NRI INSTITUTE OF TECHNOLOGY
(Approved by AICTE - New Delhi, Affiliated to JNTUK, Kakinada)
POTHAVARAPPADU-521 212 VIJAYAWADA, Krishna Dist.
Ph : 0866-2469666 Website : nrigroupofcolleges.com
E-mail: nrihitech@rediffmail.com

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

ELECTRICAL CIRCUIT ANALYSIS - II

ECA-II

II B. TECH I SEM EEE

BY

K.VENKATA KISHORE

SYLLABUS

UNIT-I: BALANCED THREE PHASE CIRCUITS

Phase sequence- star and delta connection - relation between line and phase voltages and currents - analysis of balanced three phase circuits - measurement of active and reactive power.

UNBALANCED THREE PHASE CIRCUITS

Analysis of three phase unbalanced circuits: Loop method – Star-Delta transformation technique, Two wattmetermethods for measurement of three phase power.

UNIT II:TRANSIENT ANALYSIS IN DC CIRCUITS

Transient response of R-L, R-C, R-L-C circuits for DC excitations, Solution using differential equations and Laplace transforms.

UNIT III:TRANSIENT ANALYSIS IN AC CIRCUITS

Transient response of R-L, R-C, R-L-C circuits for DC and AC excitations, Solution using differential equations and Laplace transforms.

UNIT-IV: TWO PORT NETWORKS

Two port network parameters – Z, Y, ABCD and Hybrid parameters and their relations, Cascaded networks - Poles and zeros of network functions.

Filters

UNIT-V: FILTERS

Need of Filters – Classification -Characteristic impedance- Low Pass Filter, High Pass Filter, Band Pass Filter, Band Stop or Band Elimination Filter, m- Derived Filter, Composite filters– Design of Filters.

PRE-REQUISITES:

- BASIC CURRENT AND VOLTAGE LAWS
- MESH AND NODAL ANALYSIS
- BASIC MATHAMETICAL KNOWLEDGE INCLUDING LAPLACE AND INVERSE LAPLACE TRANSFORMATION,FOURIERS AND Z-TRANFORMATIONS.

REFERNCE TEXT BOOKS:

1. Engineering Circuit Analysis by William Hayt and Jack E.Kemmerley,McGraw Hill Company,6 th edition
2. Network synthesis: Van Valkenburg; Prentice-Hall of India Private Ltd.
3. Circuits by A.Bruce Carlson , Cengage Learning Publications.

Learning Outcomes:

1. Students are able to solve three- phase circuits under balanced and unbalanced condition.
2. Students are able find the transient response of electrical networks for different types of excitations.
3. Students are able to find parameters for different types of network.
4. Students are able to realize electrical equivalent network for a given network transfer function.
5. Students are able to extract different harmonics components from the response of a electrical network.

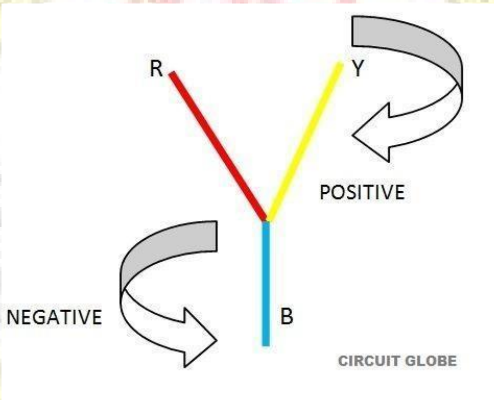
UNIT-I

BALANCED THREE PHASE CIRCUITS

Phase Sequence:

In three phase system the order in which the voltages attain their maximum positive value is called Phase Sequence. There are three voltages or EMFs in three phase system with the same magnitude, but the frequency is displaced by an angle of 120 deg electrically.

Taking an example, if the phases of any coil are named as R, Y, B then the Positive phase sequence will be RYB, YBR, BRY also called as clockwise sequence and similarly the Negative phase sequence will be RBY, BYR, YRB respectively and known as an anti-clockwise sequence.



It is essential because of the following reasons:-

1. The parallel operation of three phase transformer or alternator is only possible when its phase sequence is known.
2. The rotational direction of three phase induction motor depends upon its sequence of phase on three phase supply and thus to reverse its direction the phase sequence of the supply given to the motor has to be changed.

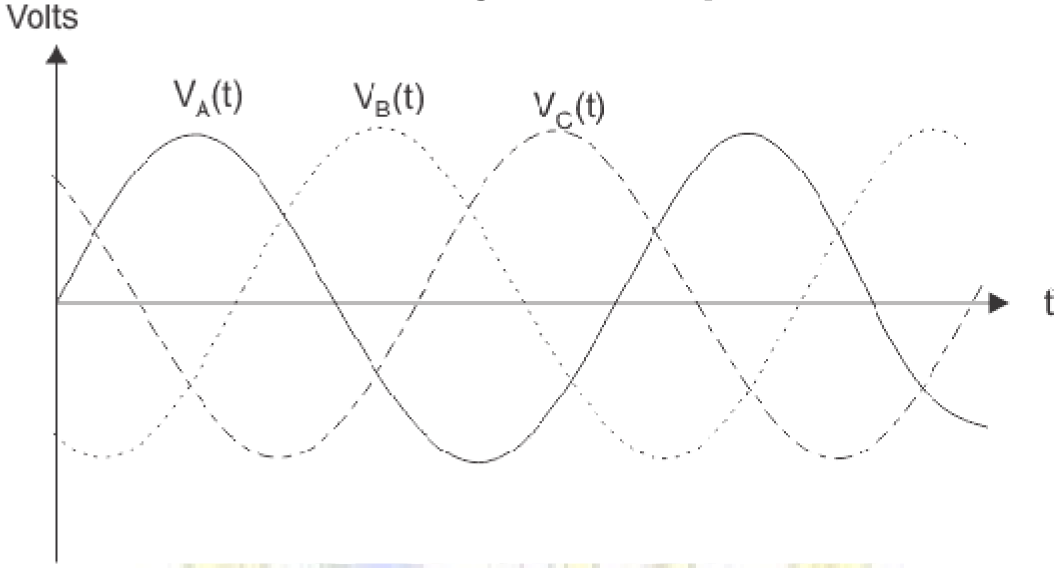
Star and Delta System:

There are two types of system available in electric circuit, single phase and three phase system. In single phase circuit, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In 1882, new invention has been done on polyphase system, that more than one phase can be used for generating, transmitting and for load system. Three phase circuit is the polyphase system where three phases are send together from the generator to the load.

Each phase are having a phase difference of 120°, i.e 120° angle electrically. So from the total of 360°, three phases are equally divided into 120° each. The power in three phase system is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below-

The three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the three phase circuit and the neutral can be used as ground to complete the circuit.



Why Three Phase is Preferred Over Single Phase?

There are various reasons for this question because there are numbers of advantages over single phase circuit. The three phase system can be used as three single phase line so it can act as three single phase system. The three phase generation and single phase generation is same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductor needed in single phase circuit. And also the instantaneous power in single phase system falls down to zero as in single phase we can see from the sinusoidal curve but in three phase system the net power from all the phases gives a continuous power to the load.

Till now we can say that there are three voltage source connected together to form a three phase circuit and actually it is inside the generator. The generator is having three voltage sources which are acting together in 120° phase difference. If we can arrange three single phase circuit with 120° phase difference, then it will become a three phase circuit. So 120°phase difference is must otherwise the circuit will not work, the three phase load will not be able to get active and it may also cause damage to the system.

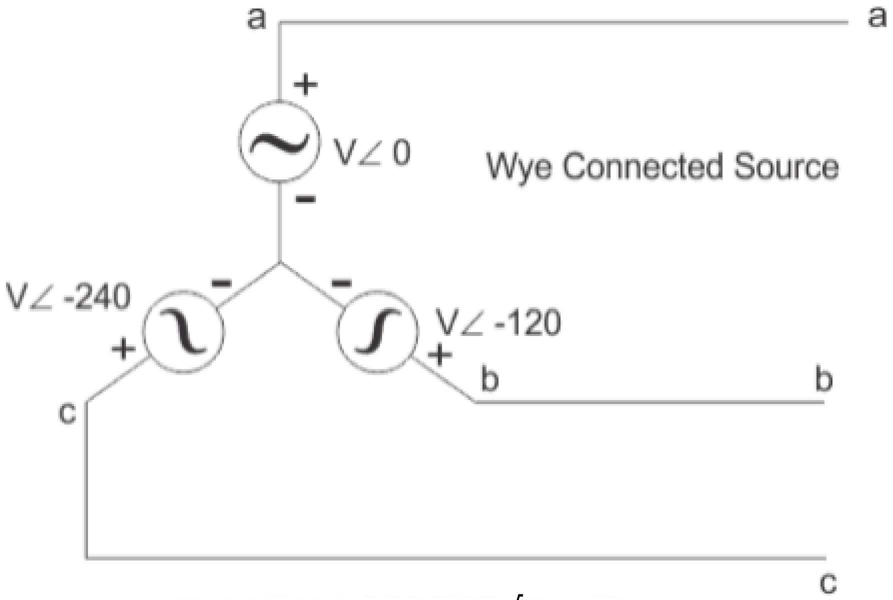
The size or metal quantity of three phase devices is not having much difference. Now if we consider the transformer, it will be almost same size for both single phase and three phase because transformer will make only the linkage of flux. So the three phase system will have higher efficiency compared to single phase because for the same or little difference in mass of transformer, three phase line will be out whereas in single phase it will be only one. And losses will be minimum in three phase circuit. So overall in conclusion the three phase system will have better and higher efficiency compared to the single phase system.
In three phase circuit, connections can be given in two types:

1. Star connection
2. Delta connection
3. Less commonly, there is also an open delta connection where two single-phase transformers are used to provide a three-phase supply. These are generally only used in emergency conditions, as their efficiency is low when compared to delta-delta (closed delta) systems (which are used during standard operations).

4. In star connection, there is four wire, three wires are phase wire and fourth is neutral which is taken from the star point. Star connection is preferred for long distance power transmission because it is having the neutral point. In this we need to come to the concept of balanced and unbalanced current in power system.

5. When equal current will flow through all the three phases, then it is called as balanced current. And when the current will not be equal in any of the phase, then it is unbalanced current. In this case, during balanced condition there will be no current flowing through the neutral line and hence there is no use of the neutral terminal. But when there will be unbalanced current flowing in the three phase circuit, neutral is having a vital role. It will take the unbalanced current through to the ground and protect the transformer. Unbalanced current affects transformer and it may also cause damage to the transformer and for this star connection is preferred for long distance transmission.
   The star connection is shown below-

The star connection is shown below-

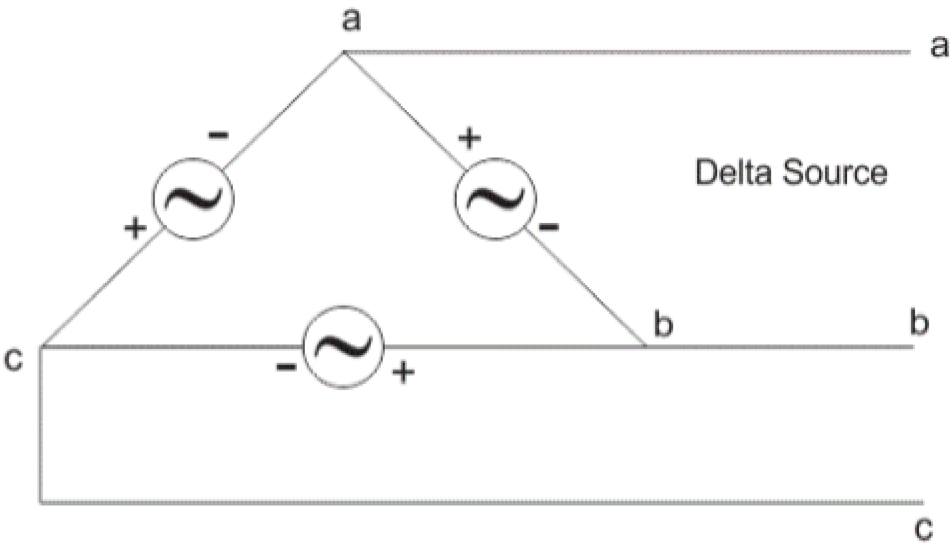


In star connection, the line voltage is √3 times of phase voltage. Line voltage is the voltagebetween two phases in three phase circuit and phase voltage is the voltage between one phase to the neutral line. And the current is same for both line and phase. It is shown as expression below

$$E_{Line} = \sqrt{3}E_{phase} \; and \; I_{Line} = I_{Phase}$$

Delta Connection

In delta connection, there is three wires alone and no neutral terminal is taken. Normally delta connection is preferred for short distance due to the problem of unbalanced current in the circuit. The figure is shown below for delta connection. In the load station, ground can be used as neutral path if required.



In delta connection, the line voltage is the same as that of phase voltage. And the line current is √3 times of phase current. It is shown as expression below,

$$E_{Line} = E_{phase} \; and \; I_{Line} = \sqrt{3}I_{Phase}$$

In a three-phase circuit, star and delta connection can be arranged in four different ways:

1. Star-Star connection
2. Star-Delta connection
3. Delta-Star connection
4. Delta-Delta connection

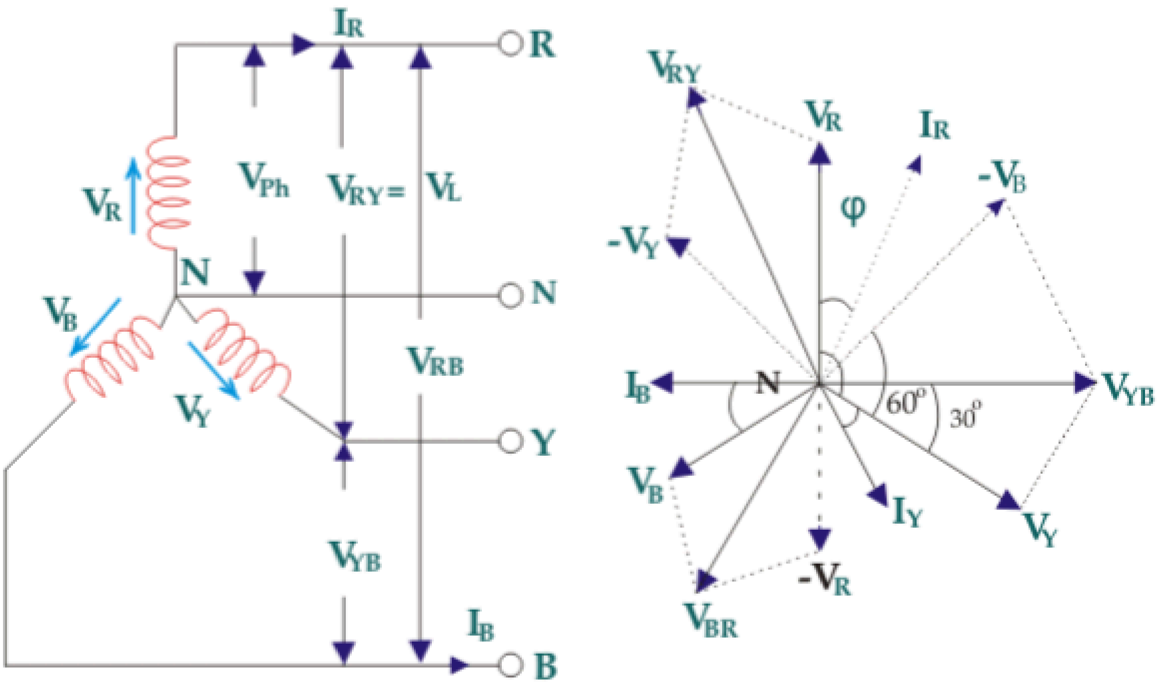
But the power is independent of the circuit arrangement of the three phase system. The net power in the circuit will be same in both star and delta connection. The power in three phase circuit can be calculated from the equation below,

$P_{Total} = 3 \times E_{phase} \times I_{phase} \times PF$

Since, there is three phases, so the multiple of 3 is made in the normal power equation and the PF is power factor. Power factor is a very important factor in three phase system and some times due to certain error, it is corrected by using capacitors.

Relation between line and phase voltages and currents:

To derive the relations between line and phase currents and voltages of a star connected system, we have first to draw a balanced star connected system.



Suppose due to load impedance the current lags the applied voltage in each phase of the system by an angle **ϕ**. As we have considered that the system is perfectly balanced, the magnitude of current and voltage of each phase is the same. Let us say, the magnitude of the voltage across the red phase i.e.

magnitude of the voltage between neutral point (N) and red phase terminal

(R) is V_R.
Similarly, the magnitude of the voltage across yellow phase is V_Y and the magnitude of the voltage across blue phase is V_B.
In the balanced star system, magnitude of phase voltage in each phase is V_{ph}.
∴ $V_R = V_Y = V_B = V_{ph}$
We know in the star connection, line current is same as phase current. The magnitude of this current is same in all three phases and say it is I_L.
∴ $I_R = I_Y = I_B = I_L$, Where, I_R is line current of R phase, I_Y is line current of Y phase and I_B is line current of B phase. Again, phase current, I_{ph} of each phase is same as line current I_L in star connected system.
∴ $I_R = I_Y = I_B = I_L = I_{ph}$.

Now, let us say, the voltage across R and Y terminal of the star connected circuit is V_{RY}.
The voltage across Y and B terminal of the star connected circuit is V_{YB} The voltage across B and R terminal of the star connected circuit is VBR.
From the diagram, it is found that
$V_{RY} = V_R + (-V_Y)$
Similarly, $V_{YB} = V_Y + (-V_B)$
And, $V_{BR} = V_B + (-V_R)$
Now, as angle between V_R and V_Y is 120°(electrical), the angle between V_R and – V_Y is 180° – 120° = 60°(electrical).

$$V_L = |V_{RY}| = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^o}$$
$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph} \times \frac{1}{2}}$$
$$= \sqrt{3}V_{ph}$$
$$\therefore V_L = \sqrt{3}V_{ph}$$

Thus, for the star-connected system line voltage = √3 × phase voltage.
Line current = Phase current
As, the angle between voltage and current per phase is φ, the electric power per phase is

$$V_{ph}I_{ph}cos\phi = \frac{V_L}{\sqrt{3}}I_L \cos\phi$$

So the total power of three phase system is

$$3 \times \frac{V_L}{\sqrt{3}}I_L \cos\phi = \sqrt{3}V_L I_L \cos\phi$$

Analysis of Balanced 3 Phase Circuit:

It is always better to solve the balanced three phase circuits on per phase basis. When the three phase supply voltage is given without reference to the line or phase value, then it is the line voltage which is taken into consideration.

The following steps are given below to solve the balanced three phase circuits.

Step 1 – First of all draw the circuit diagram.

Step 2 – Determine $X_{LP} = X_L/\text{phase} = 2\pi f_L$.

Step 3 – Determine $X_{CP} = X_C/\text{phase} = 1/2\pi f_C$.

Step 4 – Determine $X_P = X/\text{ phase} = X_L - X_C$

Step 5 – Determine $Z_P = Z/\text{phase} = \sqrt{R^2{}_P + X^2{}_P}$

Step 6 – Determine $\cos\phi = R_P/Z_P$; the power factor is lagging when $X_{LP} > X_{CP}$ and it is leading when $X_{CP} > X_{LP}$.

Step 7 – Determine V phase.

For star connection $V_P = V_L/\sqrt{3}$ and for delta connection $V_P = V_L$

Step 8 – Determine $I_P = V_P/Z_P$.

Step 9 – Now, determine the line current I_L.

For star connection $I_L = I_P$ and for delta connection $I_L = \sqrt{3}\ I_P$

Step 10 – Determine the Active, Reactive and Apparent power.

Analysis of Unbalanced 3 Phase Circuit:

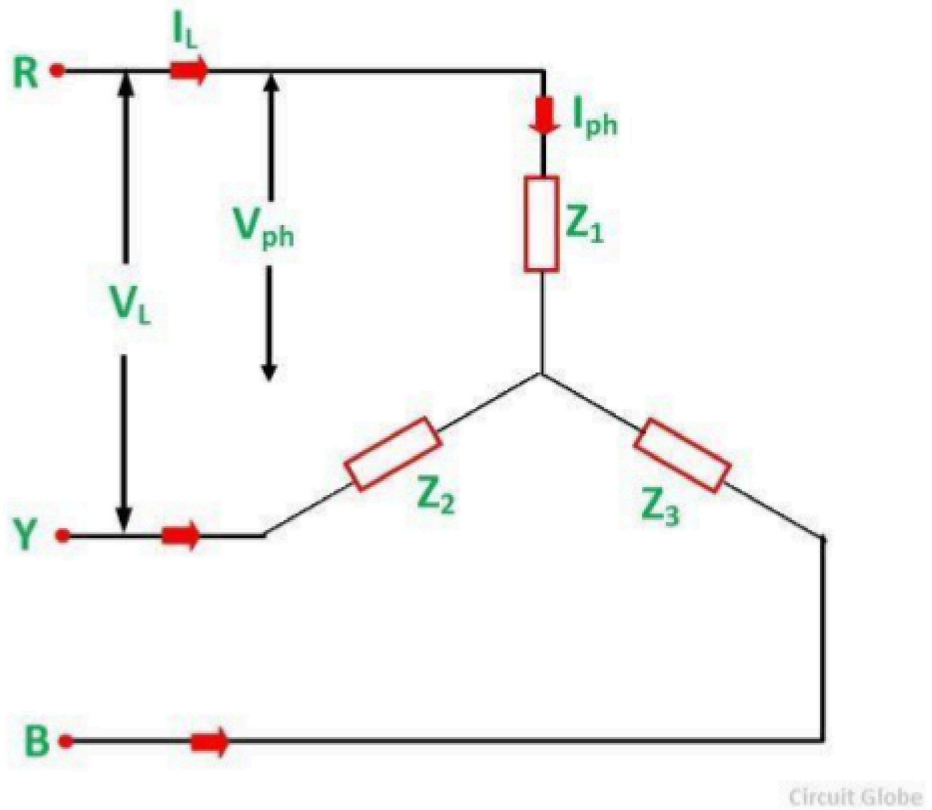
The analysis of the 3 Phase unbalanced system is slightly difficult, and the load is connected either as Star or Delta. The topic is discussed in detail in the article named as Star to Delta and Delta to Star Conversion.

