BC Calculus　　　　　　　　　　　　　　　　　　Name:___________________________

UNIT 6 Review D

1) A function f has Maclaurin series given by $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$. Which of the following is an expression for $f(x)$?

(A) $\cos x$

(B) $e^x - \sin x$

(C) $e^x + \sin x$

(D) $\frac{1}{2}\left(e^x + e^{-x}\right)$

(E) e^{x^2}

2) The coefficients of the power series $\sum_{n=0}^{\infty} a_n (x-2)^n$ satisfy $a_0 = 5$ and $a_n = \left(\frac{2n+1}{3n-1}\right) a_{n-1}$ for all $n \geq 1$. The radius of convergence of the series is

(A) 0　　(B) $\frac{2}{3}$　　(C) $\frac{3}{2}$　　(D) 2　　(E) infinite

3) AP 2007B #6

Let f be the function given by $f(x) = 6e^{-x/3}$ for all x.

(a) Find the first four nonzero terms and the general term for the Taylor series for f about $x = 0$.

(b) Let g be the function given by $g(x) = \int_0^x f(t)\,dt$. Find the first four nonzero terms and the general term for the Taylor series for g about $x = 0$.

(c) The function h satisfies $h(x) = k f'(ax)$ for all x, where a and k are constants. The Taylor series for h about $x = 0$ is given by

$$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots.$$

Find the values of a and k.

4) AP 2003B #6

The function f has a Taylor series about $x = 2$ that converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n+1)!}{3^n}$ for $n \geq 1$, and $f(2) = 1$.

(a) Write the first four terms and the general term of the Taylor series for f about $x = 2$.

(b) Find the radius of convergence for the Taylor series for f about $x = 2$. Show the work that leads to your answer.

(c) Let g be a function satisfying $g(2) = 3$ and $g'(x) = f(x)$ for all x. Write the first four terms and the general term of the Taylor series for g about $x = 2$.

(d) Does the Taylor series for g as defined in part (c) converge at $x = -2$? Give a reason for your answer.