students exhibit in class, the teacher fosters a classroom climate of growth mindset and multiple learning styles. This helps to mitigate the sense that some students are just “not good” at math.

When do I use this strategy?

This strategy can be used whenever students are working in collaborative groups (partners, trios, or other). It is especially useful when establishing classroom and groupwork norms, and when doing tasks that do not require much teacher assistance.

How do I use this strategy?

1. Analyze the math task or activity.

Is the task or activity accessible, challenging, and important? Does it require students to read and talk together? Does it require a variety of math abilities? If a task is too hard, the teacher may spend more time answering group questions than observing. However, if a task is too routine, students will naturally work on it alone since little collaboration may be required.

2. Decide on a focus.

The teacher decides which group norms and target math students should focus on. Early in the year, the teacher may focus on establishing norms, such as getting a quick start (reading problem promptly and making sure group understands), working together (heads leaning in and working in the middle of the group), and asking the group questions before asking the teacher. Later in the year, the teacher may focus on refining a norm that the class is struggling with, such as one student rephrasing what another student said. Additionally, the teacher may choose to highlight strengths of specific students who are not generally seen by their peers as being strong in math.

3. Communicate the focus to students.

The teacher lets his or her class know that the lesson will be structured as a Groupwork Feedback class. This means that the teacher will focus mainly on how the groups are interacting together and working on the math instead of re-teaching or working with individuals or small groups. The teacher is clear about what he or she is looking for and uses language that students understand. The teacher explains that he or she will publically record how the students work in their groups, how they collaborate and approach the mathematics, and that there will be a discussion at the end of class based on these observations.

For example, the teacher can say that he or she is looking for:

- Quick Start
- “Because” statements (addresses the norm: “We make statements with reasons”).
- Students “leaning in” (addresses the norm: “We work together on the same problem”).
- Group questions only.

As students work, the teacher publically records statements about how groups are working together. This can be done on a document camera, whiteboard, chart paper, or overhead projector. The recording sheet is split into as many spaces as there are groups (see example Notetaking Form below). Making the recording public allows student groups to be reinforced for behavior that supports the norms and to notice where their behavior can change to meet the norms. Sometimes groups do not notice this public documentation, while other times they pay attention and feel reinforced or change their behaviors.

4. Debrief the notes taken.

The teacher takes time before the end of class or in the middle of the task to debrief. Time is given for students to read comments or for the teacher to read comments aloud. The teacher highlights key evidence that supports the groupwork norms. The focus on groupwork norms to start and end the lesson can be a powerful way to reinforce the kinds of cooperative behaviors that teachers want to establish.

Grade Span	Modifications
All Grades	<ul style="list-style-type: none">• Teacher may take digital photos of collaborative work and show them on projector during debrief.• Teacher may have sheets of paper with the goals of the day's Participation Quiz written on them and he or she may stamp the group's sheet when evidence of the goals is observed.
Third - Fifth	<ul style="list-style-type: none">• Teacher may write down his or her observations on sticky notes and leave them at a table group during an activity.• Teacher may write down his or her observations on sticky notes and post notes under the document camera when convenient. This allows the feedback to be public and real-time.• Teacher may record observations on a Groupwork Feedback Notetaking Form, projecting the feedback using a document camera as much as possible.

More at <http://www.sfusdmath.org/participation-quiz--groupwork-feedback.html>

Sample Groupwork Feedback Notetaking Form

Task:

Math Goal(s):

Groupwork Goal(s):

Group	What they did mathematically	What they did collaboratively

Classroom Structures for the SFUSD Mathematics Curriculum

Class Norms

What is this structure?

Class norms are a set of statements of value or behavior that support active and equitable participation in the classroom. They set expectations for all members of the classroom community, adults and students. Class norms are stated in a positive tone, apply to all situations all the time, and are few in number.

Why do I use this structure?

Class norms help students, and give them opportunities, to be accountable to the community and to act on the prosocial values of responsibility, respect, fairness, caring, and helpfulness. They support equitable participation, engagement with the practices of doing math, and establishment of a positive classroom climate.

When do I use this structure?

Establish classroom norms at the beginning of the school year and continue to reinforce them every day.