Interconnection of 3 Phase System

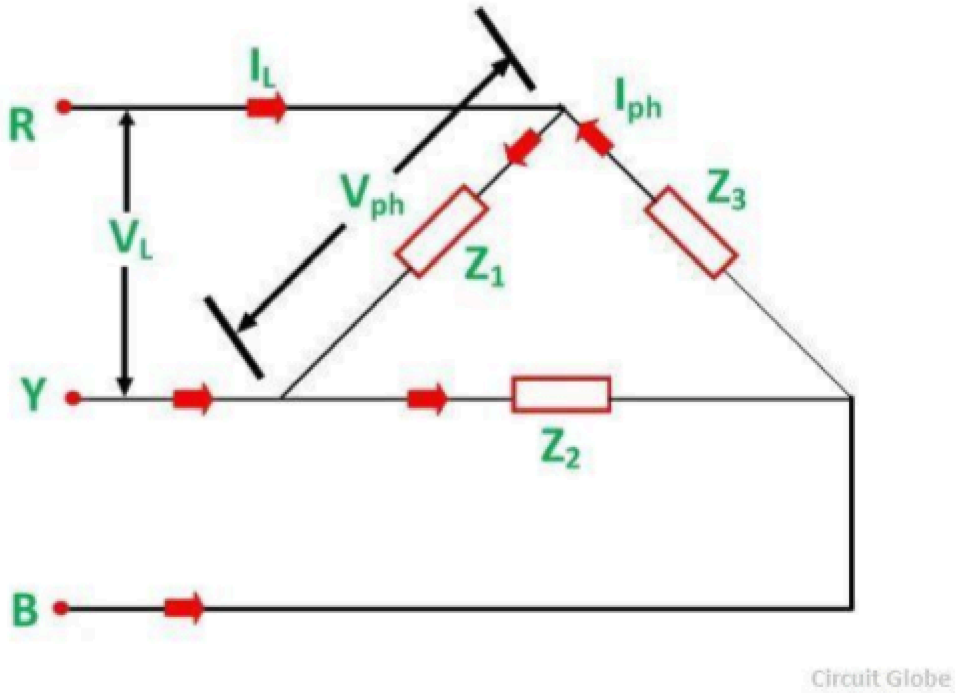
In a three-phase AC generator, there are three windings. Each winding has two terminals (start and finish). If a separate load is connected across each phase winding as shown in the figure below, then each phase supplies as independent load through a pair of wires. Thus, six wires will be required to connect the load to a generator. This will make the whole system complicated and costly.

Connection of 3 Phase Loads in 3 Phase System

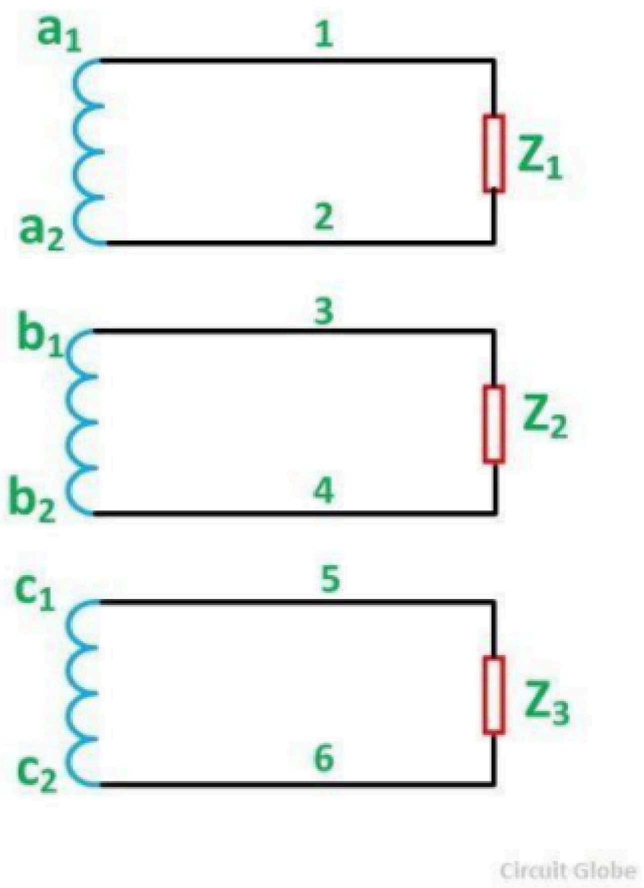
As the three phase supply is connected in star and delta connections. Similarly, the three-phase loads are also connected either as Star connection or as Delta Connection. The three phase load connected in the star is shown in the figure below.



The delta connection of three phase loads is shown in the figure below.



The three phase loads may be balanced or unbalanced as discussed above. If the three loads Z_1, Z_2 and Z_3 have the same magnitude and phase angle, then the 3 phase load is said to be a balanced load. Under such connections, all the phase or line currents and all the phase or line voltages are equal in magnitude.



Therefore, in order to reduce the number of line conductors, the three phase windings of an AC generator are interconnected.The interconnection of the windings of a three phase system .

measurement of active and reactive power:

Reactive Power Measurement

The power which exists in the circuit when the voltage and current are out of phase to each other, such type of power is known as the reactive power. The formula measures the reactive power in the circuit

$$Q = VIsin\emptyset$$

The measurement of reactive power is essential because the value of reactive power shows the total power loss in the circuit. If the value of reactive power is low, the power factor of the load becomes poorer and more loss occurs in the system. The electrical system is classified by the number of phases used in the circuit, and according to these phases, the varmeter is categorised into two types.

1. Single Phase Varrneter
2. Polyphase Varmeter

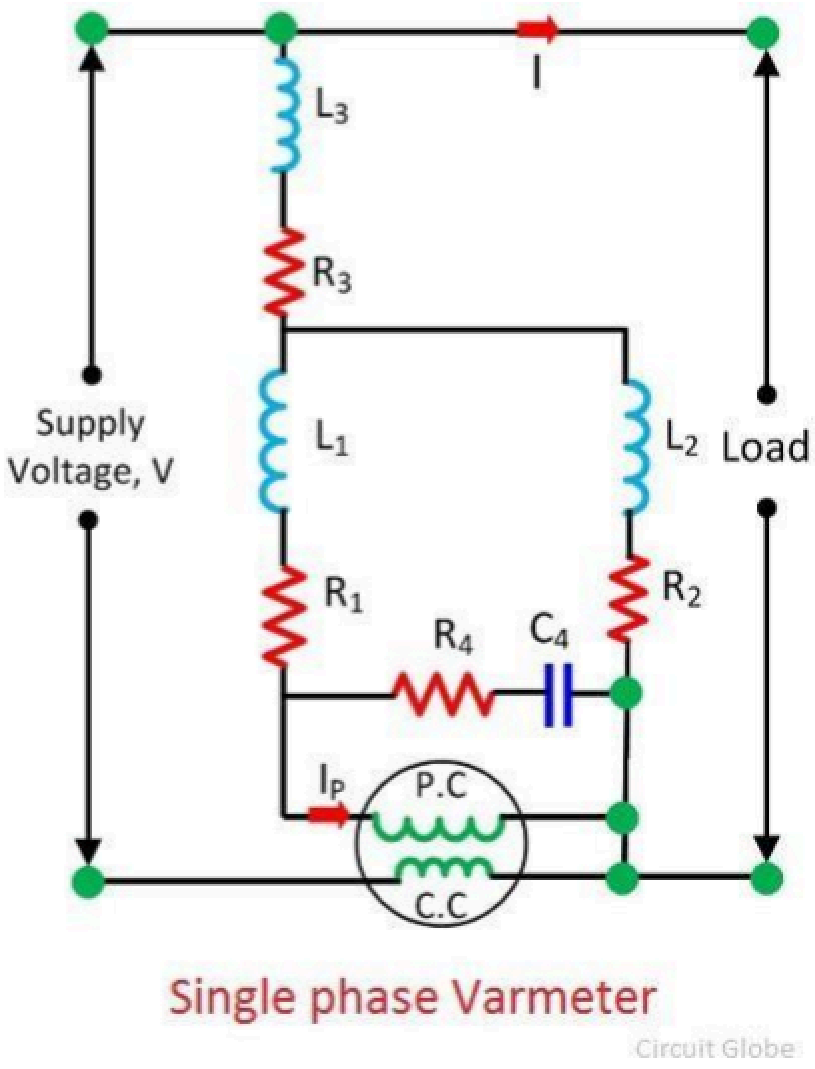
Single Phase Varmeter:

The reactive power of the single phase circuit is measured by the Varmeter (Volt ampere reactive meter). The varmeter is a type of the Electrodynamometer Wattmeter in which the pressure coil of the meter is made highly inductive. The terms "highly inductive" means, the voltage of the pressure coils lags at an angle of 90° with that of the current coil.

The current which passes through the current coil is the load current. The load current has a phase difference of 90° concerning that of the supply voltage, and it is given by the equation shown below.

$$VI\,\mathrm{Cos}(90^{\circ} - \emptyset) = VIsin\emptyset = reactive\ power$$

The circuit diagram of the single phase varmeter is shown in the figure below.

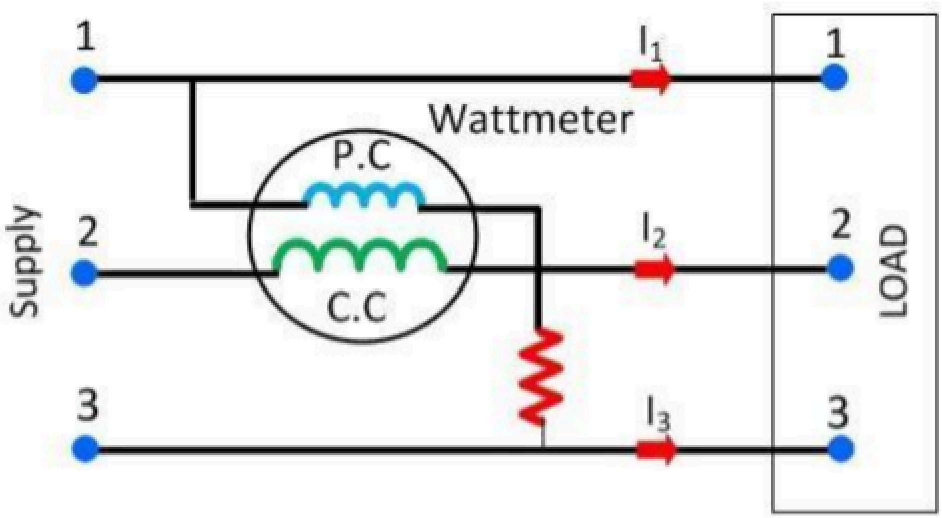


Single phase Varmeter

The Single Phase Varmeter gives the incorrect result because of the presence of harmonics. If the varmeter measures the reading at a different frequency from that which we used during the calibration then also the varmeter gives the inaccurate result.

Reactive Power Measurement in Balanced Three-Phase Circuit:

The single wattmeter method is used for measuring the power of the balanced three-phase circuit. The current coil of the Wattmeter is connected to one phase, and the pressure coil is connected to the other phase of the line.



Reactive Power Measurement with One Wattmeter

Circuit Globe

Let the current through the current coil – I_2

Voltage across the pressure coil – V_{13}

$$Reading\ of\ Wattmeter = V_{13} I_2 \mathrm{Cos}(90^{\circ} + \emptyset)$$

$$= \sqrt{3} VI \mathrm{Cos}(90^{\circ} + \emptyset)$$

$$= \sqrt{3} VISin90^{\circ}$$

Total reactive volt amperes of the circuit

$$Q = 3VISin\emptyset$$

$$= \left(-\sqrt{3}\right) \times reading\ of\ Wattmeter$$

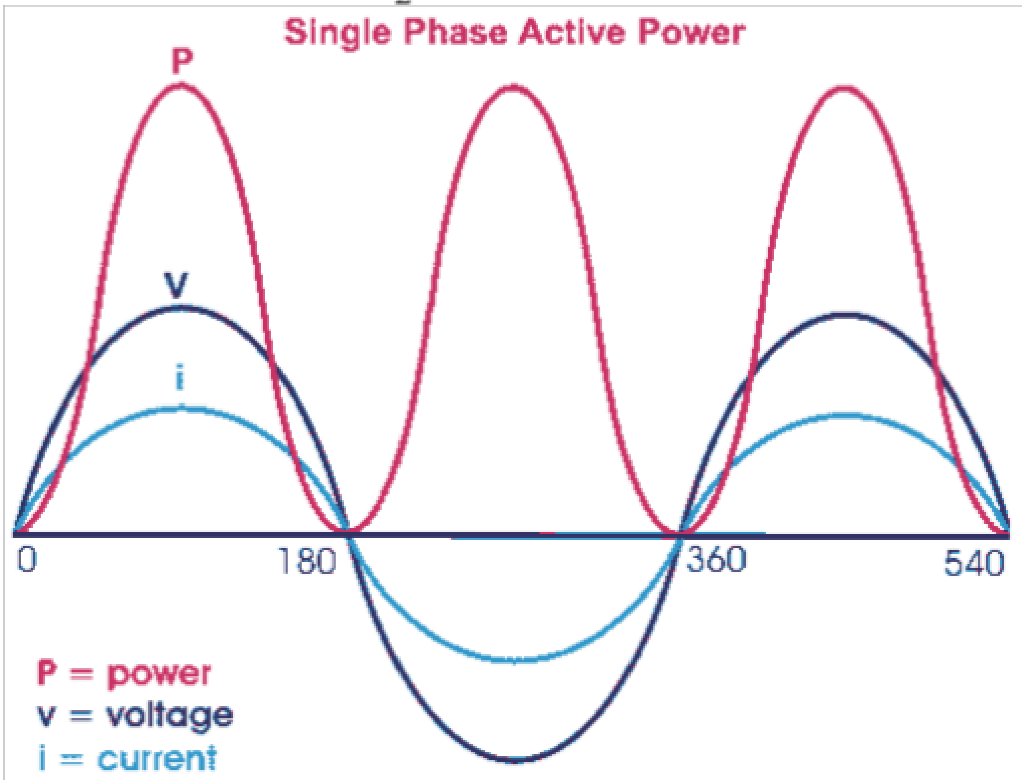
The phase angle

$$\emptyset = \tan^{-1}\frac{Q}{P}$$

Active Power:
Resistive Power

Let's take the condition first where the single phase power circuit is fully resistive in nature, that means the phase angle between voltage and current i.e. φ = 0 and hence,

$$p = \frac{1}{2}V_m.I_m.[1 - \cos 2\omega t]$$



From the above equation it is clear that, whatever may be the value of ωt the value of cos2ωt cannot be greater than 1; hence the value of p cannot be negative. The value of p is always positive irrespective of the instantaneous direction of voltage v and currenti, that means the energy is flowing in its conventional direction, i.e. from source to load and p is the rate of energy consumption by the load and this is called active power. As this power is consumed due to resistive effect of an electrical circuit, hence sometimes it is also called Resistive Power.

UNBALANCED THREE PHASE CIRCUITS

Analysis of three phase unbalanced circuits:

An unbalanced three-phase circuit is one that contains at least one source or load that does not possess three-phase symmetry. A source with the three source-function magnitudes unequal and/or the successive phase displacements different from 120° can make a circuit unbalanced. Similarly, a three-phase load with unequal phase impedance values can make a circuit unbalanced.

The single-phase equivalent circuit technique of analysis does not work for unbalanced three-phase circuits. General circuit analysis techniques like mesh analysis or nodal analysis will have to be employed for analysing such circuits.

Loop method:

Introduction

The Loop Current Method is a small variation on the Mesh Current Method. It accounts for two special cases that are bothersome for the Mesh method. In this article we describe the special cases and show how to deal with them using the Loop method.

The Loop Current Method, just like the Mesh Current Method, is based on Kirchhoff's Voltage Law (KVL).

What we're building to

The two special cases are a non-planar circuit (one that can't be drawn without crossing wires) and a circuit with a current source shared between two meshes.

To analyze circuits like this, you include equations for some non-mesh loops. Make sure every loop includes a circuit element that is not part of any other loop. The steps in the Loop Current Method are otherwise the same as the Mesh Current Method.

Special case: non-planar circuit

The Mesh Current Method defines equations based on meshes. This works for circuits that are planar.

Planar vs. non-planar

- A circuit is planar if it can be drawn on a flat surface without crossing wires. All the schematics you have seen up to now are planar. The schematic below on the left is planar. For planar circuits, we use the Mesh Current Method and write the equations based on meshes. This always works for planar circuits.
- A non-planar circuit is shown below on the right. It has has to be drawn with at least one crossing wire, meaning it cannot be drawn flat. Since there is no way to redraw the circuit to avoid a crossing wire, the circuit on the right is non-planar.

When faced with a non-planar circuit, we must use the Loop Current Method (described below).

Another special case: current source shared by two meshes

A second special case comes up when you have a current source shared between two meshes. This is another time when you may want to include a non-mesh loop in the system of equations.

Both mesh \goldD{\text I}IstartcolorgoldD, I, end colorgoldD and mesh \goldD{\text{II}}IIstartcolorgoldD, I, I, end colorgoldD go through the current source. It is possible (but irksome) to write and solve mesh equations for this configuration. (Try it and see what it's like. It is quite awkward to figure out the voltage at the node above the current source.) This is a time when you might want to use a loop. You can drop one of the meshes and replace it with the loop that goes around both meshes, as shown here for loop \greenD{\text{III}}IIIstartcolorgreenD, I, I, I, end colorgreenD.

You then solve the system of equations exactly the same as the Mesh Current Method.
You may see loop \greenD{\text{III}}IIIstartcolorgreenD, I, I, I, end colorgreenD referred to as a supermesh.

Selecting loops:

We can make a small adjustment to the Mesh Current Method to help us with the two special cases: We allow loops to participate in the equation-building step (not just meshes). This isn't a big deal. When selecting which loops to include:

- Make sure every element is included in a loop or mesh. Every element needs to have a chance to influence the solution.
- Make sure at least one element in each loop is not part of any other loop or mesh. This assures the loop equations are independent.

These rules generate just the right number of independent equations to solve the circuit.

Loop Current Method:

The Loop Current Method is a small variation on the Mesh Current Method. The changes are highlighted in this list of steps.

- Identify the meshes, (the open windows of the circuit) and loops (other closed paths).
- Assign a current variable to each mesh or loop, using a consistent direction (clockwise or counterclockwise).
- Write Kirchhoff's Voltage Law equations around each mesh and loop.
- Solve the resulting system of equations for all mesh and loop currents.
- Solve for any element currents and voltages you want using Ohm's Law.

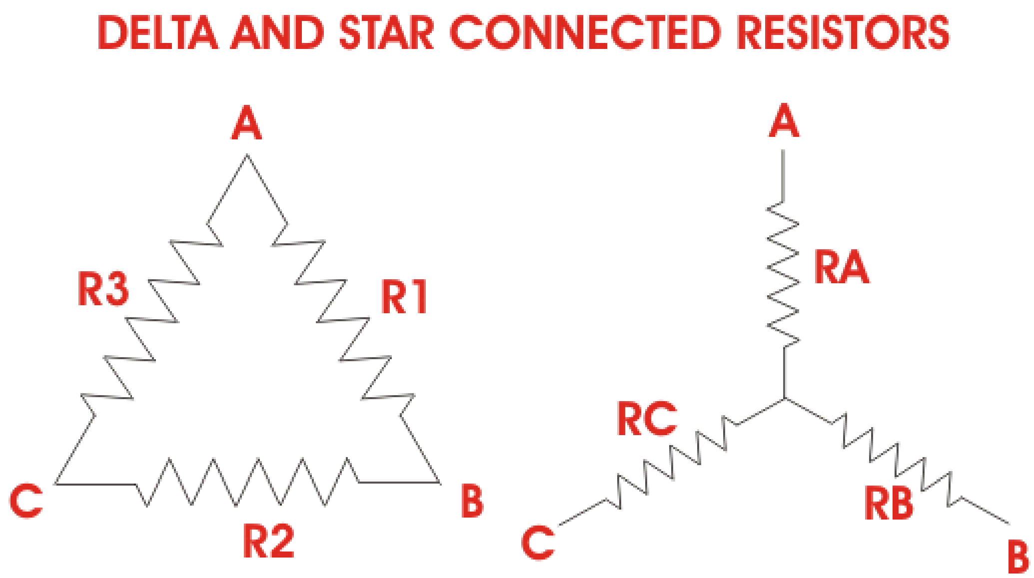
If the circuit is non-planar, or there is a current source shared by two meshes, it is beneficial to switch to the Loop method. Just make sure every loop includes a circuit element that is not parts ofany other loop.

