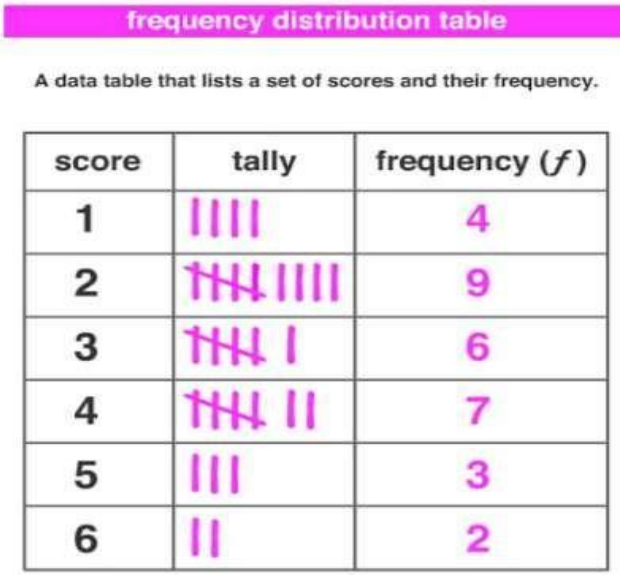
DATA INTERPRETATION

Introduction

- This is the branch of mathematics that deals with the collection, organization, representation and interpretation of data. Data is the basic information.

Frequency Distribution Table

- A data table that lists a set of scores and their frequency



frequency distribution table

A data table that lists a set of scores and their frequency.

score	tally	frequency (f)
1	IIII	4
2	𝍸 IIII	9
3	𝍸 I	6
4	𝍸 II	7
5	III	3
6	II	2

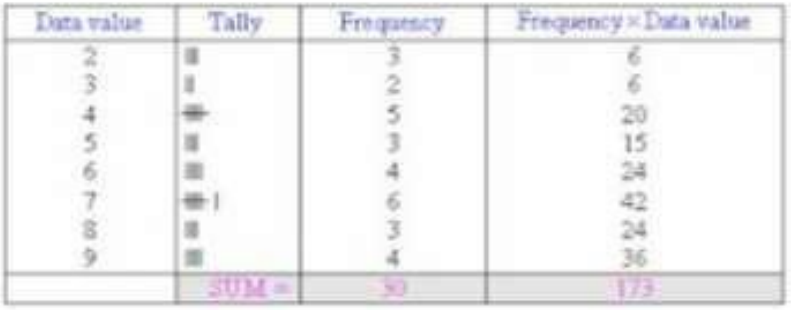
Tally

- In tallying each stroke represent a quantity.

Frequency

Mean

This is usually referred to as arithmetic mean, and is the average value for the data

- 

Data value	Tally	Frequency	Frequency × Data value
2	II	3	6
3	I	2	6
4	IIII	5	20
5	III	3	15
6	IIII	4	24
7	IIII I	6	42
8	III	3	24
9	IIII	4	36
	SUM =	30	173

The mean ($\bar{x}$)

=total marks scored

total number of students

48. $\frac{\sum fx}{\sum f}$

$=\frac{173}{30}$

= 5.767

Mode

- This is the most frequent item or value in a distribution or data. In the above table its 7 which is the most frequent.

Median

- To get the median arrange the items in order of size. If there are N items and N is an odd number, the item occupying $(^{n+1}/_{2})^{th}$
- If N is even, the average of the items occupying $^{n}/_{2}$

Grouped Data

- Then difference between the smallest and the biggest values in a set of data is called the range. The data can be grouped into a convenient number of groups called classes. 30 – 40 are called class boundaries.
- The class with the highest frequency is called the modal class. In this case its $50 \leq m < 60$, the class width or interval is obtained by getting the difference between the class limits. In this case, 30 – 40 = 10, to get the mid-point you divide it by 2 and add it to the lower class limit.

Mass (m) kg	Midpoint	Frequency	Midpoint × Frequency
$30 \leq m < 40$	35	7	245
$40 \leq m < 50$	45	6	270
$50 \leq m < 60$	55	8	440
$60 \leq m < 70$	65	4	260
	Totals:	25	1,215

The mean mass in the table above is $\sum f = 25$, $\sum fx = 1215$ Mean

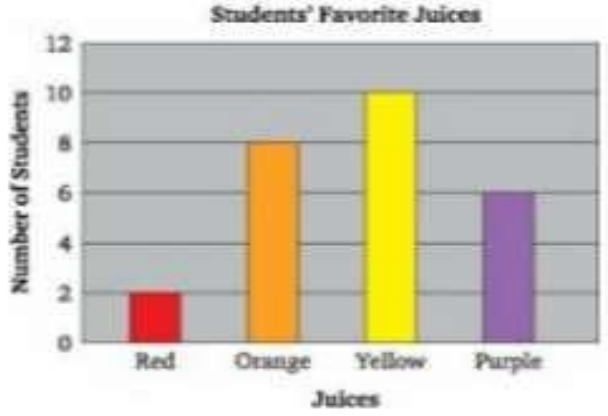
$^{1215}/_{25} = 48.6$

Representation of Statistical Data

- The main purpose of representation of statistical data is to make collected data more easily understood.
- Methods of representation of data include.