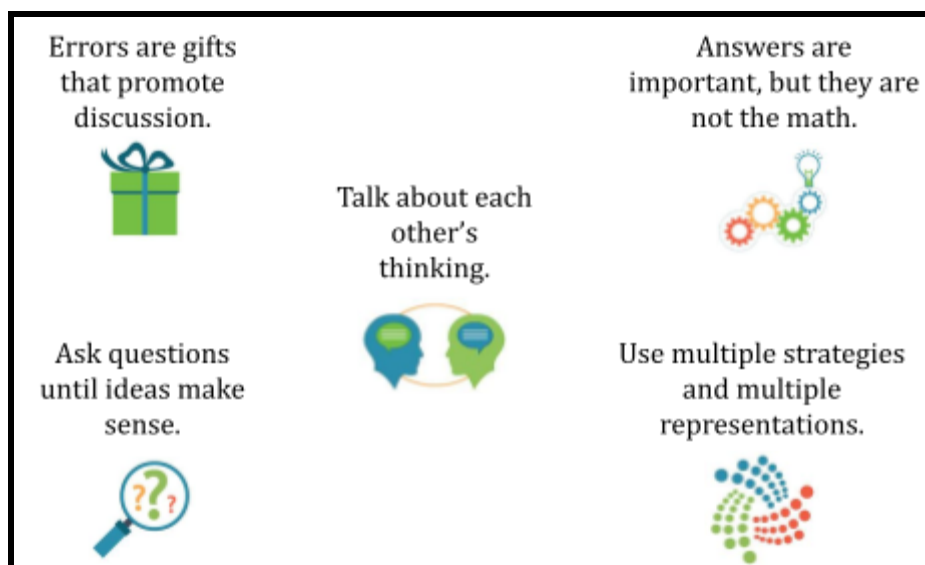
How do I use this structure?

Establishing class norms:

- ❖ Hold a discussion of what the community wants the classroom to look and feel like. The norms could come out of this discussion, or this discussion could lay the groundwork for discussion of teacher-chosen norms.
- ❖ It takes time and effort for students to follow the norms consistently. Provide clear expectations and examples of what norms look like. Have students brainstorm and share your own powerful vision.
- ❖ Distinguish between rules and norms. Rules, which establish classroom safety and efficiency, can support norms. Rules should be established by the teacher on the first day of school. Phrasing the rules using a positive tone contributes to a sense of community.
- ❖ Give, or have students generate, specific examples of what the norms look like and sound like.

Reinforcing class norms:

- ❖ Continue to remind and reinforce the positive behaviors that you see throughout the year. Include specific examples of what the norms look like and sound like.
- ❖ Participation Quiz and Groupwork Feedback strategies can be used when establishing the norms and, as the year continues, to support attention to the norms.
- ❖ Encourage students to notice themselves and others supporting the norms. Celebrate positive mathematical interactions.



More at <http://www.sfusdmath.org/class-norms.html>

Class Norms

Choose 4-6 norms to use with your class, including one or two from each of the following value categories, depending on the needs of your students. Note that some norms support multiple values. Make the norms your own by discussing them with your students and changing the language if necessary.

Collaboration	<ul style="list-style-type: none"> • We will respect each other. • We will help each other. • We will solve problems together. • We will ask our group members for help before the teacher. • Talk about each other's thinking. • Ask questions until ideas make sense. • Make decisions together. • Justify your ideas. • Together, work to answer questions. • Ask questions and share ideas. • Members of your team are your first resource. • Smarter together than apart. • You have the responsibility to ask for help, and the responsibility to offer it. • No one is done until everyone is done.
Growth Mindset	<ul style="list-style-type: none"> • Errors are gifts that promote discussion. • Answers are important, but they are not the math. • We learn from mistakes. • We learn from trying new things. • We learn from taking risks.
Communication	<ul style="list-style-type: none"> • We will ask our group members for help before the teacher. • Ask questions until ideas make sense. • Justify your ideas. • Explain and give reasons. • Ask questions and share ideas.
Persistence	<ul style="list-style-type: none"> • We will ask our group members for help before the teacher. • Ask questions until ideas make sense. • Smarter together than apart. • You have the responsibility to ask for help, and you the responsibility to offer it. • No one is done until everyone is done.
Process	<ul style="list-style-type: none"> • We will solve problems together. • We will ask our group members for help before the teacher. • Answers are important, but they are not the math. • Talk about each other's thinking. • Ask questions until ideas make sense. • Use multiple strategies and representations. • Make decisions together. • Justify your ideas. • Together, work to answer questions. • Explain and give reasons. • Members of your team are your first resource. • You have the responsibility to ask for help, and the responsibility to offer it. • No one is done until everyone is done.

5 Practices for Orchestrating Productive Mathematics Discussions²

What is this structure?

The 5 Practices for Orchestrating Productive Mathematics Discussion are a set of teacher actions for organizing mathematically powerful and productive discussions that are rooted in student thinking.

Why do I use this structure?