Star To Delta Conversion Formula (Delta to Wye):

Three branches in an electrical network can be connected in numbers of forms but most common among them is either star or delta form. In delta connection, three branches are so connected, that they form a closed loop. As these three branches are connected nose to tail, they form a triangular closed loop, this configuration is referred as delta connection. On the other hand, when either terminal of three branches is connected to a common point to form a Y like pattern is known as star connection. But these star and delta connections can be transformed from one form to another. For simplifying complex network, delta to star or star to delta transformation is often required.

Delta To Star Conversion

The replacement of delta or mesh by equivalent star connection is known as delta – star transformation. The two connections are equivalent or identical to each other if the impedance is measured between any pair of lines. That means, the value of impedance will be the same if it is measured between any pair of lines irrespective of whether the delta is connected between the lines or its equivalent star is connected between that lines.



Consider a delta system that's three corner points are A, B and C as shown in the figure. Electrical resistance of the branch between points A and B, B and C and C and A are R_1, $R_2$and R_3 respectively.

The resistance between the points A and B will be,

$$R_{AB} = R_1||(R_2 + R_3) = \frac{R_1.(R_2 + R_3)}{R_1 + R_2 + R_3}$$

Now, one star system is connected to these points A, B, and C as shown in the figure. Three arms R_A, R_B and R_C of the star system are connected with A, B and C respectively. Now if we measure the resistance value between points A and B, we will get,

$$R_{AB} = R_A + R_B$$

Since the two systems are identical, resistance measured between terminals A and B in both systems must be equal.

$$R_A + R_B = \frac{R_1.(R_2 + R_3)}{R_1 + R_2 + R_3} \cdots\cdots\cdots\cdot(i)$$

Similarly, resistance between points B and C being equal in the two systems,

$$R_B + R_C = \frac{R_2.(R_3 + R_1)}{R_1 + R_2 + R_3} \cdots\cdots\cdots\cdot(ii)$$

And resistance between points C and A being equal in the two systems,

$$R_C + R_A = \frac{R_3.(R_1 + R_2)}{R_1 + R_2 + R_3} \cdots\cdots\cdots\cdot(iii)$$

Adding equations (I), (II) and (III) we get,

$$2(R_A + R_B + R_C) = \frac{2(R_1.R_2 + R_2.R_3 + R_3.R_1)}{R_1 + R_2 + R_3}$$

$$R_A + R_B + R_C = \frac{R_1.R_2 + R_2.R_3 + R_3.R_1}{R_1 + R_2 + R_3}..........(iv)$$

Subtracting equations (I), (II) and (III) from equation (IV) we get,

$$R_A = \frac{R_3.R_1}{R_1 + R_2 + R_3}..................(v)$$

$$R_B = \frac{R_1.R_2}{R_1 + R_2 + R_3}..................(vi)$$

$$R_C = \frac{R_2.R_3}{R_1 + R_2 + R_3}..................(vii)$$

The relation of delta – star transformation can be expressed as follows.
The equivalent star resistance connected to a given terminal, is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta connected resistances.
If the delta connected system has same resistance R at its three sides then equivalent star resistance r will be,

$$r = \frac{R.R}{R + R + R} = \frac{R}{3}$$

Star To Delta Conversion:

For star – delta transformation we just multiply equations (v), (VI) and (VI), (VII) and (VII), (V) that is by doing (v) × (VI) + (VI) × (VII) + (VII) × (V) we get,

$$R_AR_B + R_BR_C + R_CR_A = \frac{R_1.{R_2}^2.R_3 + R_1.R_2.{R_3}^2 + {R_1}^2.R_2.R_3}{(R_1+R_2+R_3)^2}$$

$$= \frac{R_1.R_2.R_3(R_1+R_2+R_3)}{(R_1+R_2+R_3)^2}$$

$$= \frac{R_1.R_2.R_3}{R_1+R_2+R_3} \ldots\ldots\ldots\ldots\ldots(viii)$$

Now dividing equation (VIII) by equations (V), (VI) and equations (VII) separately we get,

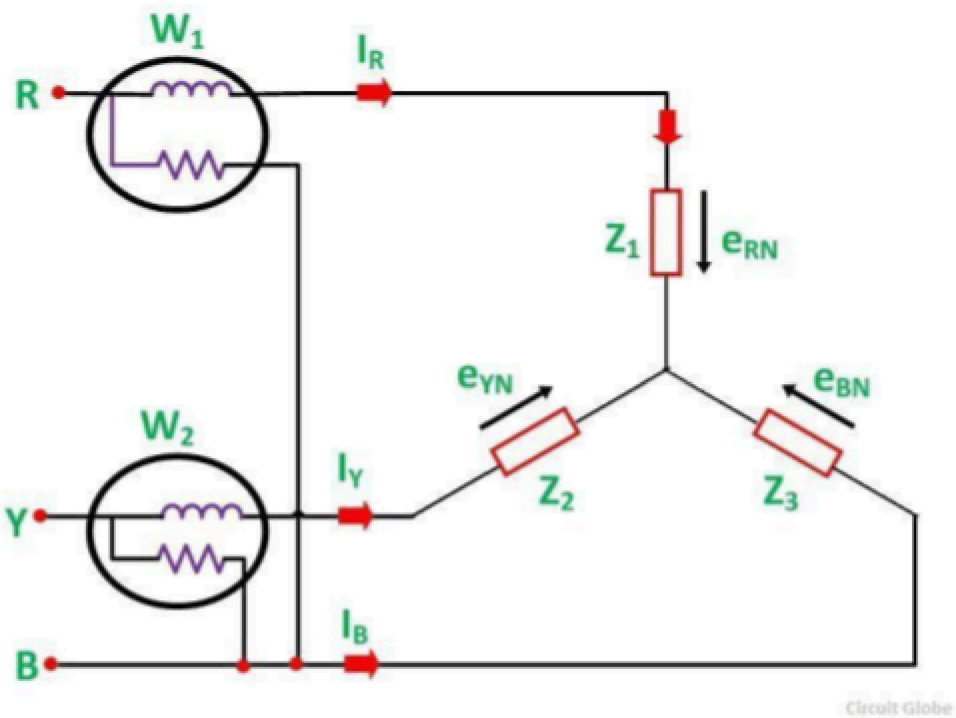
$$R_3 = \frac{R_AR_B + R_BR_C + R_CR_A}{R_A}$$

$$R_1 = \frac{R_AR_B + R_BR_C + R_CR_A}{R_B}$$

$$R_2 = \frac{R_AR_B + R_BR_C + R_CR_A}{R_C}$$

Two wattmeter methods for measurement of three phase power:

Two Wattmeter Method can be employed to measure the power in a 3 phase, three wire star or delta connected the balanced or unbalanced load. In Two wattmeter method the current coils of the wattmeter are connected with any two lines, say R and Y and the potential coil of each wattmeter is joined on the same line, the third line i.e. B as shown below in figure (A).
me line, the third line i.e. B as shown below in figure (A).



The total instantaneous power absorbed by the three loads Z_1, Z_2 and Z_3, are equal to the sum of the powers measured by the Two wattmeters, W_1 and W_2.

Measurement of Power by Two Wattmeter Method in Star Connection:

Considering the above figure (A) in which Two Wattmeter W_1 and W_2 are connected, the instantaneous current through the current coil of Wattmeter, W_1 is given by the equation shown below.

$$W_1 = i_R$$

Instantaneous potential difference across the potential coil of Wattmeter, W_1 is given as

$$W_1 = e_{RN} - e_{BN}$$

Instantaneous power measured by the Wattmeter, W_1 is

$$W_1 = i_R \left(e_{RN} - e_{BN}\right) \quad \ldots\ldots\ldots (1)$$

The instantaneous current through the current coil of Wattmeter, W_2 is given by the equation

$$W_2 = i_Y$$

Instantaneous potential difference across the potential coil of Wattmeter, W_2 is given as

$$W_2 = e_{YN} - e_{BN}$$

Instantaneous power measured by the Wattmeter, W_2 is

$$W_2 = i_Y (e_{YN} - e_{BN}) \quad \ldots\ldots\ldots\ldots(2)$$

Therefore, the Total Power Measured by the Two Wattmeters W_1 and W_2 will be obtained by adding the equation (1) and (2).

$$W_1 + W_2 = i_R (e_{RN} - e_{BN}) + i_Y (e_{YN} - e_{BN})$$

$$W_1 + W_2 = i_R e_{RN} + i_Y e_{YN} - e_{BN} (i_R + i_Y) \quad \text{or}$$

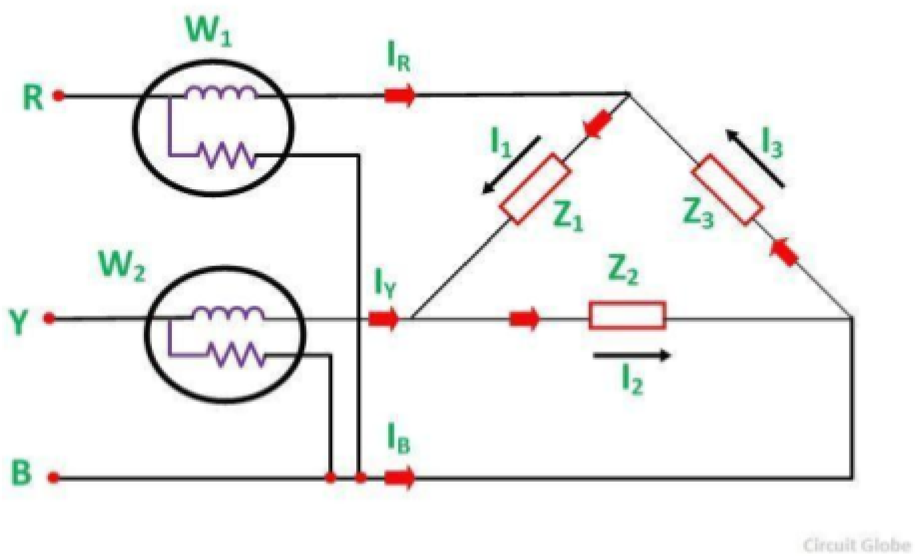
$$W_1 + W_2 = i_R e_{RN} + i_Y e_{YN} + i_B e_{BN} \quad (\text{i.e. } i_R + i_Y + i_B = 0)$$

$$W_1 + W_2 = P$$

Where P – the total power absorbed in the three loads at any instant.

Measurement of Power by Two Wattmeter Method in Delta Connection:

Considering the delta connected circuit shown in the figure below.



The instantaneous current through the coil of the Wattmeter, W_1 is given by the equation

$$W_1 = i_R = i_1 - i_3$$

Instantaneous Power measured by the Wattmeter, W_1 will be

$$W_1 = e_{RB}$$

Therefore, the instantaneous power measured by the Wattmeter, W_1 will be given as

$$W_1 = e_{RB}(i_1 - i_3) \; \ldots\ldots\ldots (3)$$

The instantaneous current through the current coil of the Wattmeter, W_2 is given as

$$W_2 = i_Y = i_2 - i_1$$

The instantaneous potential difference across the potential coil of Wattmeter, W_2 is

$$W_2 = e_{YB}$$

Therefore, the instantaneous power measured by Wattmeter, W_2 will be

$$W_2 = e_{YB}(i_2 - i_1) \; \ldots\ldots\ldots (4)$$

Hence, to obtain the total power measured by the Two Wattmeter the two equations, i.e. equation (3) and (4) has to be added.

$$W_1 + W_2 = e_{RB}(i_1 - i_3) + e_{YB}(i_2 - i_1)$$

$$W_1 + W_2 = i_1 e_{RB} + i_1 e_{YB} - i_3 e_{RB} - i_1 e_{YB}$$

$$W_1 + W_2 = i_2 e_{YB} + i_3 e_{BR} - i_1(e_{YB} + e_{BR}) \qquad (\text{i.e.} -e_{RB} = e_{RB})$$

$$W_1 + W_2 = i_1 e_{RY} + i_2 e_{YB} + i_3 e_{BR} \quad (\text{i.e } e_{RY} + e_{YB} + e_{BR} = 0)$$

$$W_1 + W_2 = P$$

Where P is the total power absorbed in the three loads at any instant.

The power measured by the Two Wattmeter at any instant is the instantaneous power absorbed by the three loads connected in three phases. In fact, this power is the average power drawn by the load since the Wattmeter reads the average power because of the inertia of their moving system.

UNIT-2

TRANSIENT ANALYSIS IN DC CIRCUITS

TRANSIENT RESPONSE OF RL CIRCUITS:

So far we have considered dc resistive network in which currents and voltages were independent of time. More specifically, Voltage (cause input) and current (effect output) responses displayed simultaneously except for a constant multiplicative factor (VR). Two basic passive elements namely, inductor and capacitor are introduced in the dc network. Automatically, the question will arise whether or not the methods developed in lesson-3 to lesson-8 for resistive circuit analysis are still valid. The voltage/current relationship for these two passive elements are defined by the derivative (voltage across the inductor

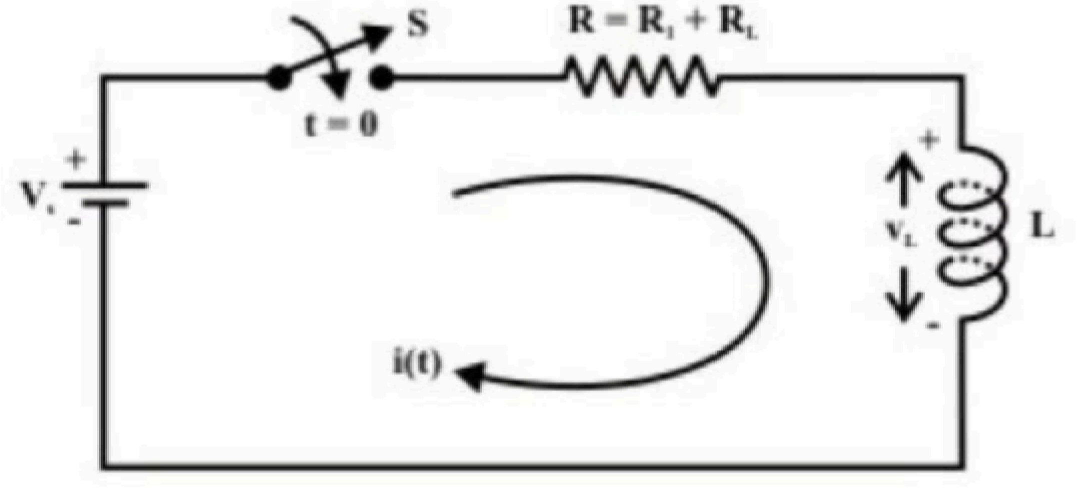
$$v_L(t) = L\frac{di_L(t)}{dt}$$

where =current flowing through the inductor ; current through the capacitor

$$i_C(t) = C\frac{dv_C(t)}{dt},$$

voltage across the capacitor) or in integral form as ()C

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)dt + i_L(0) \quad or \quad v_C(t) = \frac{1}{C}\int_0^t i(t)dt + v_C(0)$$



Our problem is to study the growth of current in the circuit through two stages, namely; (i) dc transient response (ii) steady state response of the system

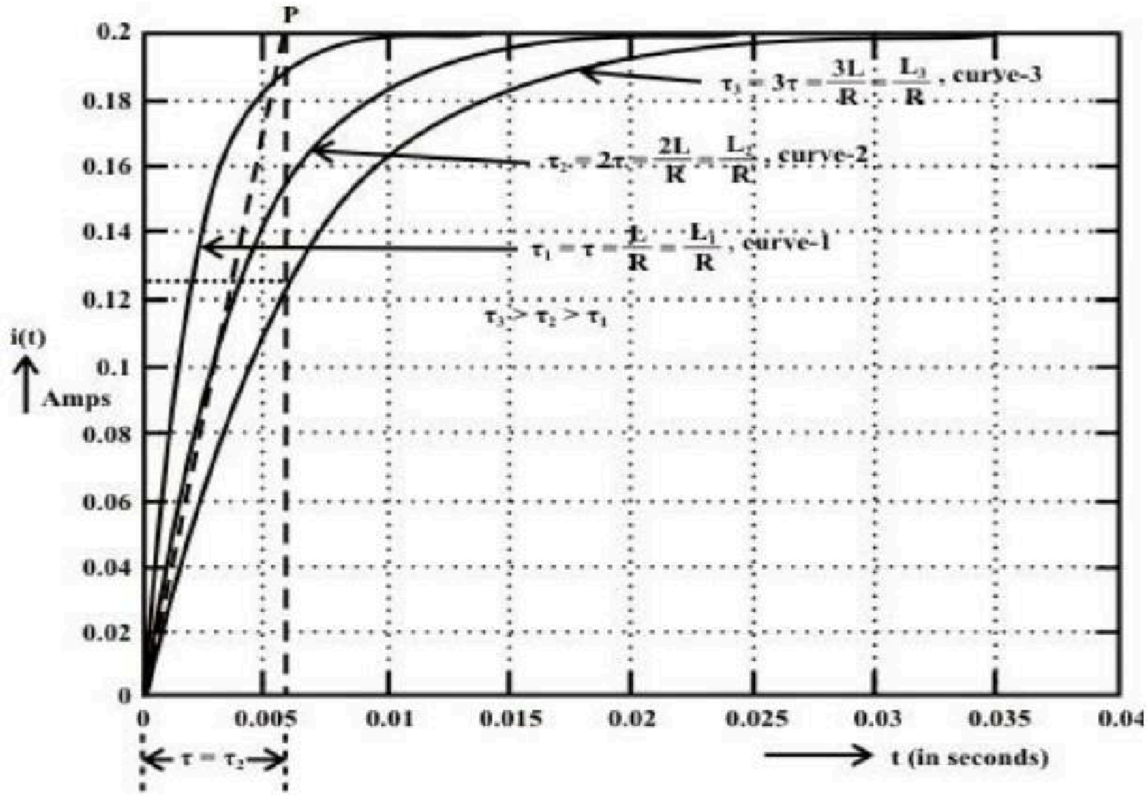
D.C Transients: The behavior of the current and the voltage in the circuit switch is closed until it reaches its final value is called dc transient response of the concerned circuit. The response of a circuit (containing resistances, inductances, capacitors and switches) due to sudden application of voltage or current is called transient response. The most common instance of a transient response in a circuit occurs when a switch is turned on or off –a rather common event in an electric circuit.

Growth or Rise of current in R-L circuit

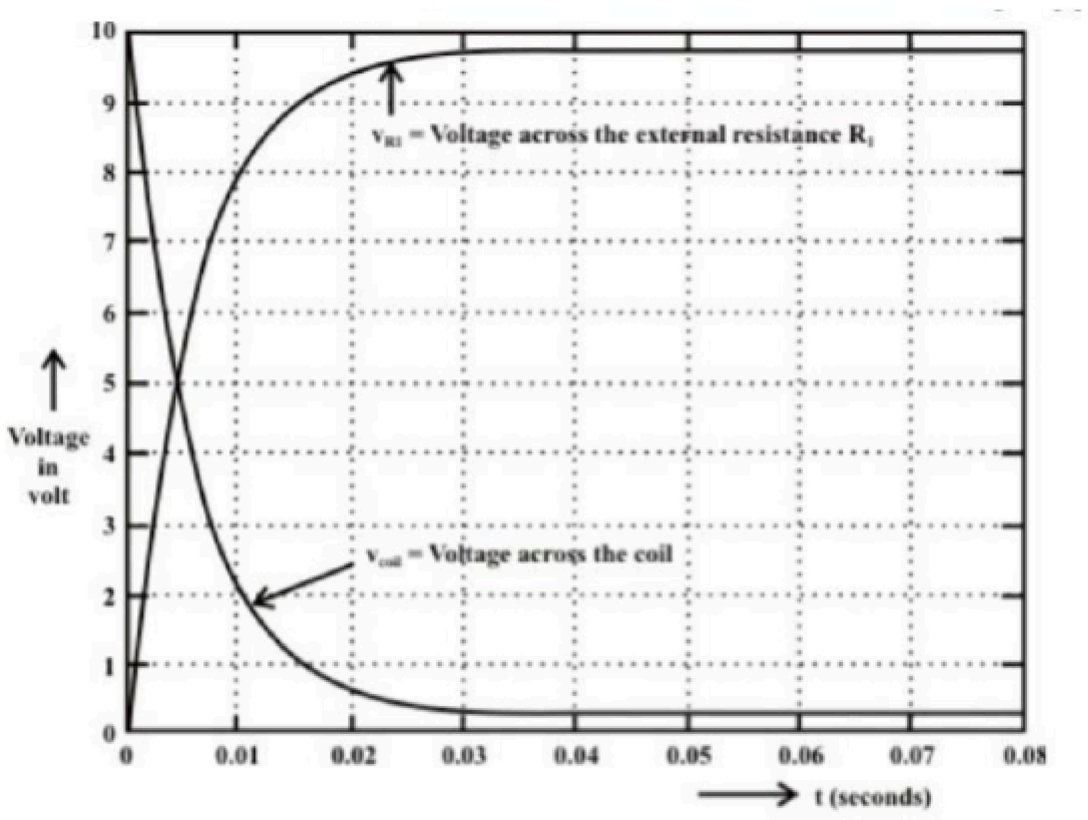
To find the current expression (response) for the circuit shown in fig. 10.6(a), we can write the KVL equation around the circuit The

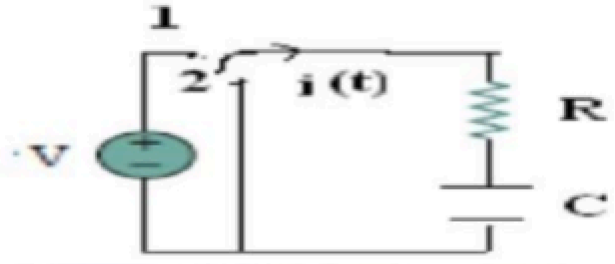
table shows how the current i(t) builds up in a R-L circuit.

Actual time (t) in sec	Growth of current in inductor (Eq.10.15)
$t=0$	$i(0)=0$
$t=\tau\left(=\frac{L}{R}\right)$	$i(\tau)= 0.632 \times\frac{V_s}{R}$
$t=2\tau$	$i(2\tau)= 0.865 \times\frac{V_s}{R}$
$t=3\tau$	$i(3\tau)= 0.950 \times\frac{V_s}{R}$
$t=4\tau$	$i(4\tau)= 0.982 \times\frac{V_s}{R}$
$t=5\tau$	$i(5\tau)= 0.993 \times\frac{V_s}{R}$



P
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
i(t)
Amps
$\tau_3 = 3\tau = \frac{3L}{R} = \frac{L_3}{R}$, curve-3
$\tau_2 = 2\tau = \frac{2L}{R} = \frac{L_2}{R}$, curve-2
$\tau_1 = \tau = \frac{L}{R} = \frac{L_1}{R}$, curve-1
$\tau_3 > \tau_2 > \tau_1$
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
$\tau = \tau_2$
t (in seconds)





Consider network shown in fig. the switch k is moved from position 1 to 2 at reference time t = 0.

Now before switching take place, the capacitor C is fully charged to V volts and it discharges through resistance R. As time passes, charge and hence voltage across capacitor i.e. Vc decreases gradually and hence discharge current also decreases gradually from maximum to zero exponentially.

After switching has taken place, applying kirchoff's voltage law,

$$0 = V_R + V_C$$

Where VR is voltage across resistor and VC is voltage across capacitor.

$$\therefore V_C = -V_R = -i \cdot R$$

$$i = C\frac{dv_C}{dt}$$

$$\therefore V_C = -R \cdot C \cdot \frac{dv_C}{dt}$$

Above equation is linear, homogenous first order differential equation. Hence rearranging we have,

$$\frac{dt}{RC} = -\frac{dv_C}{V_C}$$

Integrating both sides of above equation we have

$$\frac{t}{RC} = -\ln V_C + k'$$

Now at t = 0, VC =V which is initial condition, substituting in equation we have,

$$\therefore 0 = -\ln V + k'$$

$$\therefore k' = \ln V$$

Substituting value of k' in general solution, we have

$$\frac{t}{RC} = -\ln V_C + \ln V$$

$$\therefore \frac{t}{RC} = \ln\frac{V}{V_C}$$

$$\therefore \frac{V}{V_C} = e^{\frac{t}{RC}}$$

$$\therefore V_C = V \cdot e^{-\frac{t}{RC}}$$

$$V = \frac{Q}{C}$$

Where Q is total charge on capacitor

Similarly at any instant, VC = q/c where q is instantaneous charge.

So we have, $\frac{q}{c} = \frac{Q}{C}e^{-\frac{t}{RC}}$

$$q = Q \cdot e^{-\frac{t}{RC}}$$

Thus charge behaves similarly to voltage across capacitor.
Now discharging current i is given by

$$i = \frac{V_C}{R}$$

but $V_R = V_C$ when there is no source in circuit.

$$\therefore i = \frac{V_C}{R}$$

$$\therefore i = \frac{V}{R} e^{-\frac{t}{RC}}$$

but VR = VC when there is no source in circuit.

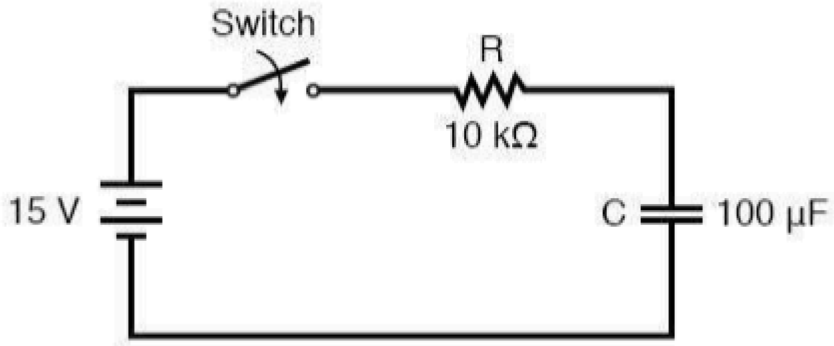
The above expression is nothing but discharge current of capacitor. The variation of this current with respect to time is shown in fig.

This shows that the current is exponentially decaying. At point P on the graph. The current value is (0.368) times its maximum value. The characteristics of decay are determined by values R and C, which are 2 parameters of network.

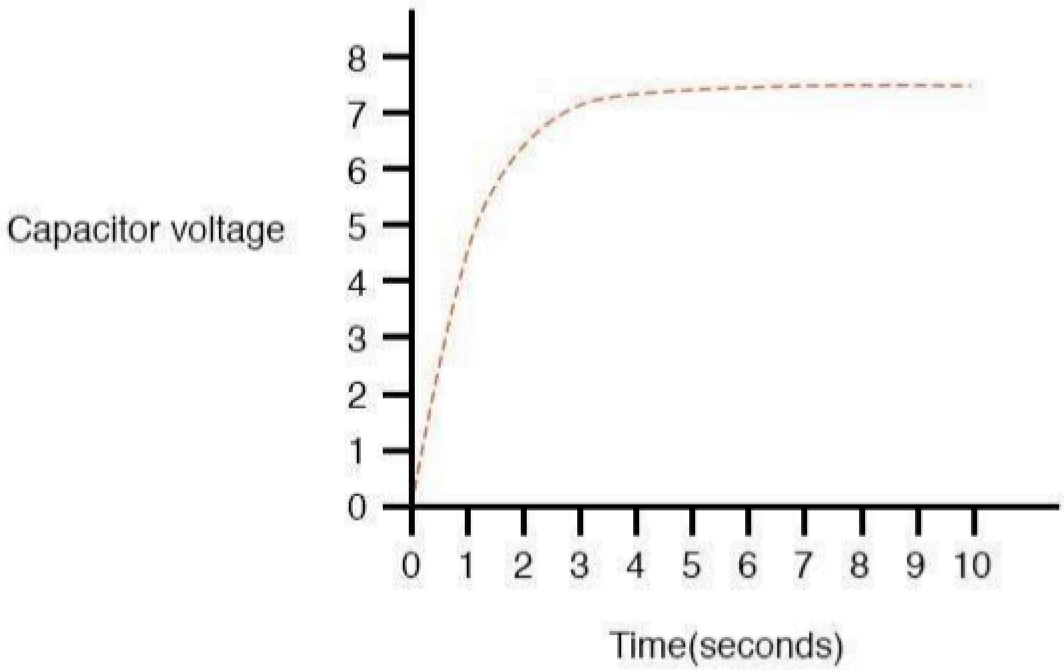
For this network, after the instant t = 0, there is no driving voltage source in circuit, hence it is called undriven RC circuit.

Capacitor Transient Response:

capacitors store energy in the form of an electric field, they tend to act like small secondary-cell batteries, being able to store and release electrical energy. A fully discharged capacitor maintains zero volts across its terminals, and a charged capacitor maintains a steady quantity of voltage across its terminals, just like a battery. When capacitors are placed in a circuit with other sources of voltage, they will absorb energy from those sources, just as a secondary-cell battery will become charged as a result of being connected to a generator. A fully discharged capacitor, having a terminal voltage of zero, will initially act as a short-circuit when attached to a source of voltage, drawing maximum current as it begins to build a charge. Over time, the capacitor's terminal voltage rises to meet the applied voltage from the source, and the current through the capacitor decreases correspondingly. Once the capacitor has reached the full voltage of the source, it will stop drawing current from it, and behave essentially as an open-circuit.



When the switch is first closed, the voltage across the capacitor (which we were told was fully discharged) is zero volts; thus, it first behaves as though it were a short-circuit. Over time, the capacitor voltage will rise to equal battery voltage, ending in a condition where the capacitor behaves as an open-circuit. Current through the circuit is determined by the difference in voltage between the battery and the capacitor, divided by the resistance of 10 kΩ. As the capacitor voltage approaches the battery voltage, the current approaches zero. Once the capacitor voltage has reached 15 volts, the current will be exactly zero. Let's see how this works using real values:



The capacitor voltage's approach to 15 volts and the current's approach to zero over time is what a mathematician would describe as asymptotic: that is, they both approach their final values, getting closer and closer over time, but never exactly reach their destinations. For all practical purposes, though, we can say that the capacitor voltage will eventually reach 15 volts and that the current will eventually equal zero. Using the SPICE circuit analysis program, we can chart this asymptotic buildup of capacitor voltage and decay of capacitor current in a more graphical form (capacitor current is plotted in terms of voltage drop across the resistor, using the resistor as a shunt to measure current):

Transient Response of RLC Circuits:

In the preceding lesson, our discussion focused extensively on dc circuits having resistances with either inductor () or capacitor () (i.e., single storage element) but not both. Dynamic response of such first order system has been studied and discussed in detail. The presence of resistance, inductance, and capacitance in the dc circuit introduces at least a second order differential equation or by two simultaneous coupled linear first order differential equations. We shall see in next section that the complexity of analysis of second order circuits increases significantly when compared with that encountered with first order circuits. Initial conditions for the circuit variables and their derivatives play an important role and this is very crucial to analyze a second order dynamic system.

Response of a series R-L-C circuit

Consider a series RLcircuit as shown in fig.11.1, and it is excited with a dc voltage source C−−sV.

Applying around the closed path for ,

$$L\frac{di(t)}{dt}+Ri(t)+v_c(t)=V_s$$

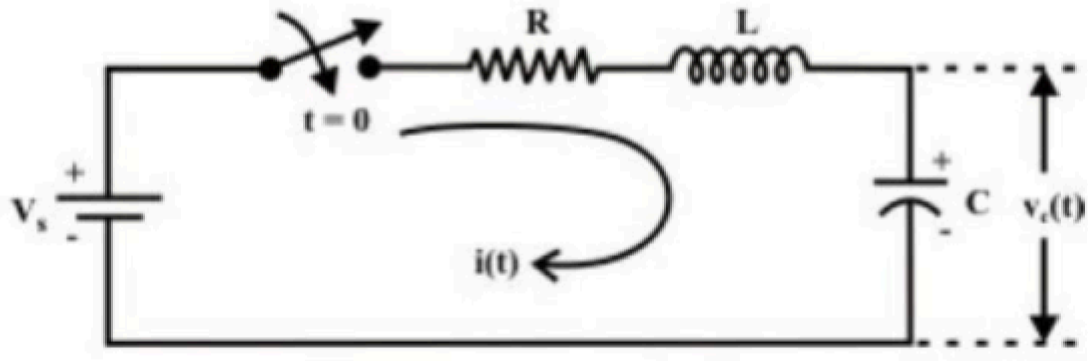


Fig. 11.1: A Simple R-L-C circuit excited with a dc voltage source

The current through the capacitor can be written as Substituting the current ‘’expression in eq.(11.1) and rearranging the terms,

$$i(t) = C\frac{dv_c(t)}{dt}$$

$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

The above equation is a 2nd-order linear differential equation and the parameters associated with the differential equation are constant with time. The complete solution of the above differential equation has two components; the transient response and the steady state response. Mathematically, one can write the complete solution as

$$v_c(t) = v_{cn}(t) + v_{cf}(t) = \left(A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}\right) + A$$

$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = 0 \Rightarrow \frac{d^2v_c(t)}{dt^2} + \frac{R}{L}\frac{dv_c(t)}{dt} + \frac{1}{LC}v_c(t) = 0$$

$$a\,\frac{d^2v_c(t)}{dt^2} + b\,\frac{dv_c(t)}{dt} + c\,v_c(t) = 0 \text{ (where } a = 1,\ b = \frac{R}{L} \text{ and } c = \frac{1}{LC})$$

Since the system is linear, the nature of steady state response is same as that of forcing function (input voltage) and it is given by a constant value. Now, the first part of the total response is completely dies out with time while and it is defined as a transient or natural response of the system. The natural or transient response (see Appendix in Lesson-10) of second order differential equation can be obtained from the homogeneous equation (i.e., from force free system) that is expressed by

$$\alpha^2 + \frac{R}{L}\alpha + \frac{1}{LC} = 0 \quad \Rightarrow a\alpha^2 + b\alpha + c = 0 \text{ (where } a = 1,\ b = \frac{R}{L} \text{ and } c = \frac{1}{LC})$$

and solving the roots of this equation (11.5) on that associated with transient part of the complete solution (eq.11.3) and they are given below.

$$\alpha_1 = \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right) = \left(-\frac{b}{2a} + \frac{1}{a}\sqrt{\left(\frac{b}{2}\right)^2 - ac}\right);$$

$$\alpha_2 = \left(-\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right) = \left(-\frac{b}{2a} - \frac{1}{a}\sqrt{\left(\frac{b}{2}\right)^2 - ac}\right)$$

where, $b = \frac{R}{L}$ *and* $c = \frac{1}{LC}$.

The roots of the characteristic equation are classified in three groups depending upon the values of the parameters „RLand of the circuit

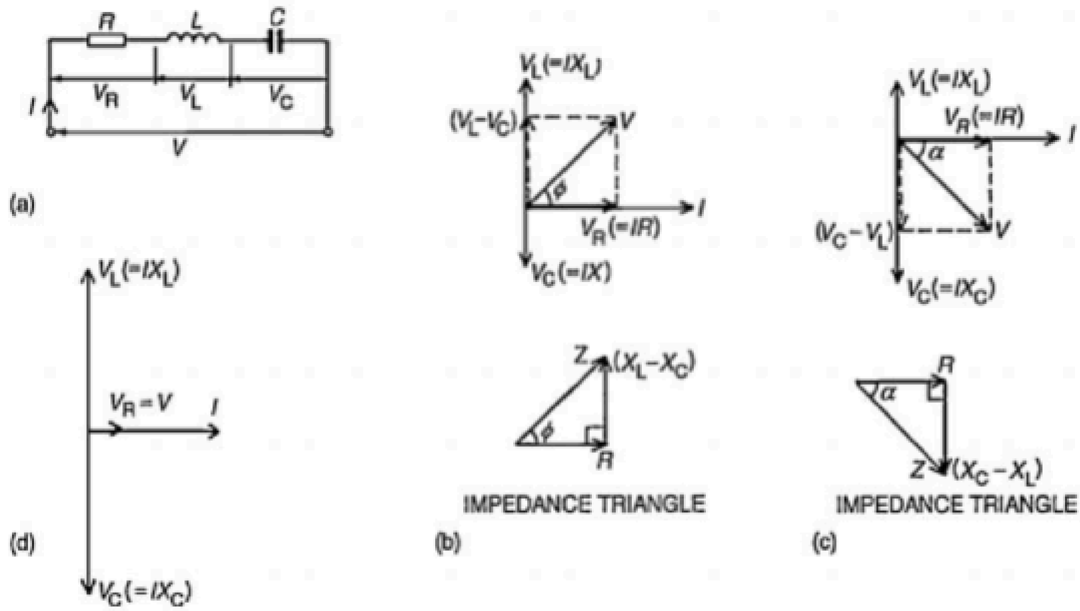
Case-A (overdamped response): That the roots are distinct with negative real parts. Under this situation, the natural or transient part of the complete solution is written as

$$v_{cn}(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

and each term of the above expression decays exponentially and ultimately reduces to zero as and it is termed as overdamped response of input free system. A system that is overdamped responds slowly to any change in excitation. It may be noted that the exponential term t→∞11tAeαtakes longer time to decay its value to zero than the term21tAeα. One can introduce a factorξ that provides an information about the speed of system response and it is defined by damping ratio

$$(\xi) = \frac{Actual\ damping}{critical\ damping} = \frac{b}{2\sqrt{ac}} = \frac{R/L}{2/\sqrt{LC}} > 1$$

RLC Circuit:



Consider a circuit in which R, L, and C are connected in series with each other across ac supply as shown in fig.

The ac supply is given by, V = Vm sin wt
The circuit draws a current I. Due to that different voltage drops are,

1. Voltage drop across Resistance R is VR = IR

2. Voltage drop across Inductance L is VL = IXL

3. Voltage drop across Capacitance C is Vc = IXcThe characteristics of three drops are,

(i) VR is in phase with current I

(ii) VL leads I by 900

(iii) Vc lags I by 900

According to krichoff's laws

Steps to draw phasor diagram:

1. Take current I as reference
2. VR is in phase with current I
3. VL leads current by 900
4. Vc lags current by 900
5. obtain resultant of VL and Vc. Both VL and Vc are in phase opposition (1800 out of phase)
6. Add that with VRby law of parallelogram to get supply voltage.

The phasor diagram depends on the condition of magnitude of VL and Vc which ultimately depends on values of XL and Xc.
Let us consider different cases:

Case(i): XL >Xc
When X L >Xc
Also VL >Vc (or) IXL >IXc
So, resultant of VL and Vc will directed towards VL i.e. leading current I. Hence I lags V i.e. current I will lags the resultant of VL and Vc i.e. (V L - Vc). The circuit is said to be inductive in nature.
From voltage triangle,

$$V = \sqrt{(V_R^2 + (V_L - Vc)^2)} = \sqrt{((IR)^2 + (IX_L - IXc)^2)}$$
$$V = I\sqrt{(R^2 + (X_L - Xc)^2)}$$
$$V = IZ$$
$$Z = \sqrt{(R^2 + (X_L - Xc)^2)}$$

If , V = Vm Sin wt ; i = Im Sin (wt - ϕ) i.e I lags V by angle ϕ

Case(ii): $X_L < X_c$

When XL <Xc
Also VL <Vc (or) IXL <IXc

Hence the resultant of VL and Vc will directed towards Vci.e current is said to be capacitive in nature
Form voltage triangle

$$V = \sqrt{(V_R^2 + (V_c - V_L)^2)} = \sqrt{((IR)^2 + (IX_c - IX_L)^2)}$$

$$V = I\sqrt{(R^2 + (X_c - X_L)^2)}$$

$$V = IZ$$

$$Z = \sqrt{(R^2 + (Xc - X_L)^2)}$$

If , V = Vm Sin wt ; i = Im Sin (wt + ϕ)

i.e I lags V by angle ϕ

i.e I lags V by angle ϕ

Case(iii): XL = Xc
When XL = Xc
Also VL = Vc (or) IXL = IXc

So VL and Vc cancel each other and the resultant is zero. So V = VR in such a case, the circuit is purely resistive in nature.

Impedance:

In general for RLC series circuit impedance is given by, Z = R + j X
X = XL –Xc = Total reactance of the circuit

If XL >Xc ; X is positive & circuit is Inductive

If XL <Xc ; X is negative & circuit is Capacitive

If XL = Xc ; X =0 & circuit is purely Resistive

Tan ϕL - =Xc[(X)/R]
Cos ϕ = [R/Z]
Z = 2 √+(X L(R-Xc) 2)

Impedance triangle:

In both cases R = Z Cos ϕ

X = Z Sin ϕ

Power and power triangle:

The average power consumed by circuit is,

Pavg = (Average power consumed by R) + (Average power consumed by L) + (Average power consumed by C)

Pavg = Power taken by R = I2R = I(IR) = VI

V = V Cos ϕ P = VI Cos ϕ

Thus, for any condition, $XL > Xc$ or $XL < Xc$ General power can be expressed as

P = Voltage x Component in phase with voltage

Power triangle:

S = Apparent power = I2Z = VI

P = Real or True power = VI Cos ϕ = Active po Q = Reactive power = VI Sin ϕ

Analyze an RLC Circuit Using Laplace Methods:

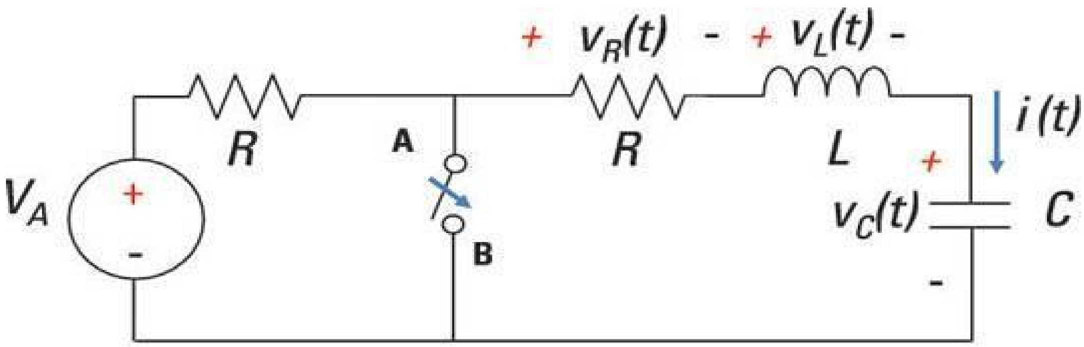
Using the Laplace transform as part of your circuit analysis provides you with a prediction of circuit response. Analyze the poles of the Laplace transform to get a general idea of output behavior. Real poles, for instance, indicate exponential output behavior.