This set of instructional practices helps teachers use student work as the launching point for discussions in which important ideas are surfaced, contradictions are explored, and understandings are consolidated. The use of student voice and the focus on the content objectives support the achievement of those objectives.

When do I use this structure?

This structure is intended to be used in lessons in which students are working together in small groups to complete some mathematical task and a whole class discussion is expected to be the culminating event of the lesson.

How do I use this structure:

This structure specifically identifies the things that a teacher will be doing before the lesson (Anticipation), during group work (Monitoring, Selecting, and Sequencing) and during discussion (Connecting) to help all students access the grade-level mathematics.

1. Anticipating likely student responses to mathematical tasks.

Involves envisioning potential student responses, strategies (correct or incorrect), representations, procedures, and interpretations. This occurs during the planning stage of a lesson.

2. Monitoring students' actual responses to the tasks.

Involves paying close attention to students' mathematical thinking as they work on a problem. This is usually done by circulating around the classroom during group work.

3. Selecting student response to feature during the discussion.

Involves choosing particular students or student groups to present their work because of the mathematical responses. These responses need not be chosen solely because they are correct, but rather because they emphasize different approaches to the problem. In fact, it may be advantageous to choose incorrect responses to highlight how and why they are incorrect, and help surface and clarify misconceptions. The selections can highlight a variety of responses or strategies for a task, or can show a progression from simple to complex representation. Make sure over time that all students feel they are authors of mathematical ideas.

4. Sequencing student responses during the discussions.

Involves purposeful ordering of the featured student responses in order to make the mathematics accessible to all students. This also helps build a mathematically coherent story line during whole class discussion.

5. Connecting student responses during the discussions.

Involves helping students to make mathematical connections between different student responses. This helps ensure that key mathematical ideas remain the focus of the lesson debrief.

More at <http://www.sfusdmath.org/5-practices-for-orchestrating-productive-math-discussions.html>

²Based on NCTM publication *5 Practices for Orchestrating Productive Mathematics Discussions* by Margaret Smith and Mary Kay Stein

Formative Assessment

What is formative assessment?

Formative assessment describes a range of teacher actions which begin with the gathering of evidence of student thinking. This information helps teachers to adapt their teaching and meet student learning needs through targeted feedback. The evidence-gathering can be an informal, in-the-moment observation of student work and behavior, or a formal, rigorous analysis of student thinking through written work or observations of discourse. Teachers use the evidence to identify student strengths and needs, provide targeted feedback, and re-engage with challenging concepts (see Re-engagement). This feedback and shift in instruction can happen in the moment, in planning for the following day, or in planning for a later unit.

Why do I use formative assessment?

Research has consistently shown that the use of assessment to inform instruction and give feedback increases student learning. When teachers know what their students strengths and needs are, they can tailor their teaching to build on the strengths and support the needs. They can also identify which teaching structures work and which need to be modified to better support students.

When should I use formative assessment?

Informal formative assessment should be used daily to inform instructional decisions. More formal and deliberate formative assessment should be done at least once every unit. The tasks that form the backbone of the SFUSD Core Curriculum Units are convenient opportunities for formative assessment. They can give teachers information about how students are taking up and applying new content, and help them plan upcoming lessons which re-engage students with the material (see Re-engagement) in the unit.

How do I use formative assessment?

Formative Assessment has six key steps³:

- Make the objectives of the lesson explicit: share the objectives with students and occasionally ask students to produce evidence that they can achieve the objectives.
- Observe and listen to groups as well as individual students: as groups communicate, misconceptions become visible.
- Watch and listen before intervening: when you do intervene, begin by asking students to explain their thinking.
- Ask open-ended questions that allow students to explain their thinking and reasoning.
- Give constructive, useful feedback that pushes students to think: make comments, oral or written, that help students recognize what they can do, what they need to be able to do, and how they might narrow the gap.
- Give students the opportunity to improve their work: students can either revise work that they have done or do a similar mathematical activity to experience and show growth.

Opportunities for formative assessment occur in every lesson. For example, students can respond on mini-whiteboards for individual assessment. Teachers can also take the opportunity to look over shoulders and listen in on student conversations, or to join student conversations with open-ended questions that allow students to explain their reasoning.

Teachers can also collect student work in a variety of ways. Classwork, homework, and exit slips provide opportunities for students to show individual and group thinking. Student conversations can also be recorded, with an iPad, for example, for later review. Students, alone or in groups, can demonstrate their level of understanding by creating a poster that shows one or more solutions to a problem, summarizes what they know about a given topic, or shows the connections between mathematical ideas.