Follow these basic steps to analyze a circuit using Laplace techniques:

1. Develop the differential equation in the time-domain using Kirchhoff's laws and element equations.
2. Apply the Laplace transformation of the differential equation to put the equation in the s-domain.
3. Algebraically solve for the solution, or response transform.
4. Apply the inverse Laplace transformation to produce the solution to the original differential equation described in the time-domain.

To get comfortable with this process, you simply need to practice applying it to different types of circuits such as an RC (resistor-capacitor) circuit, an RL (resistor-inductor) circuit, and an RLC (resistor-inductor-capacitor) circuit.

Here you can see an RLC circuit in which the switch has been open for a long time. The switch is closed at time t = 0.



$R = 800\ \Omega$ $\qquad C = \frac{1}{4.1} \bullet 10^{-5}\ \text{F} = 2.439\ \mu\text{F}$

$L = 1\ \text{H}$ $\qquad V_A = 5\ \text{V}$

In this circuit, you have the following KVL equation:

$v_R(t) + v_L(t) + v(t) = 0$

Next, formulate the element equation (or i-v characteristic) for each device. Ohm's law describes the voltage across the resistor (noting that $i(t) = i_L(t)$ because the circuit is connected in series, where $I(s) = I_L(s)$ are the Laplace transforms):

$v_R(t) = i(t)R$

The inductor's element equation is given by

$$v_L(t) = L\frac{di_L(t)}{dt}$$

And the capacitor's element equation is

$$v_c(t) = \frac{1}{C}\int_0^t i(\tau) + v_c(0)$$

Here, $v_C(0) = V_0$ is the initial condition, and it's equal to 5 volts.

Substituting the element equations, $v_R(t)$, $v_C(t)$, and $v_L(t)$, into the KVL equation gives you the following equation (with a fancy name: the integro-differential equation):

$$L\frac{di_L(t)}{dt} + i_L(t)R + \frac{1}{C}\int_0^t i(\tau) + v_c(0) = 0$$

The next step is to apply the Laplace transform to the preceding equation to find an $I(s)$ that satisfies the integro-differential equation for a given set of initial conditions:

$$\mathcal{L}\left[L\frac{di_L(t)}{dt} + i(t)R + \frac{1}{C}\int_0^t i(\tau)d\tau + V_0\right] = 0$$

$$\mathcal{L}\left[L\frac{di_L(t)}{dt}\right] + \mathcal{L}[i(t)R] + \mathcal{L}\left[\frac{1}{C}\int_0^t i(\tau)d\tau + V_0\right] = 0$$

The preceding equation uses the linearity property allowing you to take the Laplace transform of each term. For the first term on the left side of the equation, you use the differentiation property to get the following transform:

$$\mathcal{L}\left[L\frac{di(t)}{dt}\right] = L\left[sI(s) - I_0\right]$$

This equation uses $I_L(s) = \mathcal{L}[i(t)]$, and I_0 is the initial current flowing through the inductor. Because the switch is open for a long time, the initial condition I_0 is equal to zero.

For the second term of the KVL equation dealing with resistor R, the Laplace transform is simply

$$\mathcal{L}[i(t)R] = I(s)R$$

For the third term in the KVL expression dealing with capacitor C, you have

$$\mathcal{L}\left[\frac{1}{C}\int_0^t i(\tau)d\tau + V_0\right] = \frac{I(s)}{sC} + \frac{V_0}{s}$$

The Laplace transform of the integro-differential equation becomes

$$L\left[sI(s) - I_0\right] + I(s)R + \frac{I(s)}{sC} + \frac{V_0}{s} = 0$$

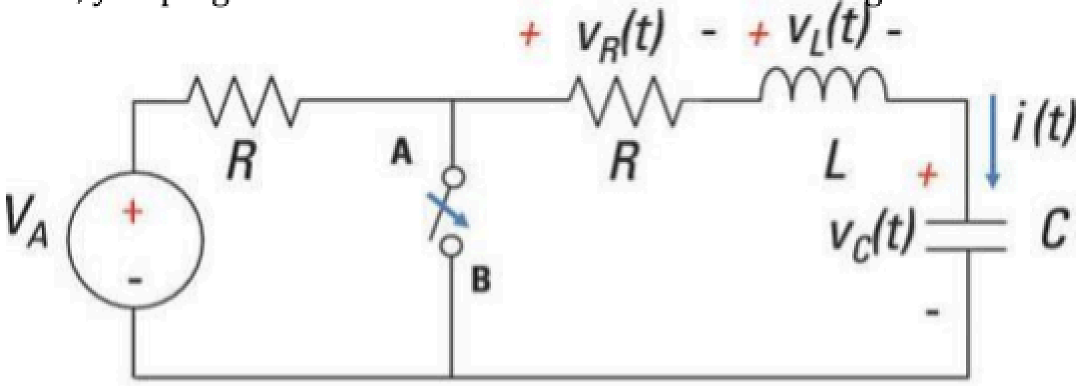
Rearrange the equation and solve for I(s):

$$I(s) = \frac{sI_0 - \frac{V_0}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

To get the time-domain solution i(t), use the following table, and notice that the preceding equation has the form of a damping sinusoid.

Signal Description	Time-Domain Waveform, f(t)	s-Domain Waveform, F(s)
Step	$u(t)$	$\frac{1}{s}$
Exponential	$[e^{-\alpha t}]u(t)$	$\frac{1}{s+\alpha}$
Impulse	$\delta(t)$	1
Ramp, *r(t)*	$tu(t)$	$\frac{1}{s^2}$
Sine	$[\sin\beta t]u(t)$	$\frac{\beta}{s^2+\beta^2}$
Cosine	$[\cos\beta t]u(t)$	$\frac{s}{s^2+\beta^2}$
Damped Pairs		
Damped ramp	$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$
Damped sine	$[e^{-\alpha t}\sin\beta t]u(t)$	$\frac{\beta}{(s+\alpha)^2+\beta^2}$
Damped cosine	$[e^{-\alpha t}\cos\beta t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$

Now, you plug in $I_0 = 0$ and some numbers from this figure:



$R = 800\ \Omega$ $\quad C = \frac{1}{4.1} \cdot 10^{-5}\ \text{F} = 2.439\ \mu\text{F}$

$L = 1\ \text{H}$ $\quad V_A = 5\ \text{V}$

Now you've got this equation:

$$I(s) = -\frac{5}{s^2+800s+401\cdot 10^5}$$

$$= -\frac{5}{500}\left[\frac{500}{(s+400)^2+(500)^2}\right]$$

You wind up with the following solution:

$i(t) = [-0.01e^{-400t}\sin 500t]u(t)$

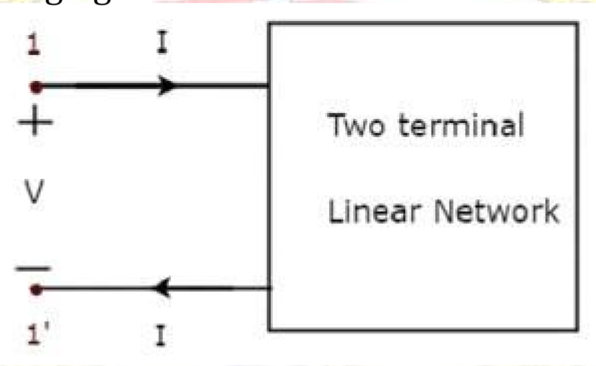
For this RLC circuit, you have a damping sinusoid. The oscillations will die out after a long period of time. For this example, the time constant is 1/400 and will die out after $5/400 = 1/80$ seconds.

UNIT-4

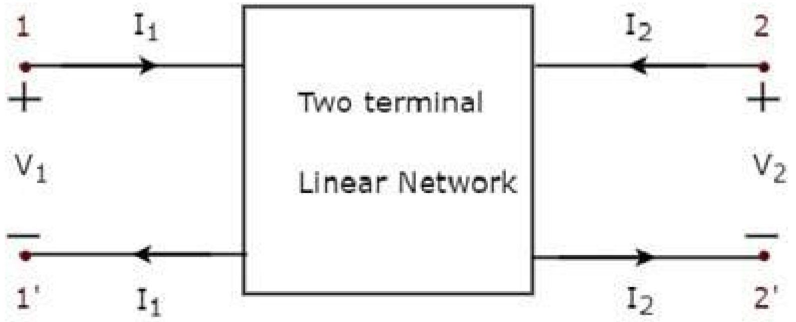
TWO PORT NETWORK

Two port network parameters:

In general, it is easy to analyze any electrical network, if it is represented with an equivalent model, which gives the relation between input and output variables. For this, we can use two port network representations. As the name suggests, two port networks contain two ports. Among which, one port is used as an input port and the other port is used as an output port. The first and second ports are called as port1 and port2 respectively.
One port network is a two terminal electrical network in which, current enters through one terminal and leaves through another terminal. Resistors, inductors and capacitors are the examples of one port network because each one has two terminals. One port network representation is shown in the following figure.



Here, the pair of terminals, 1 & 1' represents a port. In this case, we are having only one port since it is a one port network.
Similarly, two port network is a pair of two terminal electrical network in which, current enters through one terminal and leaves through another terminal of each port. Two port network representation is shown in the following figure.



Here, one pair of terminals, 1 & 1' represents one port, which is called as port1 and the other pair of terminals, 2 & 2' represents another port, which is called as port2.

There are four variables V_1, V_2, I_1 and I_2 in a two port network as shown in the figure. Out of which, we can choose two variables as independent and another two variables as dependent. So, we will get six possible pairs of equations. These equations represent the dependent variables in terms of independent variables. The coefficients of independent variables are called as parameters. So, each pair of equations will give a set of four parameters.

Two Port Network Parameters:

The parameters of a two port network are called as two port network parameters or simply, two port parameters. Following are the types of two port network parameters.

- Z parameters
- Y parameters
- G parameters
- H parameters
- ABCD parameters

Now, let us discuss about these two port network parameters one by one.

Z parameters:

We will get the following set of two equations by considering the variables V_1& V_2 as dependent and I_1 & I_2 as independent. The coefficients of independent variables, I_1 and I_2 are called as Z parameters.

$$V1=Z11I1+Z12I2V1=Z11I1+Z12I2$$

$$V2=Z21I1+Z22I2V2=Z21I1+Z22I2$$

The Z parameters are

$$Z11=V1I1,whenI2=0Z11=V1I1,whenI2=0$$
$$Z12=V1I2,whenI1=0Z12=V1I2,whenI1=0$$
$$Z21=V2I1,whenI2=0Z21=V2I1,whenI2=0$$
$$Z22=V2I2,whenI1=0Z22=V2I2,whenI1=0$$

Z parameters are called as impedance parameters because these are simply the ratios of voltages and currents. Units of Z parameters are Ohm (Ω).

We can calculate two Z parameters, Z_{11} and Z_{21}, by doing open circuit of port2. Similarly, we can calculate the other two Z parameters, Z_{12} and Z_{22} by doing open circuit of port1. Hence, the Z parameters are also called as open-circuit impedance parameters.

Y parameters:

We will get the following set of two equations by considering the variables I_1 & I_2 as dependent and V_1 & V_2 as independent. The coefficients of independent variables, V_1 and V_2 are called as Y parameters.

I1=Y11V1+Y12V2I1=Y11V1+Y12V2

I2=Y21V1+Y22V2I2=Y21V1+Y22V2

The Y parameters are

Y11=I1V1,whenV2=0Y11=I1V1,whenV2=0
Y12=I1V2,whenV1=0Y12=I1V2,whenV1=0
Y21=I2V1,whenV2=0Y21=I2V1,whenV2=0
Y22=I2V2,whenV1=0Y22=I2V2,whenV1=0

Y parameters are called as admittance parameters because these are simply, the ratios of currents and voltages. Units of Y parameters are mho. We can calculate two Y parameters, Y_{11} and Y_{21} by doing short circuit of port2. Similarly, we can calculate the other two Y parameters, Y_{12} and Y_{22} by doing short circuit of port1. Hence, the Y parameters are also called as short-circuit admittance parameters.

Hybrid Parameters or h Parameters:

Hybrid parameters (also known as h parameters) are known as 'hybrid' parameters as they use Z parameters, Y parameters, voltage ratio, and current ratios to represent the relationship between voltage and current in a two port network.

H parameters are useful in describing the input-output characteristics of circuits where it is hard to measure Z or Y parameters (such as a transistor). H parameters encapsulate all the important linear characteristics of the circuit, so they are very useful for simulation purposes. The relationship between voltages and current in h parameters can be represented as:

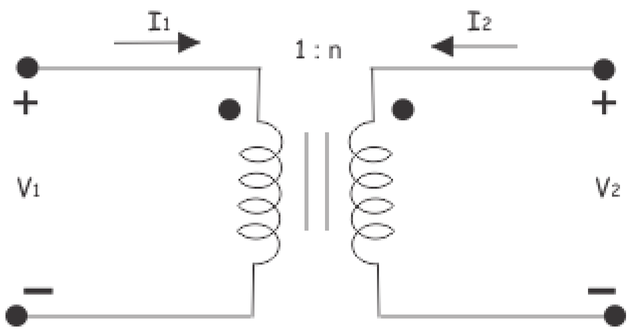
$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

This can be represented in matrix form as:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

To illustrate where h parameters are useful, take the case of an ideal transformer, where Z parameters cannot be used. Since here, the relations between voltages and current in that ideal transformer would be,

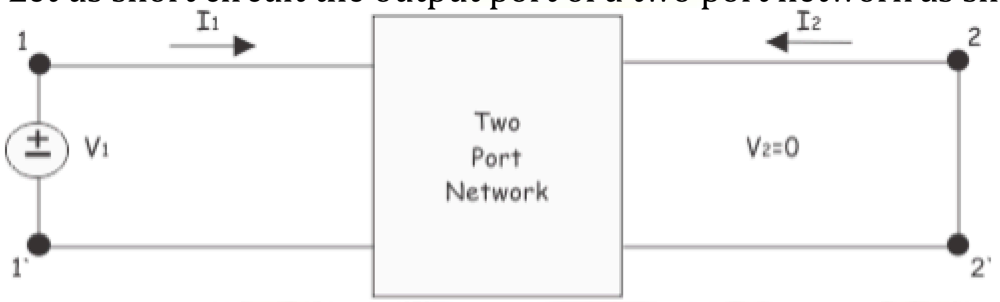
$$V_1 = \frac{1}{n}V_2 \; and \; I_1 = -nI_2$$



Since, in an ideal transformer voltagescan not be expressed in terms of current, it is impossible to analyze a transformer with Z parameters because a transformer does not have Z parameters. The problem can be solved by using hybrid parameters (i.e. h parameters).

Determining h Parameters:

Let us short circuit the output port of a two port network as shown below,



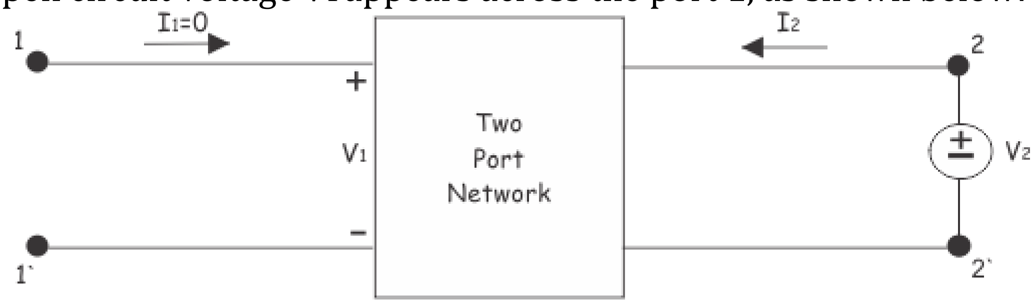
Now, ratio of input voltage to input current, at short circuited output port is:

$$\left.\frac{V_1}{I_1}\right|_{V_2=0} = h_{11}$$

This is referred to as the short circuit input impedance. Now, the ratio of the output current to input current at the short-circuited output port is:

$$\left.\frac{I_2}{I_1}\right|_{V_2=0} = h_{21}$$

This is called short-circuit current gain of the network. Now, let us open circuit the port 1. At that condition, there will be no input current ($I_1=0$) but open circuit voltage V_1 appears across the port 1, as shown below:



Now:

$$\left.\frac{V_1}{V_2}\right|_{I_1=0} = h_{12} = open\ circuit\ reverse\ voltage\ gain$$

This is referred as reverse voltage gain because, this is the ratio of input voltage to the output voltage of the network, but voltage gain is defined as the ratio of output voltage to the input voltage of a network.
Now:

$$\left.\frac{I_2}{V_2}\right|_{I_1 = 0} = h_{21}$$

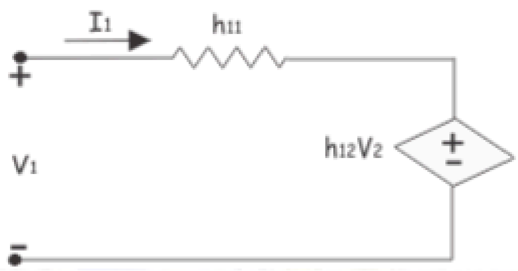
It is referred as open circuit output admittance.

h Parameter Equivalent Network of Two Port Network:

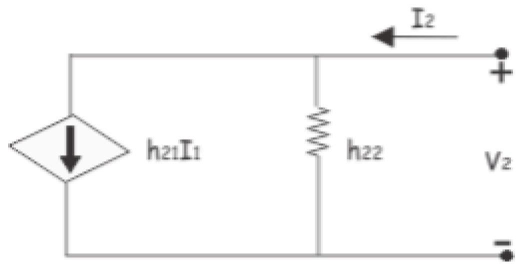
To draw h parameter equivalent network of a two port network, first we have to write the equation of voltages and currents using h parameters. These are:

$$V_1 = h_{11}I_1 + h_{12}V_2 \cdots\cdots\cdots\cdots\cdots (i)$$
$$I_2 = h_{21}I_1 + h_{22}V_2 \cdots\cdots\cdots\cdots\cdots (ii)$$

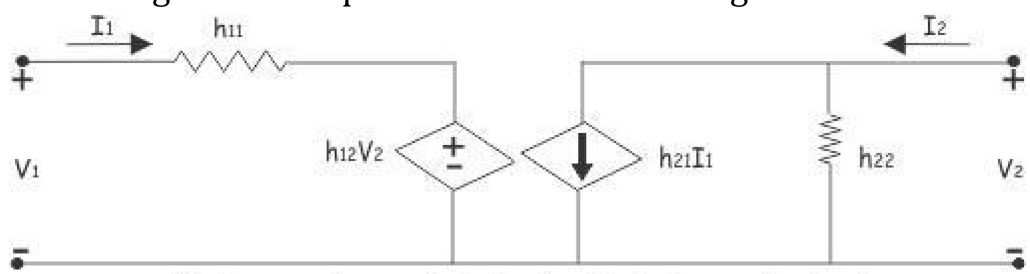
Equation (i) can be represented as a circuit based on Kirchhoff Voltage Law:



Equation (ii) can be represented as a circuit based on Kirchhoff Current Law:



Combining these two parts of the network we get:



The h parameters equivalent network of a two port network

Inverse Hybrid Parameters or g Parameters:

There is another set of parameters which is closely related to the h parameters. These parameters are called inverse hybrid parameters or g parameters. The relations between currents and voltages with g parameters are represented as:

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

In matrix form:

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

where:

$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2 = 0} = open\ circuit\ input\ admittance$$

$$g_{12} = \frac{I_1}{I_2}\Big|_{V_1 = 0} = short\ circuit\ reverse\ current\ gain$$

$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2 = 0} = open\ circuit\ voltage\ gain$$

$$g_{22} = \frac{V_2}{I_2}\Big|_{V_1 = 0} = short\ circuit\ output\ impedance$$

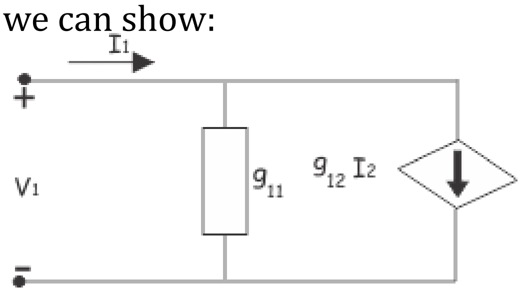
G Parameter Equivalent Network of Two Port Network:

g parameters can be obtained in a similar way to h parameters. The equivalent circuit of two port network using g parameter can be constructed.