In all cases, teachers should use the evidence they gather to provide constructive, useful feedback, including information about what students can do already, what they need to be able to do, and how they can move forward in their learning.

³ Adapted from the Mathematics Assessment Project,
http://map.mathshell.org/static/draft/pd/modules/1_Formative_Assessment/html/index.htm

Re-engagement

What is this structure?

Re-engagement is using student work for the purpose of uncovering misconceptions, providing feedback on student thinking, and helping students to go deeper into the mathematics. Students have the opportunity to reflect on their own learning while make connections between mathematical ideas. There is a focus on metacognitive development as student analyze other student work in the search for possible mathematical misconceptions.

Why do I use this structure?

Re-engagement pushes students to address their conceptual understanding of a topic in order to make connections and eliminate misconceptions. Research has shown it to be more effective than re-teaching the same content, because it engages all students in a metacognitive activity with high cognitive load.

When do I use this structure?

Re-engagement should be used when students have had some opportunities to learn about a topic. At some point, some students will be developing a strong understanding, while others will have significant misconceptions. A re-engagement lesson allows all students to think again about the topic, deepening their understanding through a collaborative experience.

How do I use this structure?

First, formatively assess students to identify common errors. Then, for each common error, ask, “What might students have been thinking?” By trying to understand the students’ reasoning, a teacher can identify next steps. Opportunities for re-engagement range from small scale (10 minutes) to large scale (2 class periods).

Examples of Re-engagement Activities

Re-engagement Activity / Time	Description
Math Talk 10 minutes	If many students in a class are making a common error, lead a Math Talk that will allow students to make that error. As part of the Math Talk discussion, have students who do not make the error share their reasoning and engage all students in discussing the misconception.
Revising Work Based on Feedback 20 minutes	After students complete a piece of work, provide constructive feedback for revision. Then provide time, either in class or for homework, for students to revise their work. The feedback should help students recognize what they can do, what they need to be able to do, and how they might narrow the gap. One way to do this is to ask them to find their own mistakes, eg.: “You have 3 calculation errors. Find and fix them.” Another is to ask a pointed question, eg.: “What does this 4 represent?”).
Math Hospital 10 minutes - 1 class period	The teacher creates a sheet compiling common mistakes that students make about a given topic, sometimes including problems that are completed correctly. Student teams identify what mistake is made in each problem, if any, explain why that mistake is a common one, and correct the mistake. This structure allows students to correct mistakes that they might make by addressing the mistakes directly but in an impersonal context.

Re-engagement Activity	Description
5 Practices 1 class period	Use the 5 Practices for Orchestrating Productive Mathematics Discussion to Anticipate likely student responses to a task, including what errors they might make, Monitor students' actual responses to the task, Select student responses that will allow discussion of misconceptions, Sequence the responses for discussion, and Connect student strategies and thinking during discussion. This purposeful preparation for conversation allows you to address known and emerging misconceptions.
MARS Tools 1 class period	The MARS tasks, included in many of our units, come with extensive teacher materials that include guidance for analyzing student work, implications for instruction, and direction for how to construct a re-engagement lesson based on that analysis. More guidance is provided within the MARS teacher materials.
Formative Re-engagement Lesson (FRL) Formative Assessment Lesson (FAL) 2 class periods	These highly-structured lessons come in many varieties, but the format for most are similar: ⁴ <ol style="list-style-type: none"> 1. Start with a problem to assess current understanding. 2. Students work in collaborative teams to develop a problem solving strategy. 3. Students examine and make sense of other strategies, think about why strategies work, and decide which is most efficient or useful. 4. Students analyze errors and, in the process, let go of misconceptions and clarify their thinking about the big ideas. 5. Students return to the original problem or a similar problem to apply their learning through revising their work.

Re-teaching... ⁵	vs.	Re-engagement...
<ul style="list-style-type: none"> • is teaching the unit again • is addressing missing basic skills • is doing the same problems over • is more practice • is focused on procedures • is focused on underachievers • has lower cognitive load 		<ul style="list-style-type: none"> • is revisiting student thinking • is addressing conceptual understanding • is examining the task from different perspectives • is critiquing approaches • is making connections • engages the entire class in mathematics • has higher cognitive load

More at <http://www.sfusdmath.org/reengagement.html>

⁴ adapted from "General Guide: Elementary Formative Assessment Re-engagement Lessons," http://www.svmimac.org/images/GeneralGuide_Elementary_FRL.pdf

⁵ Adapted from Illustrative Mathematics, <http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons>

Effective Questioning⁶

You can promote discourse and stimulate student thinking through effective questioning. This, in turn, develops the habits of mind suggested by the Standards for Mathematical Practice. Here is a list of questions from the *Professional Standards in Teaching Mathematics*, grouped into categories that reflect the mathematical practices.

- **Helping students work together to make sense of mathematics:**
 - "What do others think about what Janine said?"
 - "Do you agree? Disagree?"
 - "Does anyone have the same answer but a different way to explain it?"
 - "Do you understand what they are saying?"
- **Helping students to rely more on themselves to determine whether something is mathematically correct:**
 - "Why do you think that?"
 - "Why is that true?"
 - "How did you reach that conclusion?"
 - "Can you make a model to show that?"
- **Helping student learn to reason mathematically:**
 - "Does that always work?"
 - "Can you think of a counterexample?"
 - "How can you prove that?"
 - "What assumptions are you making?"
- **Helping students learn to conjecture, invent, and solve problems:**
 - "What would happen if...? What if not?"
 - "Do you see a pattern?"
 - "What is alike and what is different about your method and her method to solve the problem?"
 - "Can you predict the next one? What about the last one?"
- **Helping students to connect mathematics, its ideas, and its applications:**
 - "How is this process like others that you have used?"
 - "How does this relate to _____?"
 - "Have you ever solved a problem like this before?"
 - "Can you give me an example of _____?"

More at <http://www.sfusdmath.org/effective-questioning.html>

⁶ Adapted from *NCSM Great Tasks for Mathematics, 6-12* by Schrock, Norris, Pugalee, Seitz, and Hollingshead, 2013

Gallery Walk

What is this structure?

A Gallery Walk allows students to display their work for discussion much like artists would display their artistic pieces in an exhibit. It also allows students to reflect and comment on the work that others have done on the same task. Finally, it gives students feedback from peers on their own work that can give them insights and help them revise and improve it.



Why do I use this structure?

A Gallery Walk is a way for students to receive feedback on their work, check their understanding, and see multiple solution paths for a given math task. It is an opportunity for students to see and discuss multiple ways of approaching and representing math thinking.

When do I use this structure?

Gallery Walks can be used at any time during a unit, but are most commonly used as a way of sharing students' thinking and starting a debrief of a math task.

How do I use this structure?

There are many ways to conduct a Gallery Walk. Here is one way:

After teams of students work on an assigned task or activity, they show their work on a collaborative poster.

- When the teams finish their posters, they display their work either on their work space or on wall space spread around the classroom.
- Inform teams that they will visit each poster to discuss the math work displayed on the poster. Consider giving teams specific prompts to respond to as they observe each other's work. For example,
 - What part of the math do you agree with? Why?
 - What part of the math work do you disagree with? Why?
 - What looks similar to what you did?
 - What looks different from what you did?

Teams might post responses to these questions on the posters using adhesive notes.

- Students may fill out a Gallery Walk Response Sheet instead of posting their thinking directly on their classmates' work. In this case, you may choose to direct students to pick one or more posters to focus their thinking on. Questions might include all of those above plus:
 - What are some mathematical characteristics of the work that fit...? (Insert an aspect of the math content that you are focusing on.)
 - What do you notice that is similar among all or most of the posters?
 - What do you notice that is different on all or most of the posters?

After the Gallery Walk, teams return to their own poster to discuss the comments others left for them. If there is time, and after considering the work that they saw on others' posters and the comments and questions that were left, they may revise their work. Decide a focus for the debrief based on these questions. It is not necessary to address all the topics that are raised. Instead, focus the discussion on aspects of the core mathematics you want to highlight in this lesson.

More at <http://www.sfusdmath.org/gallery-walk.html>

Group Roles

Collaborative groups benefit from clear roles for their members. Here are some sample role cards, with questions for each role to ask their team. A poster of these roles could be posted in the room to support daily student interactions. These questions could also be modified to fit the particular work of the day. Group Roles can be a focus of Participation Quizzes or Groupwork Feedback.

<p style="text-align: center;">Questioner</p> <p>Keep track of questions that the group has as you work on the math problem.</p> <ul style="list-style-type: none"> • <i>What was your question?</i> • <i>What do you mean by that?</i> 	<p style="text-align: center;">Director</p> <p>Make sure everyone is participating in the math work.</p> <ul style="list-style-type: none"> • <i>What do you think?</i> • <i>What do you mean by that?</i>
<p style="text-align: center;">Illustrator</p> <p>Draw diagrams, tables, or other illustrations that show what the group is doing.</p> <ul style="list-style-type: none"> • <i>Is this what you were thinking?</i> • <i>What do you mean by that?</i> 	<p style="text-align: center;">Connector</p> <p>Make connections between what people in the group think and say.</p> <ul style="list-style-type: none"> • <i>How does this idea connect to that idea?</i> • <i>What do you mean by that?</i>

Adapted from Literature Circles.