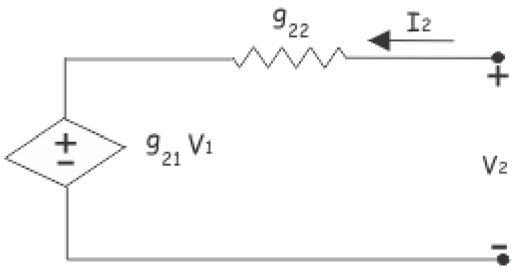
The relations between currents and voltages in g parameter are:

$$I_1 = g_{11}V_1 + g_{12}I_2 \cdots\cdots\cdots\cdots\cdots (iii)$$
$$V_2 = g_{21}V_1 + g_{22}I_2 \cdots\cdots\cdots\cdots\cdots (iv)$$

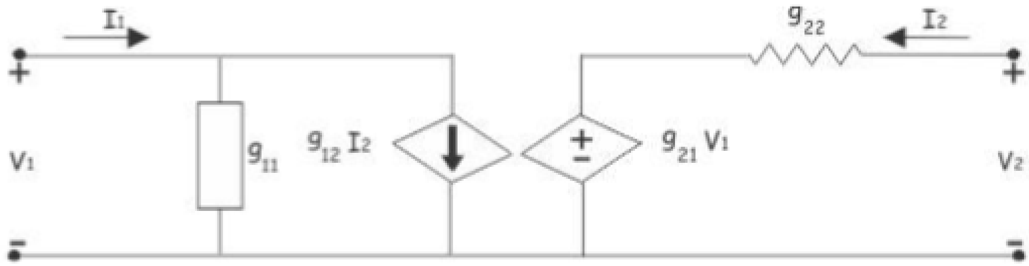
As per equation (iii), we can show:



As per equation (iv) we can show:



Combining these two circuits, we get,

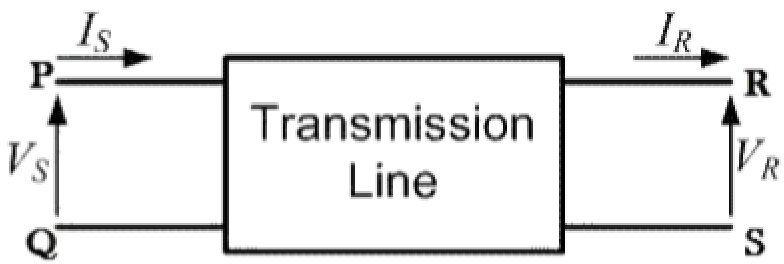


The g parameters equivalent network of a two port network

The h parameters are used to analyzingBipolar Junction Transistor or BJT. Whereas, g parameter are used to analyzingJunction Field Effect Transistor or JFET.

ABCD PARAMETERS:

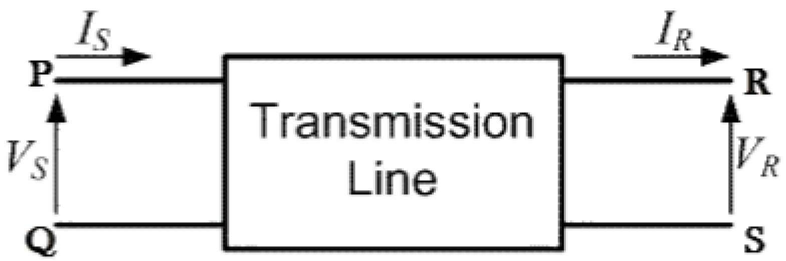
ABCD parameters (also known as chain or transmission line parameters) are generalized circuit constants used to help model transmission lines. More specifically, ABCD parameters are used in the two port network representation of a transmission line. The circuit of such a two-port network is shown below:



A major section of power system engineering deals in the transmission of electrical powerfrom one place (eg. generating station) to another (e.g. substations or residential homes) with maximum efficiency. So it's important for power system engineers to be thorough with the mathematical modeling of how this power is transmitted. ABCD parameters and a two-port model is used to simplify these complex calculations.

To maintain the accuracy of this mathematial model, transmission lines are classified into three types: short transmission lines, medium transmission

lines, and long transmission lines. The formula for these ABCD parameters will change depending on the length of the transmission line. This is necessary since certain electrical phenomenon – such as corona discharge and the Ferranti effect – only come into play when dealing with long transmission lines.
As the name suggests, a two port network consists of an input port PQ and an output port RS. In any 4 terminal network, (i.e. linear, passive, bilateral network) the input voltage and input current can be expressed in terms of output voltage and output current. Each port has 2 terminals to connect itself to the external circuit. Thus it is essentially a 2 port or a 4 terminal circuit, having:



$Supply\ end\ voltage = V_S$
$and\ Supply\ end\ current = I_S$ Given to the input port PQ.
$And\ there\ is\ the\ Receiving\ end\ voltage = V_R$
$and\ Receiving\ end\ current = I_R$
Given to the output port RS.

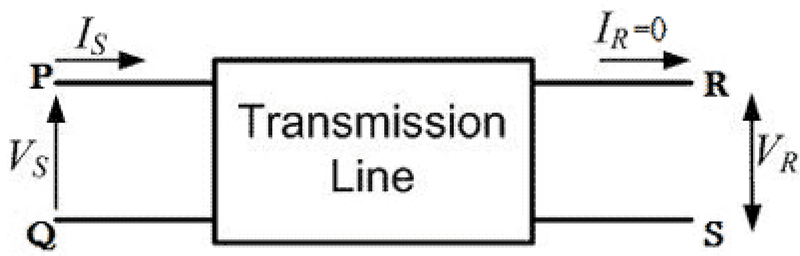
Now the ABCD parameters of transmission line provide the link between the supply and receiving end voltages and currents, considering the circuit elements to be linear in nature.
Thus the relation between the sending and receiving end specifications are given using ABCD parameters by the equations below.

$$V_S = AV_R + BI_R \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

$$I_S = CV_R + DI_R \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

Now in order to determine the ABCD parameters of transmission line let us impose the required circuit conditions in different cases.

ABCD Parameters, When Receiving End is Open Circuited:



The receiving end is open circuited meaning receiving end current $I_R = 0$.

Applying this condition to equation (1) we get,

$$V_S = A\,V_R + B\,0 \Rightarrow V_S = A\,V_R + 0$$

$$A = \left.\frac{V_S}{V_R}\right|_{I_R = 0}$$

Thus it's implied that on applying open circuit condition to ABCD parameters, we get parameter A as the ratio of sending end voltage to the open circuit receiving end voltage. Since dimension wise A is a ratio of voltage to voltage, A is a dimension less parameter.

Applying the same open circuit condition i.e $I_R = 0$ to equation (2)

$$I_S = C\,V_R + D\,0 \Rightarrow I_S = C\,V_R + 0$$

$$C = \left.\frac{I_S}{V_R}\right|_{I_R = 0}$$

Thus it's implied that on applying open circuit condition to ABCD parameters of transmission line, we get parameter C as the ratio of sending end current to the open circuit receiving end voltage. Since dimension wise C is a ratio of current to voltage, its unit is mho.

Thus C is the open circuit conductance and is given by

$C = I_S / V_R$ ohm.

ABCD Parameters, When Receiving End is Short Circuited:

0

Receiving end is short circuited meaning receiving end voltage $V_R = 0$

Applying this condition to equation (1) we get,

$$V_S = A\,0 + B\,I_R \Rightarrow V_S = 0 + B\,I_R$$

$$B = \left.\frac{V_S}{I_R}\right|_{V_R = 0}$$

Thus it's implied that on applying short circuit condition to ABCD parameters, we get parameter B as the ratio of sending end voltage to the short circuit receiving end current. Since dimension wise B is a ratio of voltage to current, its unit is Ω. Thus B is the short circuit resistance and is given by

$B = V_S / I_R$ Ω.

Applying the same short circuit condition i.e $V_R = 0$ to equation (2) we get

$$I_S = C\,0 + D\,I_R \Rightarrow I_S = 0 + D\,I_R$$

$$D = \left.\frac{I_S}{I_R}\right|_{V_R = 0}$$

Thus it's implied that on applying short circuit condition to ABCD parameters, we get parameter D as the ratio of sending end current to the short circuit receiving end current. Since dimension wise D is a ratio of current to current, it's a dimensionless parameter.

∴ The ABCD parameters of transmission line can be tabulated as:-

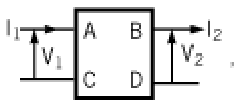
Parameter	Specification	Unit
$A = V_S / V_R$	Voltage ratio	Unit less
$B = V_S / I_R$	Short circuit resistance	Ω
$C = I_S / V_R$	Open circuit conductance	mho
$D = I_S / I_R$	Current ratio	Unit less

Cascaded networks:

Telephone subscriber lines have become a topic of intense interest for organizations attempting to transmit Internet signals on telephone lines. Basic loop standards exist, and tables of twisted-pair primary constants extending to 20 MHz are in the literature. Asymmetric digital-subscriber lines (ADSLs) are now available for high-speed Internet service. The telephone industry has always used two-port networks in the form of ABCD matrices to cascade sections of telephone cable. These configurations allow you to join the sections by matrix multiplication. However, because the elements in the matrices are complex numbers and several sections make up a chain, you need to organize the process of matrix multiplication. A short computer program performs this task (Listing 1). You enter the number of matrices in the chain and then enter the real and imaginary parts for each A, B, C, and D element. The product then appears on the screen.

The two-port matrix equation with the ABDC parameters has the following format:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & C \\ B & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

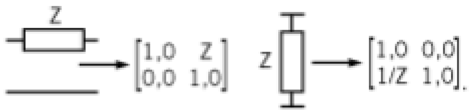


where

$$A=\frac{V_1}{V_2}\bigg|_{I_2=0}\quad B=\frac{V_2}{I_2}\bigg|_{V_2=0}\quad C=\frac{I_1}{V_2}\bigg|_{I_2=0}\quad D=\frac{I_2}{I_2}\bigg|_{V_2=0}.$$

A represents the open-circuit transfer function, B represents the short-circuit transfer impedance, C represents the open-circuit transfer admittance, and D represents the short-circuit current ratio. You can find the ABCD matrix for a ladder network composed of RLC elements by slicing the network into series and shunt matrices. You then multiply the product of the first two by the third, and the progression continues with additional subscripts identifying the components. In the final product, the elements A, B, C, and D—with all their component symbols—may become quite cumbersome. To avoid these complicated expressions in the matrix for the ladder network, don't use them. Rather, enter the numerical values for the series and shunt components along with the necessary 0,0s and 1,0s. You can use this concept for any network comprising passive components that you can separate into isolated two-port sections. You can also use the method to predict the degradation a bridged tap produces in a telephone cable.

The progression of entering values for cascaded networks is normally from left to right, or from source to load with the current pointing toward the load. If you reverse the progression, the direction of the current is usually reversed, and the location of elements A and D is reversed to conform to the reciprocity theorem. The matrices for a single series component and a single shunt component are as follows:



You can use these simple matrices to represent RLC components, transformers, and bridged taps in telephone cables. However, for a telephone cable, you must consider the reflected wave. Thus, you must express the A, B, C, D elements by the following hyperbolic functions:

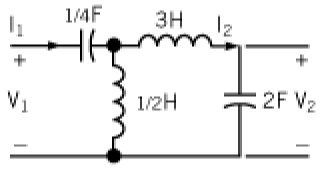
$A=\cosh(\gamma d)$,

$B=Z_0\sinh(\gamma d)$,

$C=(1/Z_0)\sinh(\gamma d)$, and

$D=\cosh(\gamma d)$.

where the propagation constant , $\gamma=\gamma+\gamma$, d is the electrical length of the cable, and Z_0 is the characteristic impedance. In the United States, the nominal value for Z_0 is 100V, which is also the specified value for the source and load impedance. Because γ is a complex number, A, B, C, and D are also complex numbers, or the polar equivalent thereof. These numbers are not difficult to calculate; however, the parameters depend on the application (References 1 and 2). The following example shows how to enter the data and read the results:



$$N_1=\begin{bmatrix}1,0 & 0,-4\\ 1/Z & 1,0\end{bmatrix} \qquad N_2=\begin{bmatrix}1,0 & 0,0\\ 0,-2 & 1,0\end{bmatrix}$$

$$N_3=\begin{bmatrix}1,0 & 0,3\\ 0,0 & 1,0\end{bmatrix} \qquad N_4=\begin{bmatrix}1,0 & 0,0\\ 0,2 & 1,0\end{bmatrix}$$

$$N_1\cdot N_2=\begin{bmatrix}-7,0 & 0,-4\\ 0,2 & 1,0\end{bmatrix} \qquad N_1\cdot N_2\cdot N_3=\begin{bmatrix}-7,0 & 0,-25\\ 0,-2 & 7,0\end{bmatrix}$$

$$N_1\cdot N_2\cdot N_3\cdot N_4=\begin{bmatrix}43,0 & 0,-25\\ 0,12 & 7,0\end{bmatrix}$$

Poles and zeros of network functions:

INTRODUCTION:

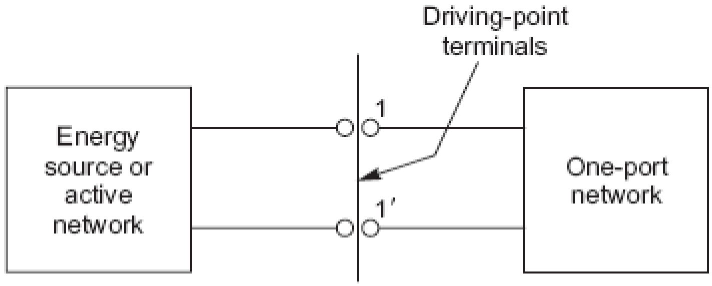
In this chapter we introduce the concept of transfer functions relating currents
and voltages in different sections of a network. These functions are
mathematically similar to the transform impedance or admittance functions
and are included in the broader category of functions called network functions.

Terminal Pairs or Ports:

Any network may be represented schematically

by a rectangle or box as shown in Figure 9.1. A network may be used for
a variety of purposes. Thus consider its use as a load connected to some other
network. In order to connect it to the active network, there must be available
two terminals of this passive network. Figure 9.1 shows a network with one
pair of terminals 1-1' or with one port. Such a network may be called a one-port
network or one terminal-pair network. When such a one-port network
is connected to an energy source or an active network at its pair of terminals,
the energy source provides the driving force for this one-port network and
the pair of terminals constitute the driving-point of the network. One pair of

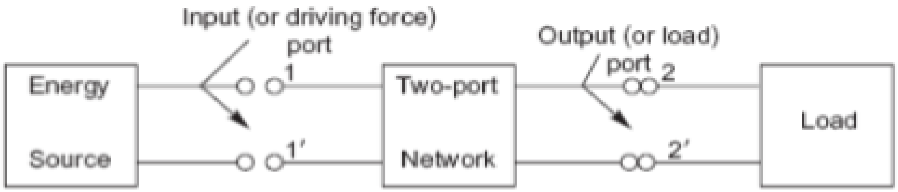
terminals is known as a port.



One-port network driven by a source:

shows a two-port network or two terminal-pair network. In this case, port 1 is connected to the driving force or the input and is called the input
port or the driving port. On the other hand, port 2 (i.e., terminal-pair 2-2'),

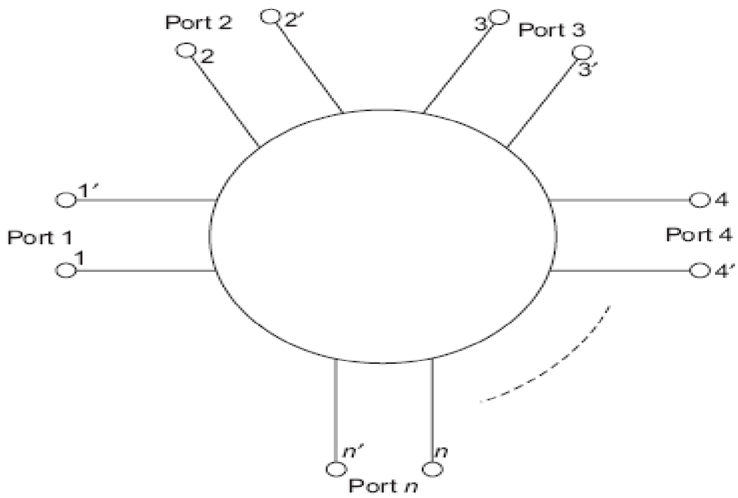
is connected to the load and is called the output port or the load port.



Two-port network driven by an energy source:

shows a general n-port network. Here the driving force

(energy source) may be connected to one or more ports and other networks
may be connected at the remaining ports.



NETWORK SYNTHESIS

Positive real function:

Any function which is in the form of F(s) will be called as a positive real function if fulfill these four important conditions:

1. F(s) should give real values for all real values of s.
2. P(s) should be a Hurwitz polynomial.
3. If we substitute s = jω then on separating the real and imaginary parts, the real part of the function should be greater than or equal to zero, means it should be non negative. This most important condition and we will frequently use this condition in order to find out the whether the function is positive real or not.
4. On substituting s = jω, F(s) should posses simple poles and the residues should be real and positive.

Properties of Positive Real Function

There are four very important properties of positive real functions and they are written below:

1. Both the numerator and denominator of F(s) should be Hurwitz polynomials.
2. The degree of the numerator of F(s) should not exceed the degree of denominator by more than unity. In other words (m-n) should be less than or equal to one.
3. If F(s) is positive real function then reciprocal of F(s) should also be positive real function.
4. Remember the summation of two or more positive real function is also a positive real function but in case of the difference it may or may not be positive real function.

Following are the four necessary but not the sufficient conditions for the functions to be a positive real function and they are written below:

1. The coefficient of the polynomial must be real and positive.
2. The degree of the numerator of F(s) should not exceed the degree of denominator by more than unity. In other words (m – n) should be less than or equal to one.
3. Poles and zeros on the imaginary axis should be simple.
4. Let us consider the coefficients of denominator of F(s) is b_n, $b_{(n-1)}$, $b_{(n-2)}$ b_0.Here it should be noted that b_n, $b_{(n-1)}$, b_0 must be positive and b_n and $b_{(n-1)}$ should not be equal to zero simultaneously.

Now there two necessary and sufficient conditions for the functions to be a positive real function and they are written below:

1. F(s) should have simple poles on the jω axis and the residues of these poles must be real and positive.
2. Summation of both numerator and denominator of F(s) must be a

Hurwitz polynomial.

basic synthesis procedure:

An electric network is any possible interconnection of electric circuit elements, while an electric circuit is a closed energised network. An electric circuit is characterised by currents (I) in the elements and voltage (V) across them. A network is not necessarily and circuit.

Circuit's elements may be as follows

- Active or passive
- Unilateral or bilateral
- Linear or non linear
- Lumped or distributed.

Network analysis includes study of the network condition and its changes due to change by applied source or change in circuit elements. During this process branch currents and element voltage changes from earlier form to new one. This is called "transient" after this it attains study state.

V (voltage) I (current) relation is linear first order differential equation. R (resistance), L (inductance), C (capacitance) is parallel and series combinations or second order equations and can be solved and analysed.

Network analysis includes number of input and output terminals with components like resistance inductance and capacitance etc...It also includes driving signals like voltage source and current source. Signals may be time variant or time invariant. Analysis always includes response of input signal with components inters of solutions such analysis demands various methods like mathematical equations (first order and second order differential equations, Laplace transformations and Z transformations).

Different standard input signals are used like continuous AC sinusoidal signal, Step signal, Ramp signal, impulse signal, exponential signal and rectangular gate signal.

Circuits to be analysed are usually combination of series, parallel or combination of both. Tools used for analysis are Kirchhoff voltage law, Kirchhoff current law mesh and node analysis using various network theorems.

Network to be analysed may be fed with AC signal or DC signal and tools are available for both. Since all electrical electronics and instrumentation networks are combination of static or dynamic components with different signals and hence network analysis finds its importance in every field. Analysis provides information about the stability of the system.

In synthesis preliminary problem is to determine the response given the excitation in the network.

Synthesis procedure is to determine whether T(s) can be realised as physical passive network. Important considerations are causality and stability.

Fundamentals of network analysis synthesis:

Network analysis includes analysis of active and passive elements like RLC with various combinations like series parallel and input source. It includes basic fundamental system properties as follows,

- Continuous time and discreet time system: Continuous time system results in continuous time output signal. Discreet time input signal transfers into discreet time output. Continuous time signals can be solved using differential equations and Laplace transformations. Discreet time systems are described by difference equations and can be solved by Z transformations.
- Time variant and time invariant system: Time invariant system are fixed and its input output relation does not change with the time. Time varying system varies with the time.
- Linear and nonlinear system: A linear system holds the principle of superposition and homogeneity. It has the property of additively and scaling failing which it is said to be non linear.
- Instantaneous and dynamic system: In a system output is a function of input is said to be instantaneous at present time. In a dynamic system output depends on past or future values of the input.

The fundamentals also include casual and non casual systems invariability and inverse system.

It also includes Ideal voltage source and current source.

The fundamentals of synthesis include the realisation using reliability theory. Different task to synthesize a network are causality and stability. Test whether the network function can be realised as physical passive network. It considers causality and stability by causality mean voltage cannot appear between any pair of terminals in a network before current is impressed. For stability no poles on RHS of S-plane, cannot have multiple poles etc...

Another element of reliability is Hurwitz polynomial.

What is the difference between analysis and synthesis:

Difference between analysis and synthesis are as follows

- Analysis is the process of reducing bigger one into small. It breaks down complex into smaller concepts for better understanding.
- Synthesis resolves conflicts set between anti synthesis and thesis by setting a common truth between them and finally provides new proposition. Analysis is dependent on logic and mathematics.
- Synthesis is a higher process that creates something new it will be done at the end of study .Analysis starts from the beginning breaks into different simple ideas to gain a better understanding of entire network.
- In simple words network analysis is considered with determining the response, given the excitation and the network.
- On the other hand network synthesis, the problem is to design the network given the excitation and the desired response.

Note:

(Excitation and response are given in terms of voltage and current.

Signal can be described well in terms of spectral or frequency information.

Time and frequency translation can be done by Fourier series, Fourier integral and the Laplace transform.)