<p style="text-align: center;">Resource Manager</p> <p>Call the teacher over when your team is stuck. Make sure that all questions are team questions.</p> <ul style="list-style-type: none"> • <i>What team question can we ask the teacher?</i> • <i>Are we sure that no one here can answer the question?</i> <p>Do not let your team stay stuck! Get materials your team needs for a task, and organize the cleanup effort.</p>	<p style="text-align: center;">Facilitator</p> <p>Make sure your team understands the entire task before you begin.</p> <ul style="list-style-type: none"> • <i>Who wants to read? Does everyone understand what we are being asked to do?</i> <p>Keep your team together, and keep track of time.</p> <ul style="list-style-type: none"> • <i>Are we all ready to move on to the next problem?</i> • <i>We need to finish this part in 5 minutes, so we have time for...</i>
<p style="text-align: center;">Recorder/Reporter</p> <p>Help your team organize their written work. The work needs to show everyone's ideas and be well organized. Use color, arrows, and other math tools to communicate your mathematics, reasons, and connections.</p> <ul style="list-style-type: none"> • <i>How can we show that on the graph?</i> • <i>How can we show that connection?</i> • <i>Did everyone write that down?</i> 	<p style="text-align: center;">Team Captain</p> <p>Make sure your team is following the class norms, and that everyone's ideas are heard.</p> <p>Make sure that all talking is within your team and is helping you accomplish the task. Eliminate side conversations.</p> <ul style="list-style-type: none"> • <i>Does anyone see it in a different way?</i> • <i>Let's hear from ____.</i>

Adapted from College Preparatory Mathematics (CPM).

More at <http://www.sfusdmath.org/group-roles.html>

Suggestions for Daily Math Schedules - Elementary

	K-2	3-5
Option #1 Opening/Math Talk Lesson Introduction Whole Group Group Work Closing	60 minutes 5 - 10 minutes 10 minutes 10 - 20 minutes 15 minutes 5 minutes	60 minutes 10 - 15 minutes 10 minutes 10 minutes 20 minutes 5 - 10 minutes
Option #2 Centers (differentiation) Whole Group Group Work Math Talk	60 minutes 10 minutes 20 minutes 20 minutes 5 - 10 minutes	60 minutes 15 minutes 10 minutes 20 minutes 10 - 15 minutes
Option #3 - task day <i>Launch:</i> Task Intro <i>Explore:</i> Individual work/think time on task Group work on task Finishing up task <i>Summarize</i>	60 minutes 10 minutes 5 minutes 10 minutes 5 minutes 15 minutes 10 minutes	60 min. 10 minutes 5 minutes 10 minutes 10 minutes 15 minutes 10 minutes

Suggestions for Daily Math Schedules - Secondary

Option #1 Warm-Up/Do Now Launch Explore (Group Work) Closure	55 minutes 5 minutes 10 minutes 25 minutes 15 minutes
Option #2 Math Talk Launch Explore (Group Work) Closure	55 minutes 10 minutes 5 minutes 25 minutes 15 minutes
Option #3 - Block Math Talk Launch Explore (Group Work) Closure	80 minutes 10 minutes 10 minutes 45 minutes 15 minutes
Option #4 - Block Math Talk Launch Explore (Group Work) Mid-Talk Explore (Group Work) Closure	80 minutes 10 minutes 5 minutes 20 minutes 10 minutes 20 minutes 15 minutes

More at <http://www.sfusdmath.org/group-roles.html>

Classroom Tools for the SFUSD Mathematics Curriculum

Math Notebooks

What is this tool?

A math notebook is a place where students keep their math work. They may be composition notebooks, spiral notebooks, or even construction paper stapled into a booklet.

There are a variety of uses for math notebooks. Two examples are:

- As a place to hold **all math work** and learning.
- As a record of **significant math learning** milestones.



Why do I use this tool?

Notebooks are a powerful way to organize student work and learning. Notebooks give students ownership over how they demonstrate and organize their own learning. They offer a concrete way for students, families, and teachers to reflect on student learning. Notebooking provides training for organization and responsibility in school and in life.

Notebooks have four major functions:

- as a record of mathematical development
- as a reference for students
- as a source for formative assessment, and
- as an organizational tool for materials and math learning.

When do I use this tool?