- In network analysis in the time domain voltage-current relationship are given in terms of differential equations.
- While in synthesis the complex frequency domain are in algebraic equations.
- The algebraic equation is the more easily solved than differential equation.
- In network analysis characteristic signals may be time variant or time invariant.
- In synthesis it may be periodic or periodic or it may be even or odd.

(Periodic signal repeats itself with the minimum fundamental period whereas periodic pulse pattern do not repeat).

Six things every student should know to solve problems in network analysis and synthesis.

- Usually electrical networks are complex in nature analysis requires techniques important techniques are remembering all theorems they are superposition theorem, the venin, Norton, maximum power transfer theorem ,reciprocity theorem and millmans theorem. Basic electrical parameters can be solved in simple way. To use theorems circuit must be linear in basic theorem like superposition theorem replace voltages and current sources except one of them .Kirchhoffs law or ohms law will be used to calculate voltage drop and current in the branch. Procedure must be repeated for every source and finally add all the voltage results due all sources and also current of each source.

- Basic concepts like components node, branch, mesh, port, circuit, transfer functions; component transfer function etc must be understood.
- Drawing equivalent circuits by reducing number of components can be done by replacing actual components with other components. Any two terminal networks can be reduced to single impedance by successive applications of impedance in series or parallel .Star and delta transformations must be remembered .Star to delta and series resistor transformation can be treated as special case of general resistor network node elimination algorithm.
- Nodal analysis requires labelling of nodes defining voltage variable for remaining nodes to the reference using KCL solving and obtaining the result.
- Mesh analysis leads to defining a mesh, assigning mesh current to the each window using KVL and solving the equations.

Proper understanding of circuit and its source applying proper law and obtaining the solution using appropriate techniques is most important rule to be remembered while solving any network problem. For every network problem there will be alternative technique which can be used to cross check the result. For example simultaneous equation and matrix method.

LC immittance functions:

The pole-zero distributions of the driving-point immittance functions of nonuniform lc, rcg, rlg and rlcg pseudo-distortionless transmission lines are examinedusing classical Sturn-Liouville theory. The results are applicable to any of the other two-port parameters and the conclusions parallel many of the powerful theorems concerning lumped, passive networks. The pseudo-distortionless line is a new class of line having interesting pole-zero characteristics.

RC impedance functions:

A resistor–capacitor circuit (RC circuit), or RC filter or RC network, is an electric circuit composed of resistors and capacitors driven by a voltage or current source. A first order RC circuit is composed of one resistor and one capacitor and is the simplest type of RC circuit.

RC circuits can be used to filter a signal by blocking certain frequencies and passing others. The two most common RC filters are the high-pass filters and low-pass filters; band-pass filters and band-stop filters usually require RLC filters, though crude ones can be made with RC filters.

Foster and caurier form:

The network, the excitation, and the response. If any two of the three quantities are given, the third may be found for linear networks. If the network and the excitation are given and the response is to be determined, the problem is defined as analysis. When the excitation and the response are given, and it is required to determine a network, the problem is defined as synthesis. If the network and the response are given to find the excitation there is generally no accepted name and it is not common.

Synthesis is the process of finding a network corresponding to a given driving-point impedance or admittance. In this chapter, we will consider some aspects of passive network synthesis, mainly driving-point immittance functions (impedance or admittance functions),

$$F(s) = \frac{R(s)}{E(s)} \quad (11.1)$$

where F(s) is the network function, which is the ratio of response R(s) to the excitation E(s).

FOURIER ANALYSIS AND TRANSFORMS

Sometimes all the information in time domain is not sufficient. This makes us to move to frequency domain of the signal for extracting more information about the signal. This movement from one domain to other domain is known as transformation. For changing the domain of signal from time to frequency we have many tools. Fourier Series and Fourier Transform are two of the tools in which we decompose the signal into harmonically related sinusoids. With such decomposition, a signal is said to be represented in frequency domain.
Most of the practical signals can be decomposed into sinusoids. Such a decomposition of periodic signals is called a Fourier series.

Frequency Analysis:

Just like a white light can be decomposed into seven colors, a periodic signal can also be decomposed into a liner weighted sum of harmonically related frequencies. This linear weighted sum of harmonically related sinusoids or complex exponentials is known as Fourier Series or Transform. In general, decomposition of any signal into its frequency related components is called frequency analysis. Like analysis of a light into colors is actually a form of frequency analysis, hence the Fourier series and Fourier transform are also tools of frequency analysis.

Suppose if we pass a light through a prism, it gets split into seven colors VIBGYOR. Each color has a particular frequency or a range of frequencies. In the same way, if we pass a periodic signal through a Fourier tool, which plays the role of prism, the signal is decomposed into a Fourier series.

Signals and Vectors Analogy:

An N dimension vector needs N dimensions for its representation. Like an ant moving on a table needs two dimensions for the representation of its position on the table i.e. x and y. Also we are familiar with i, j, k coordinate system for a vector representation in three dimensions. This unit vector i, j and k are orthogonal to each other. In the same way if we treat a signal as a multidimensional vector we need many more dimensions which are orthogonal to each other. It was the genius of J. B. J. Fourier who invented multi-dimensions, which are orthogonal to each other. These are sinusoids with harmonically related sinusoids or complex exponential. Consider the dimensions (also called bases)

$\sin\omega 0t$ $\sin 2\omega 0t$ $\sin 3\omega 0t$ $\sin 4\omega 0t$ $\sin n\omega 0t$

$\cos\omega 0t$ $\cos 2\omega 0t$ $\cos 3\omega 0t$ $\cos 4\omega 0t$ $\cos n\omega 0t$

Thus, all $\sin n\omega 0t$ are orthogonal with $\text{Sin} m\omega 0t$ $(n \neq m)$ and we, therefore can use $\sin\omega 0t$, $\sin 2\omega 0t \ldots \infty$ as the primary dimensions (also called bases) to express a periodic signal. Similarly, we can also use $\cos\omega_0 t$, $\cos 2\omega_0 t$, $\cos 3\omega_0 t \ldots \infty$ as the additional dimensions when $\sin\omega_0 t$ dimensions cannot be used. We will see for even signals only cosine terms will be suitable and for odd signal only sine terms will be suitable. For a periodic signal neither an odd nor even, we use both sine and cosine terms.

NOTE

Only periodic signals can be represented as Fourier series provided the signal follows the Dirichlet's conditions. For non-periodic signals, we have Fourier transform tool which transform the signal from time domain to frequency domain.

Resolution of signal into its harmonically related frequencies is known as Fourier Analysiswhile the inverse i.e. recombination, is known as Fourier Sysnthesis.

Dirichlet's Conditions:

x (t) is absolutely integrable over any period, that is,

$$\int_0^{T_p} |x(t)|dt < \infty$$

x (t) has a finite number of maxima and minima within any finite interval of t.

x (t) has a finite number of discontinuities within any finite interval of t, and each of these discontinuities are finite.

Note that the Dirichlet's conditions are sufficient but not necessary conditions for the Fourier series representation.

Trigonometric form and exponential form of Fourier series:

We have already discussed the Fourier series in exponential form. In this article we will discuss another form of Fourier series i.e. Trigonometric Fourier series.

Fourier series representation in Trigonometric form:

Fourier series in trigonometric form can be easily derived from its exponential form. The complex exponential Fourier series representation of a periodic signal x(t) with fundamental period T_0 is given by

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_o t} \quad \omega_o = \frac{2\pi}{T_o}$$

Since sine and cosine can be expressed in exponential form. Thus by manipulating the exponential Fourier series, we can obtain its Trigonometric form.

The trigonometric Fourier series representation of a periodic signal x (t) with fundamental period T, is given by

$$x(t) = a_o + \sum_{k=1}^{\infty}(a_k cosk\omega_o t + b_k sink\omega_o t) \quad \omega_o = \frac{2\pi}{T_o}$$

Where a_k and b_k are Fourier coefficients given by

$$a_k = \frac{2}{T_o}\int_{T_o} x(t)cosk\omega_o tdt, \;\; b_k = \frac{2}{T_o}\int_{T_o} x(t)sink\omega_o tdt$$

a_0 is the dc component of the signal and is given by

$$a_o = \frac{1}{T_o}\int_{T_o} x(t)dt$$

Properties of Fourier series:

1. If x(t) is an even function i.e. x(- t) = x(t), then $b_k = 0$ and

$$a_o = \frac{2}{T_o}\int_0^{T_0/2} x(t)dt, \;\; a_k = \frac{4}{T_o}\int_0^{T_0/2} x(t)cosn\omega_o tdt, \;\; n = 1,2,3,\cdots\cdots$$

2. If x(t) is an even function i.e. x(- t) = – x(t), then $a_0 = 0$, $a_k = 0$ and

$$b_k = \frac{4}{T_o}\int_0^{T_0/2} x(t)sinn\omega_o tdt, \;\; n = 1,2,3,\cdots\cdots$$

3. If x(t) is half symmetric function i.e. x (t) = -x(t ± T_0/2), then $a_0 = 0$, $a_k = b_k = 0$ for k even,

$$a_k = \frac{4}{T_o}\int_0^{T_0/2} x(t)cosn\omega_o tdt, \;\; b_k = \frac{4}{T_o}\int_0^{T_0/2} x(t)sinn\omega_o tdt, \;\; for\ n\ odd$$

4. Linearity

$$If\ x_1(t) \leftrightarrow c_k\ and\ x_2(t) \leftrightarrow d_k$$

$$Then\ Ax_1(t) + Bx_2(t) \leftrightarrow Ac_k + Bd_k$$

5. Time shifting

$$If\ x(t) \leftrightarrow c_k$$

$$Then\ x(t - t_0) \leftrightarrow c_k e^{(-jk\omega_0 t_0)}$$

6. Time reversal

$$If\ x(t) \leftrightarrow c_k$$

$$Then\ x(-t) \leftrightarrow c_{-k}$$

7. Multiplication

$$If\ x(t) \leftrightarrow c_k$$

$$Then\ x(t).y(t) \leftrightarrow \sum_{k=-\infty}^{\infty} c_k.d_k$$

8. Conjugation

$$If\ x(t) \leftrightarrow c_k$$
$$Then\ x^*(t) \leftrightarrow c^*_{-k}$$

9. Differentiation

$$If\ x(t) \leftrightarrow c_k$$

$$Then\ \frac{dx(t)}{dt} \leftrightarrow (jk\omega_0)c_k$$

10. Integration

$$If\ x(t) \leftrightarrow c_k$$

$$Then \int_{-\infty}^{t} x(t) \leftrightarrow \frac{1}{jk\omega_0} c_k$$

11. Periodic Convolution

$$If\ x(t) \leftrightarrow c_k$$

$$And\ h(t) \leftrightarrow d_k$$

$$Then\ [x(t) * h(t)] \leftrightarrow T_0.c_k.d_k$$

Relationship between coefficients of exponential form and coefficients of trigonometric form

$$a_0 = C_0,\ a_k = C_k + C_{-k},\ b_k = j(C_k - C_{-k}),$$

$$c_k = \frac{1}{2}(a_k - jb_k),\ c_{-k} = \frac{1}{2}(a_k + jb_k)$$

When x (t) is real, then a, and b, are real, we have

$$a_k = 2Re[c_k]\ \ and\ \ b_k = -2Im[c_k]$$

Effect of Shifting Axis of the Signal:

- On shifting the waveform to the left right with respect to the reference time axis t = 0 only the phase values of the spectrum changes but the magnitude spectrum remains same.
- On shifting the waveform upward or downward w.r.t time axis changes only the DC value of the function.

Before applying proper electrical protection system, it is necessary to have through knowledge of the conditions of electrical power system during faults. The knowledge of electrical fault condition is required to deploy proper different protective relays in different locations of electrical power system.

Information regarding values of maximum and minimum fault currents, voltages under those faults in magnitude and phase relation with respect to the currents at different parts of power system, to be gathered for proper application of protection relay system in those different parts of the electrical power system. Collecting the information from different parameters of the system is generally known as electrical fault calculation.

Fault calculation broadly means calculation of fault current in any electrical power system. There are mainly three steps for calculating faults in a system.

1. Choice of impedance rotations.
2. Reduction of complicated electrical power system network to single equivalent impedance.
3. Electrical fault currents and voltages calculation by using symmetrical component theory.

Impedance Notation of Electrical Power System:

If we look at any electrical power system, we will find, these are several voltage levels. For example, suppose a typical power system where electrical power is generated at 6.6 kV then that 132 kV power is transmitted to terminal substation where it is stepped down to 33 kV and 11 kV levels and this 11 kV level may further step down to 0.4 kv.
Hence from this example it is clear that a same power system network may have different voltage levels. So calculation of fault at any location of the said system becomes much difficult and complicated it try to calculate impedance of different parts of the system according to their voltage level. This difficulty can be avoided if we calculate impedance of different part of the system in reference to a single base value. This technique is called impedance notation of power system. In other wards, before electrical fault calculation, the system parameters, must be referred to base quantities and represented as uniform system of impedance in either ohmic, percentage, or per unit values.
Electrical power and voltage are generally taken as base quantities. In three phase system, three phase power in MVA or KVA is taken as base power and line to line voltage in KV is taken as base voltage. The base impedance of the system can be calculated from these base power and base voltage, as follows,

$$Z_b = \frac{(KV)^2}{KVA} \text{ ohms}$$

Per unit is an impedance value of any system is nothing but the radio of actual impedance of the system to the base impedance value.

$$\text{i.e. } Z_{pu} = \frac{Z_{actual}}{Z_b}$$

Percentage impedance value can be calculated by multiplying 100 with per unit value.

$$Z_{\%} = Z_{pu} \times 100$$

Again it is sometimes required to convert per unit values referred to new base values for simplifying different electrical fault calculations. In that case,

$$\text{New, } Z_{pu} = \text{Old } Z_{pu} \times \frac{\text{New base MVA}}{\text{Old base MVA}} \text{ or New, } Z_{pu} = \text{Old } Z_{pu} \times \frac{(\text{Old base KV})^2}{(\text{New base KV})^2}$$

The choice of impedance notation depends upon the complicity of the system. Generally base voltage of a system is so chosen that it requires minimum number of transfers.

Suppose, one system as a large number of 132 KV over head lines, few numbers of 33 KV lines and very few number of 11 KV lines. The base voltage of the system can be chosen either as 132 KV or 33 KV or 11 KV, but here the best base voltages 132 KV, because it requires minimum number of transfer during fault calculation.

Network Reduction:

After choosing the correct impedance notation, the next step is to reduce network to a single impedance. For this first we have to convert the impedance of all generators, lines, cables, transformer to a common base value. Then we prepare a schematic diagram of electrical power system showing the impedance referred to same base value of all those generators, lines, cables and transformers.

The network then reduced to a common equivalent single impedance by using star/delta transformations. Separate impedance diagrams should be prepared for positive, negative and zero sequence networks.

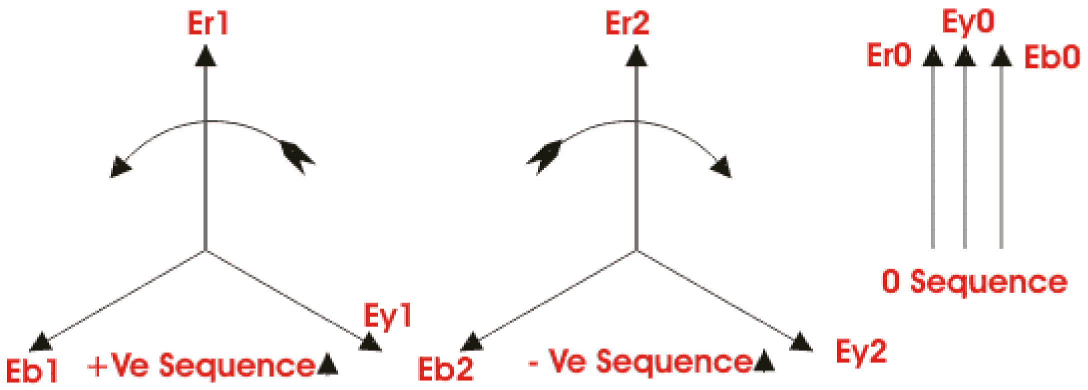
There phase faults are unique since they are balanced i.e. symmetrical in three phase, and can be calculated from the single phase positive sequence impedance diagram. Thereforethree phase fault current is obtained by,

$$I_f = \frac{V}{Z_1}$$

Where, I_f is the total three phase fault current, v is the phase to neutral voltage z_1 is the total positive sequence impedance of the system; assuming that in the calculation, impedance are represented in ohms on a voltage base.

Symmetrical Component Analysis:

The above fault calculation is made on assumption of three phase balanced system. The calculation is made for one phase only as the current and voltage conditions are same in all three phases. When actual faults occur in electrical power system, such as phase to earth fault, phase to phase fault and double phase to earth fault, the system becomes unbalanced means, the conditions of voltages and currents in all phases are no longer symmetrical. Such faults are solved by symmetrical component analysis. Generally three phase vector diagram may be replaced by three sets of balanced vectors. One has opposite or negative phase rotation, second has positive phase rotation and last one is co-phasal. That means these vectors sets are described as negative, positive and zero sequence, respectively.



The equation between phase and sequence quantities are,

$$E_r = E_{r0} + E_{r1} + E_{r2}$$

$$E_y = E_{y0} + E_{y1} + E_{y2} = E_{r0} + r^2 E_{r1} + rE_{r2}$$

$$E_b = E_{b0} + E_{b1} + E_{b2} = E_{r0} + rE_{r1} + r^2 E_{r2}$$

Therefore,

$$E_{r0} = \frac{1}{3}(E_r + E_y + E_b)$$

$$E_{r1} = \frac{1}{3}(E_r + rE_y + r^2 E_b)$$

$$E_{r2} = \frac{1}{3}(E_r + r^2 E_y + rE_b)$$

Where all quantities are referred to the reference phase r.
Similarly a set of equations can be written for sequence currents also.
From, voltage and current equations, one can easily determine the sequence impedance of the system. The development of symmetrical component analysis depends upon the fact that in balanced system of impedance, sequence currents can give rise only to voltage drops of the same sequence. Once the sequence networks are available, these can be converted to single equivalent impedance.
Let us consider Z_1, Z_2 and Z_0 are the impedance of the system to the flow of positive, negative and zero sequence current respectively.
For earth fault

$$I_{r0} = I_{r1} = I_{r2} \frac{E}{Z_0 + Z_1 + Z_2}$$

Phase to phase faults

$$I_{r1} = \frac{E}{Z_1 + Z_2}$$

$$I_{r2} = -I_{r1}$$

$$I_{r0} = 0$$

Double phase to earth faults:

Three phase faults:

If fault current in any particular branch of the network is required, the same can be calculated after combining the sequence components flowing in that branch. This involves the distribution of sequence components currents as determined by solving the above equations, in their respective network according to their relative impedance. Voltages it any point of the network can also be determine once the sequence component currents and sequence impedance of each branch are known.

Sequence Impedance

Positive Sequence Impedance:

The impedance offered by the system to the flow of positive sequence current is called positive sequence impedance.

Negative Sequence Impedance:

The impedance offered by the system to the flow of negative sequence current is called negative sequence impedance.

Zero Sequence Impedance:

The impedance offered by the system to the flow of zero sequence current is known as zero sequence impedance.
In previous fault calculation, Z_1, Z_2 and Z_0 are positive, negative and zero sequence impedance respectively. The sequence impedance varies with the type of power system components under consideration:-

1. In static and balanced power system components like transformer and lines, the sequence impedance offered by the system are the same for positive and negative sequence currents. In other words, the positive sequence impedance and negative sequence impedance are same for transformers and power lines.
2. But in case of rotating machines the positive and negative sequence impedance are different.
3. The assignment of zero sequence impedance values is a more complex one. This is because the three zero sequence current at any point in a electrical power system, being in phase, do not sum to zero but must return through the neutral and /or earth. In three phase transformer and machine fluxes due to zero sequence components do not sum to zero in the yoke or field system. The impedance very widely depending upon the physical arrangement of the magnetic circuits and winding.
    1. The reactance of transmission lines of zero sequence currents can be about 3 to 5 times the positive sequence current, the lighter value being for lines without earth wires. This is because the spacing between the go and return(i.e. neutral and/or earth) is so much greater than for positive and negative sequence currents which return (balance) within the three phase conductor groups.
    2. The zero sequence reactance of a machine is compounded of leakage and winding reactance, and a small component due to winding balance (depends on winding tritch).
    3. The zero sequence reactance of transformers depends both on winding connections and upon the construction of the core.

line spectra and phase angle spectra:

By Fourier theory, any waveform can be represented by a summation of a (possibly infinite) number of sinusoids, each with a particular amplitude and phase. Such a representation is referred to as the signal's spectrum (or it's frequency-domain representation). It is often easier to analyze signals and signal networks in terms of their spectral representations. As well, there are many instances where it is easier to synthesize a signal using a frequency-domain approach.

Why is the spectral view useful?

- It is often the case that the spectrum of a signal can indicate aspects of the signal that would otherwise not be obvious by looking only at its time-domain representation.
- For example, from the time-domain plot below it is clear that the signal is periodic, with a period of about 0.02 seconds. However, few other features of the signal are immediately clear.