Depending on the purpose of math notebooks, they can be used daily to record class work or they can be used intermittently to record important information and major math milestones.

How do I use this tool?

It is important that the teacher emphasize that a math notebook serves a variety of purposes. For example, it is a communication tool, a reference book, a reflective or interactive journal, and a record that shows how math learning is a cycle that includes revision.

Decide how you want the notebooks to be set up.

For example, your notebooks might contain these components:

- Table of Contents
- Glossary
- Page Numbers
- Anchor Chart Section

Daily entries might include:

- Date
- Heading
- Do Now
- Lesson Objectives
- Reflection or Journal

Decide how you want to set up routines for math notebooks in your classroom. Examples are:

Kinder - 2nd Grade	3rd - 5th Grades	6th - 8th Grades and High School
<ul style="list-style-type: none"> • Write in date (or teacher uses date stamp) • Paste in the problem • Show their work 	<ul style="list-style-type: none"> • Write in date • Paste in or copy the problem • Show their work • Clearly indicate the solution 	<ul style="list-style-type: none"> • Write in date • Write in lesson title • Write in problem numbers • Identify and copy in key information of a problem • Show their work • Clearly indicate their solutions

Decide what types of entries you want your students to include in their math notebooks. Examples are:

Graphic Organizers	Journal Writing	Anchor Charts	Class Work	Glossary	Other
Frayer Model T-Chart Box and T-Chart Venn Diagram Bubble Map Concept Map	Prompts to introduce a unit or lesson. Learning Logs or Reflection Open ended problems Quick write or draw	Notes on methods Addition Strategies Subtraction Strategies Multiplication Strategies Division Strategies Representations of Growth Patterns	Do Nows Problems worked on in class	Key math vocabulary Word bank	Graphs Lists or Brainstorms Multiplication chart 100's chart Equivalent fractions chart Exit slips

Math Notebooks as a Source of Formative Assessment of Student Learning

Math notebooks are an excellent tool for formative assessment. The record of student math work provides longitudinal evidence of how students are developing understanding of concepts as well as applying the Standards for Mathematical Practice.

Math notebooks are also powerful opportunities for providing feedback to students. Teachers should give their students regular feedback on their notebooks in the form of narrative response, questions, and suggestions. Teachers can also use a rubric that assesses student work in their notebooks on organization, clarity, accuracy, and how well math thinking is justified. The rubric should be provided to students so that they can improve their work over time. The purpose of notebook feedback is to help students revise their understanding and work.

More at <http://www.sfusdmath.org/math-notebooks.html>

Rule of Four

What is this tool?

The “Rule of Four” is a way to think about math both at the entry point of a task and in the representation of mathematical thinking. Showing our thinking through multiple representations helps us have a stronger and deeper understanding of the mathematics. It also allows us to see connections across concepts and topics in mathematics.

Why do I use this tool?

When we strive to represent our understand using the “Rule of Four,” we are asking ourselves to find deeper connections both within and across concepts. In addition, we are encourage approaching the math from multiple perspectives.

When do I use this tool?

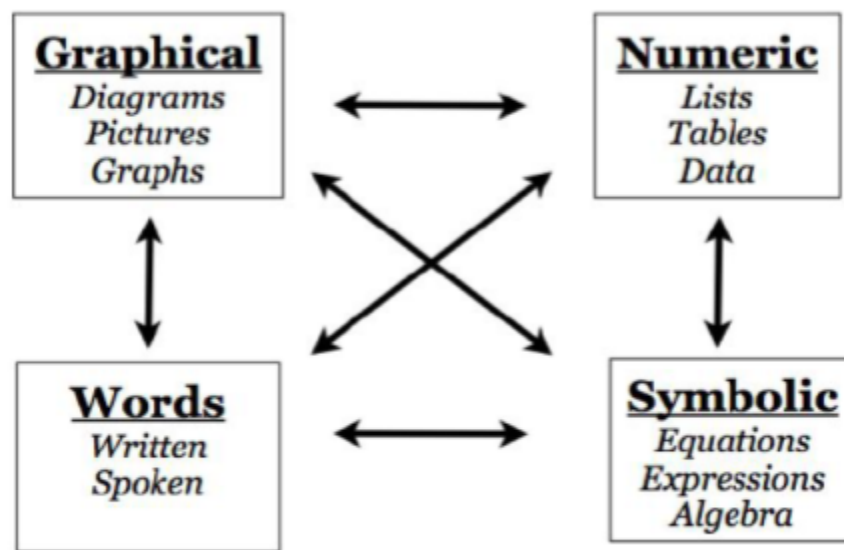
The “Rule of Four” is appropriate for most math work. How much it is emphasized depends on the context.