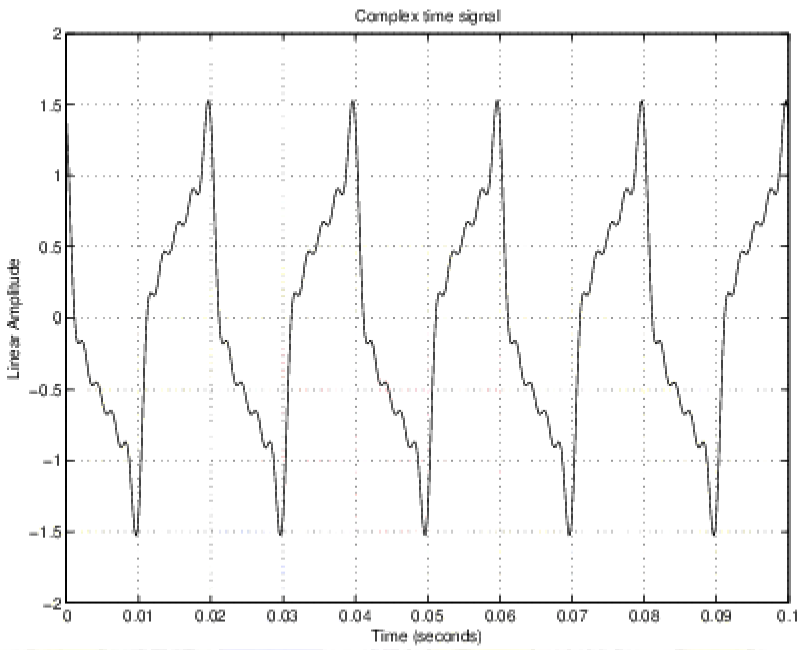


Figure 3: A complex time-domain signal.

- The plot below shows the magnitude spectrum of the signal from Fig. 3. It is now obvious that the signal is comprised of only 5 frequency components, the magnitudes of which are inversely proportional to frequency. As implied by the time-domain periodicity, the spectral components are harmonically aligned. From further analysis, we see the 5 components fall only at odd integer multiples of the fundamental frequency.

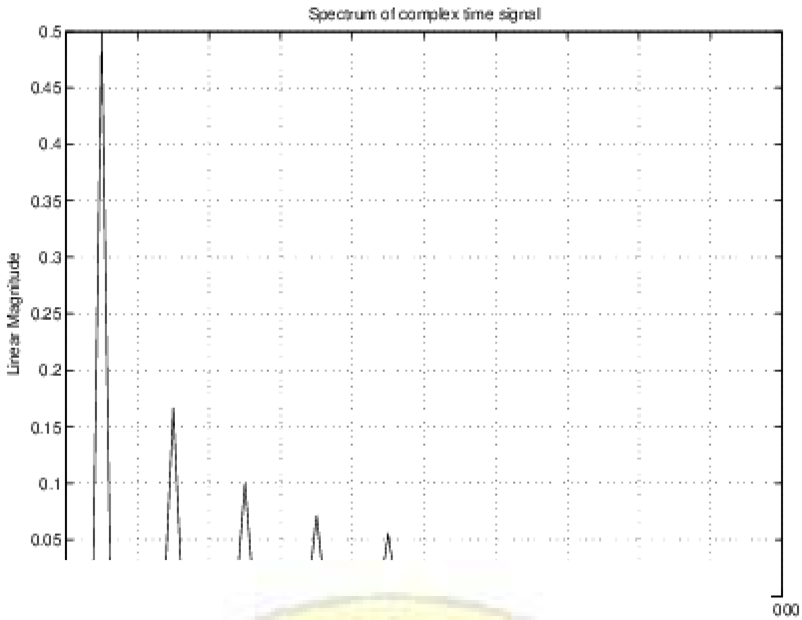


Figure 4: ' ... main signal

- Freque ... in evaluating digital
- As an e ... repres ... tainly possib ... y considering its out ... a relatively easy se ... ponse for all sinuso ... ow.

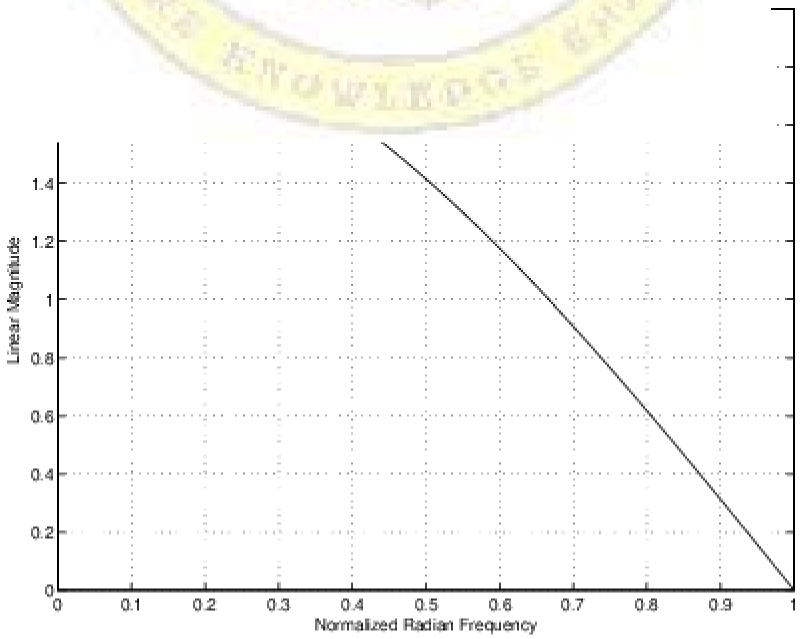


Figure 5: The magnitude frequency response of the filter y[n] = x[n] + x[n-

1].

How is a spectrum obtained?

- Given a discrete-time signal x[n], we can determine its frequency response using the Discrete Fourier Transform (DFT):

$$X[k] \triangleq \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \qquad k = 0, 1, 2, \ldots, N-1.$$

- The DFT determines sinusoidal ``weights'' via the inner product of sinusoids and the signal.
- The DFT can be interpreted as the sum of projections of x[n] onto a set of k sampled complex sinusoids or sinusoidal basis functions at (normalized) radian frequencies given by $\omega_k = 2\pi k/N$.
- In this way, the DFT and its inverse provide a ``recipe'' for reconstructing a given discrete-time signal in terms of sampled complex sinusoids.
- If the signal x[n] consists of N samples, X[k] will consist of k=N frequency weights (assuming no zero-padding). Based on the sampling theorem, however, only the first half of these frequency components are unique.
- The DFT coefficients are complex values. To plot the magnitude response of a signal's spectrum, we calculate the magnitude of each coefficient. For example, if a coefficient is equal to a + jb, its magnitude can be determined as $\sqrt{a^2 + b^2}$. The Matlab function abs performs this calculation.
- The phase response of a signal is given by the ``angles'' of its complex DFT coefficients. For a coefficient given by a + jb, the phase angle is determined as $\tan^{-1}(b/a)$. The Matlab function angle performs this calculation.
- In general, the spectra of real-world signals will not look as ``clean'' as the figures above. In fact, the spectrum of Fig. 4 was obtained by transforming an exact integer number of periods of the signal in Fig. 3. If we chose an arbitrary piece of the signal to transform, we would likely end up with a spectrum such as that shown below.

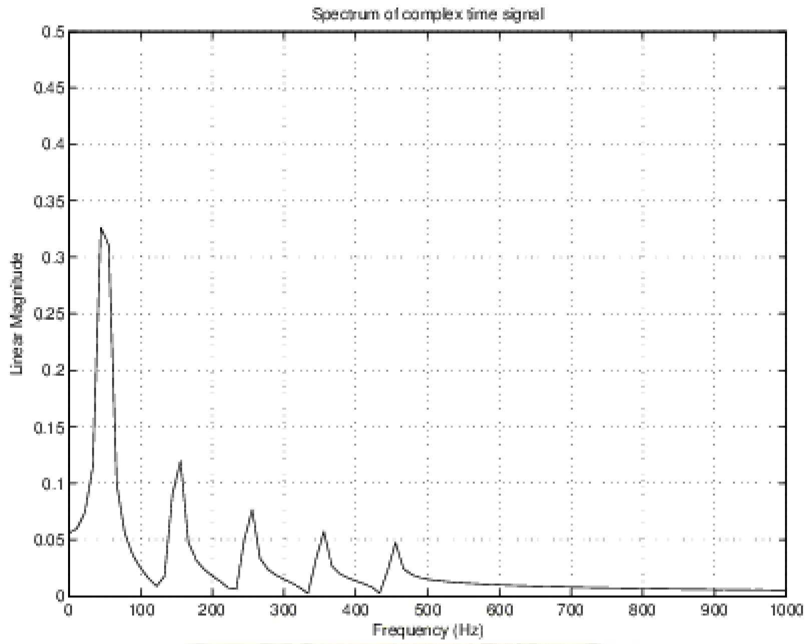


Figure 6: The magnitude spectrum of the complex time-domain signal plotted in Fig. 3 using arbitrary signal length.

The FFT:

- A brute force calculation of a length N DFT requires a number of operations roughly proportional to N^2. Thus, a 1024-point DFT would require about a million multiplications and about a million additions.
- Luckily, the DFT algorithm can be simplified to a form in which the required number of calculations is proportional to $N \log(N)$. Now, our 1024-point DFT can be calculated with only about 10,000 operations!
- This simplified algorithm is referred to as the Fast Fourier Transform or FFT.

Time / Frequency Resolution:

- As mentioned above, a length N FFT (or DFT) computes sinusoidal ``weights'' for N evenly spaced frequencies between 0 and f_s (N-1)/N. From the sampling theorem, only the first half of these frequency weights are unique.
- It follows that the larger the value of N, the more sinusoidal weights are computed and the smaller the spacing between frequency components. This spacing is given by f_s/N.
- If analyzing a fairly static sound signal, you would thus be best off using a larger value of N to get a more precise estimate of the frequency content.
- On the other hand, if the timbre of a sound changes significantly over time, you will need to segment the sound and compute separate FFTs

over each block, in order to estimate the change of the frequency content from one block to another. This may require smaller values of N, in order to isolate changes over time.

- When computing FFTs over time (blocks), the resulting data can be displayed in terms of a waterfall plot (separate spectra slightly offset from one another) or as a spectrogram (a 2D time vs. frequency plot where spectral magnitudes are displayed using color maps).

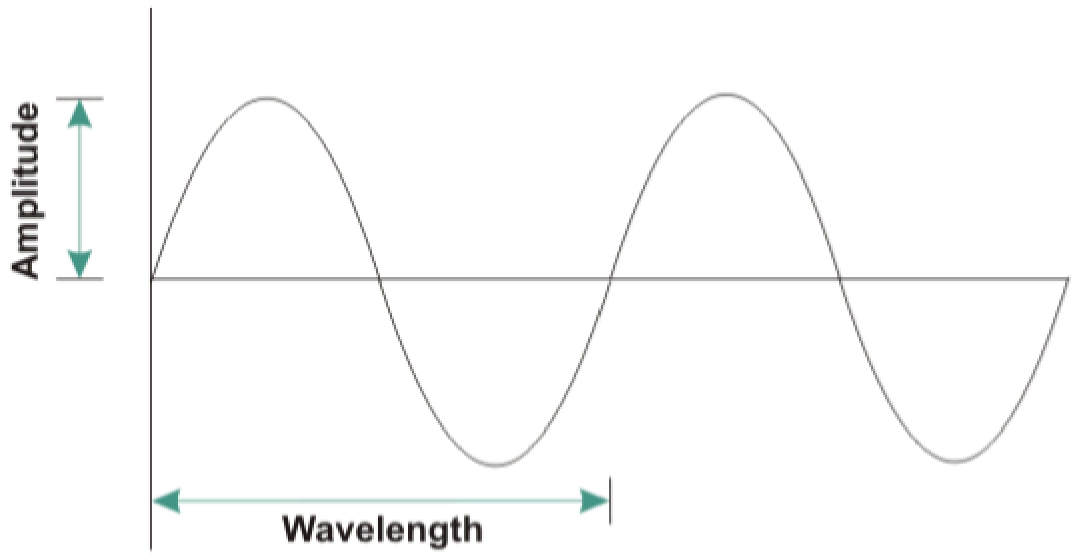
Sinusoidal Wave Signal:

To understand what is Sine wave or Sinusoidal Wave Signal, first let us try to understand what a Signal is.

What is a Signal?

There are different measurable quantities in the world surrounding us. Some quantities are constant like acceleration due to gravity, speed of light, velocity of sound in air. Some are time-varying like AC voltage, Pressure, Temperature. It means they change their value as time passes on. Signal simply means the value of any quantity taken over a period of time. Signals are usually time varying in nature. Generally a graph is plotted between values at different time instants. This is called graphical representation of signal.

What is Sine Wave or Sinusoidal Wave Signal?



Sine Wave or Sinusoidal Wave Signal is a special type of signal. It is given by the function

$$f(t) = \sin(\omega t) \quad or \quad f(t) = A\sin(\omega t + \phi)$$

$$Where, \omega = 2\pi f$$

When Sine wave starts from zero and covers positive values, reaches zero; and again covers negative values, reaches zero, it is said to have completed one cycle or single cycle.

The upper part of sine wave is called positive cycle and the lower part is called negative cycle in a single cycle.
For different values of time, the Signal gives the values of quantity at that time. Therefore Signal is a function of time. It is therefore written asf(t).
The Maximum value of the Sinusoidal Signal is also called its amplitude (A). Here ω is called Angular Frequency of Signal and f is the Frequency of Signal. Ø Is called Phase difference.
Frequency is measured in Hertz (Hz). It shows number of cycles of signal that took place in a second. Large ω or large f value indicates that the signal completes more oscillations (i.e., going from positive values to negative values) in less amount of time. Hence the Signal is more Oscillatory in nature.
Sinusoidal signal need not start at zero. It may start after certain duration of time. This is time after which Sinusoidal Signal starts is indicated with the help of phase difference (Ø). It is measured in Radians.
Periodic signals are those which repeat their pattern after certain amount of time. This time after which pattern is repeated is called time period (T) of Periodic Signal. It is inverse of frequency of Signal.

$$T = \frac{1}{f}$$

Sinusoidal signal is a periodic signal, because the pattern keeps on repeating after one Wavelength as shown in the Figure above.
All the power signals in our home, office and industries are AC sinusoidal signals. The frequency (f) in India and British countries is 50 Hz and in American countries it is 60 Hz.

Why is Sinusoidal Wave Signal so Important?

Sinusoidal signals are important in both electrical and electronic engineering domains.
According to Fourier Series Theory, any signal (Periodic Signal) can be written in terms of only sine and cosine Signals of different frequencies. Therefore a complex signal can be broken-down into simple sine and cosine signals and mathematical analysis becomes easy. Hence it is widely used in electrical and electronic analysis.
Also, in transformers the output voltage is time derivative of magnetic flux. Magnetic flux is itself time derivative of input voltage. But we want same voltage signal both at input and output. The only functions that satisfy this condition are sine and cosine functions. As sine signal starts from zero value, it is preferred. Therefore majority of power systems in the world today are using sinusoidal AC voltage. All the household equipment also work on Sinusoidal AC voltage.

Fourier integrals and Fourier transforms:

Fourier Series at a Glance

A continuous time signal x(t) is said to be periodic if there is a positive non-zero value of T for which

$$x(t+T) = x(t) \quad for\ all\ t \ ....(1)$$

As we know any periodic signal can be classified into harmonically related sinusoids or complex exponential, provided it satisfies the Dirichlet's Conditions. This decomposed representation is called FOURIER SERIES. Two type of Fourier Series representation are there. Both are equivalent to each other.

- Exponential Fourier Series
- Trigonometric Fourier Series

Both representations give the same result. Depending upon the type of signal, we choose any of the representation according to our convenience.

Exponential Fourier Series:

A periodic signal is analyzed in terms of Exponential Fourier Series in the following three stages:

- Representation of Periodic Signal.
- Amplitude and Phase Spectra of a Periodic Signal.
- Power Content of a Periodic Signal.

Representation of Periodic Signal:

A periodic signal in Fourier Series may be represented in two different time domains:

- Continuous Time Domain.
- Discrete Time Domain.

Continuous Time Domain:

The complex Exponential Fourier Series representation of a periodic signal x(t) with fundamental period T_0 is given by

$$\mathbf{x(t)} = \sum_{k=-\infty}^{\infty} \mathbf{c_k e^{jk\omega_0 t}} \quad \omega_0 = \frac{2\pi}{T_0} \quad ....(2)$$

Where, C is known as the Complex Fourier Coefficient and is given by,

$$\mathbf{C_k} = \frac{1}{T_0}\int_{T_0} \mathbf{x(t)e^{-jk\omega_0 t}dt} \qquad ....(3)$$

Where $\int_0^{T_0}$, denotes the integral over any one period and, 0 to T_0 or $-T_0/2$ to $T_0/2$ are the limits commonly used for the integration.

The equation (3) can be derived be multiplying both sides of equation (2) by $e^{(-jl\omega_0 t)}$ and integrate over a time period both sides.

$$\int_0^{T_0} \mathbf{x(t)e^{-jk\omega_0 t}} = \int_0^{T_0} \sum_{k=-\infty}^{\infty} \mathbf{C_k e^{jk\omega_0 t}.e^{-jk\omega_0 t}} \qquad ....(4)$$

On interchanging the order of summation and integration on R.H.S., we get

$$= \sum_{k=-\infty}^{\infty} \mathbf{C_k} \left[\frac{\mathbf{e^{j(k-l)\omega_0 t}}}{\mathbf{j(k-l)\omega_0 t}}\right]_0^{T_0} \qquad ....(5)$$

$$= \sum_{k=-\infty}^{\infty} C_k \int_0^{T_0} e^{j(k-1)w_0 t}$$

$$= \sum_{k=-\infty}^{\infty} \mathbf{C_k} \left[\frac{\mathbf{e^{j(k-l)\omega_0 t}}}{\mathbf{j(k-l)\omega_0 t}}\right]_0^{\mathbf{T_0}} \quad \text{...........}(\mathbf{5})$$

When, k≠l, the right hand side of (5) evaluated at the lower and upper limit yields zero. On the other hand, if k=l, we have

$$\int_0^{T_0} dt = t\,|_0^{T_0} = T_0$$

Consequently equation (4) reduces to

$$\int_0^{T_0} x(t).e^{-jlw_0 t} = C_l T_0$$

$$C_k = \frac{1}{T_0}\int_0^{T_0} x(t).e^{-jkw_0 t}$$

$$When\ \ k = 0,\ \ C_k = \frac{1}{T_0}\int_0^{T_0} x(t)$$

which indicates average value of x(t) over a period.

When x (t) is real,

$$C_{-k} = C_k^*$$

Where, * indicates conjugate

Discrete Time Domain:

Fourier representation in discrete is very much similar to Fourier representation of periodic signal of continuous time domain.

The discrete Fourier series representation of a periodic sequence x[n] with fundamental period N_0 is given by

$$x[n] = \sum_{k=0}^{N_0-1} C_k e^{jk\Omega_0 n} \qquad \Omega_0 = \frac{2\Pi}{N_0}$$

Where, C_k, are the Fourier coefficients and are given by

$$C_k = \frac{1}{N_0}\sum_{k=0}^{N_0-1} x[n]e^{-jk\Omega_0 n}$$

This can be derived in the same way as we derived it in continuous time domain.

Amplitude and Phase Spectra of a Periodic Signal:

We can express Complex Fourier Coefficient, C_k as

$$C_k = |C_k|\, e^{j\phi_k}$$

A plot of $|C_k|$ versus the angular frequency w is called the amplitude spectrum of the periodic signal x(t), and a plot of Φ_k, versus w is called the phase spectrum of x(t). Since the index k assumes only integers, the amplitude and phase spectra are not continuous curves but appear only at the discrete frequencies $k\omega_0$, they are therefore referred to as discrete frequency spectra or line spectra.

For a real periodic signal x (t) we have $C_{-k} = C_k^*$. Thus,

$$|C_{-k}| = |C_k| \quad , \quad \phi_{-k} = -\phi_k$$

Hence, the amplitude spectrum is an even function of ω, and the phase spectrum is an odd function of 0 for a real periodic signal.

Power Content of a Periodic Signal:

Average Power Content of a Periodic Signal is given by

$$P = \frac{1}{T_0} \int_0^{T_0} |x(t)|^2 dt$$

If x (t) is represented by the complex exponential Fourier Series, then

$$Power, \;\; P = \sum_{k=-\infty}^{\infty} |Ck|^2$$

This equation is known as Parseval's identity or Parseval's Theorem.

properties of Fourier transforms physical significance of the Fourier Transform and its application to electrical circuits:

he addition of silicon(Si) in iron(Fe) in right proportions with the help of certain manufacturing process significantly improves the magnetic and electrical properties of iron. By the end of 19th century, it was discovered that the addition of silicon to iron significantly improves the resistivity of iron and so silicon steel or what we know today as electrical steel was developed. It not only brought down the eddy current losses in steel, but significant improvement in magnetic permeability and reduction in magnetostriction was observed. The table below shows how certain electrical and magnetic behaviors of iron changes on addition of silicon.

Material	*Composition (wt %)*	*Initial Relative Permeability* μ_i	*Saturation Flux Density* B_s *[tesla (gauss)]*	*Hysteresis Loss/Cycle* *[J/m³ (erg/cm³)]*	*Resistivity* ρ *(Ω·m)*
Commercial iron ingot	99.95Fe	150	2.14 (21,400)	270 (2700)	1.0×10^{-7}
Silicon–iron (oriented)	97Fe, 3Si	1400	2.01 (20,100)	40 (400)	4.7×10^{-7}