How do I use this tool?

Student may not be familiar or fluent with representing their work in a variety of ways. Explicit modeling from the teacher and giving students many opportunities to practice representing their work in multiple ways is an effective way to teach students to think about their math work in this fashion. It is equally important that students make connections among the representations. These connections lead to a deeper understanding over time.

Rule of Four

Mathematical ideas can be communicated in many ways, including by using the following four representations. These four ways lead to a deeper understanding of the mathematics as well as allow for a variety of modalities for students to express their understanding.



More at <http://www.sfusdmath.org/rule-of-four.html>

Technology Tools

Technology is a growing component of the SFUSD Mathematics Core Curriculum and should always be used in the service of math learning. The guiding question for SFUSD technology and math integration is: *How might we support intentional use of technology to prioritize both discourse and target mathematics?*

This is a partial list of technology tools that SFUSD recommends for math classes. Recommendations are included in every unit of our core curriculum.

More information can be found on the SFUSD Math Department website:
<http://www.sfusdmath.org/technology-tools.html>

Rubrics

What is a rubric?

A rubric is a tool for evaluating student work. There are two main types of rubrics: holistic (a rubric that provides one overall score) and analytic (a rubric that provides scores for different categories). Most of the rubrics in the SFUSD Math Core Curriculum are 4-point holistic rubrics.

A rubric is often used in conjunction with an answer key. The rubric provides a broader picture about a student's demonstration of understanding the standards and mathematical practices, and the answer key provides specific examples of how a student might answer parts of the task.

Why do I use rubrics?

The primary purpose of a rubric is to provide specific feedback on critical elements of the task and the student work. In addition, rubrics may be used to show students the expectations before they perform a task and to give students feedback and an opportunity for revision after they perform the task. Both of these uses strongly support student learning and achievement.

Sample 4-Point Holistic Rubric

4 Meets standards at a high level	3 Meets standards	2 Approaching standards	1 Minimal understanding	0 No attempt
The student provides correct solutions and strategies. The student explains and justifies his/her thinking thoroughly and clearly. The student connects and applies the standards in complex ways.	The student provides mostly correct solutions and strategies with minor errors. The student explains and justifies his/her thinking. The student demonstrates mastery of the standards that were explicitly taught.	The student demonstrates some correct thinking about solutions and strategies. Student explains their thinking but it may be hard to follow. The student demonstrates partial understanding of the of standards that were explicitly taught.	The student demonstrates some evidence of mathematical thinking, but shows little understanding. The student offers little explanation of his/her thinking or what is offered does not make sense. The student demonstrates minimal or no understanding of the standard.	No evidence of attempting the task.

More at <http://www.sfusdmath.org/rubrics.html>

When do I use rubrics?

Rubrics are traditionally used to evaluate student work after students perform a task, especially a summative task. Rubrics are included for all the Milestone Tasks, as well as some other tasks, in the SFUSD Math Core Curriculum for this purpose. Rubrics can also be used before the task to communicate performance expectations to students and after the task to communicate feedback and provide structure for revision or re-engagement.

How do I use a rubric?

One way to give feedback to students is to make a copy of the rubric for each student, and then highlight or circle the parts of the rubric that apply to the student's work. For example, you might highlight the first and third paragraph of column 3: Meets standards, and highlight the second paragraph in column 2: Approaching standards. This feedback is useful for students because it gives them an indication of the next steps they could take to improve their work.

If you are using a holistic rubric, you will sometimes need to decide whether to give only whole points or to sometimes give half points. For example, if student work shows some elements from a score of 3 but mostly elements from a score of 2, you will need to decide whether to give that work a score of 2.5 or a score of 2.

Using a Rubric to Assign Grades

If you are using a rubric for an Entry, Apprentice, or Expert task, the rubric will be useful for informing your instruction and giving feedback to students, but it should not be used to give grades. Think about whether it is fair to expect mastery of the standards in the unit before assigning grades for student work.

If you are using a rubric for a Milestone Task, and you want to use it to assign grades, think about the score descriptors instead of converting the numbers to percents proportionally. For example, you may want to use this guide to convert rubric scores to grades:

4 Meets standards at a high level	3 Meets standards	2 Approaching standards	1 Minimal understanding	0 No attempt
A (95%)	B (85%)	C (75%)	D (65%)	F (50%)

Many teachers require that students revise their work when they receive a score of 0, 1, or 2 so that they can show progress toward mastery of the standards.

The Common Core State Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

MP8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.