Bar Graph

- Consist of a number of spaced rectangles which generally have major axes vertical. Bars are uniform
- The axes must be labelled and scales indicated



width.

- The students' favorite juices are as follows

  Red 2

  Orange 8

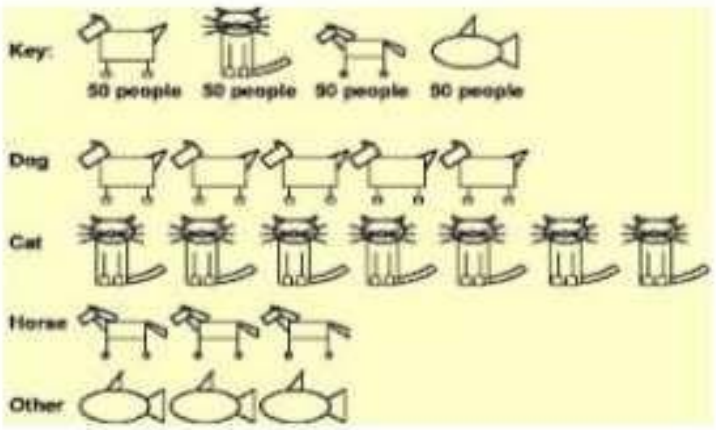
  Yellow 10

  Purple 6

Pictograms

- In a pictogram, data is represented using pictures.

  Consider the following data.



- The data shows the number of people who love the following animals Dogs

  250, Cats 350, Horses 150, other 150

Pie Chart

- A pie chart is divided into various sectors .Each sector represent a certain quantity of the item being considered the size of the sector is proportional to the quantity being measured .consider the export of US to the following countries. Canada $ 1 3390, Mexico $ 81 36, Japan $5824, France $2110.

- This information can be represented in a pie chart as follows

Canada angle

24. amount of export x

360 total population

13390 x 360 = 163.62^{o}

29460

Mexico =8136 x 360 = 99.42^{o}

29460

Japan 5824 x 360 = 71.16^{o}

29460

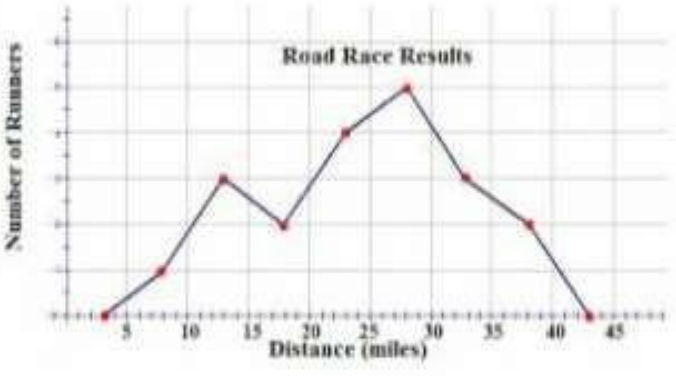
France 2110 x 360 = 25.78^{o}

29460

Line Graph

Data represented using lines

-

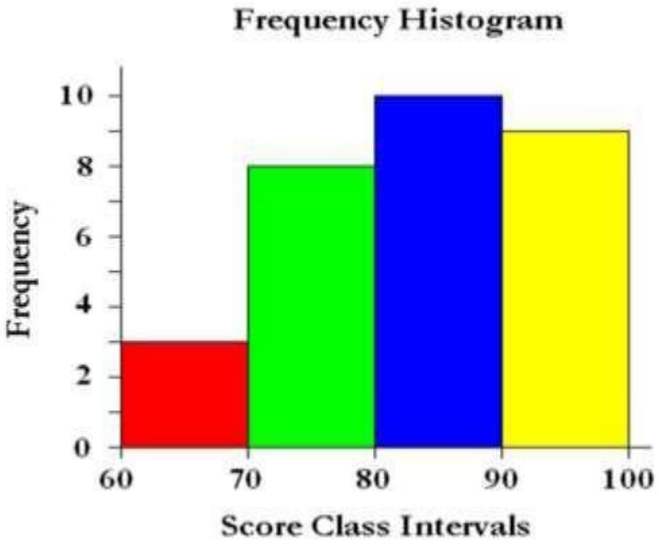


Histograms

- Frequency in each class is represented by a rectangular bar whose area is proportional to the frequency, where the bars are of the same width and the height of the rectangle is proportional to the frequency .

Note;

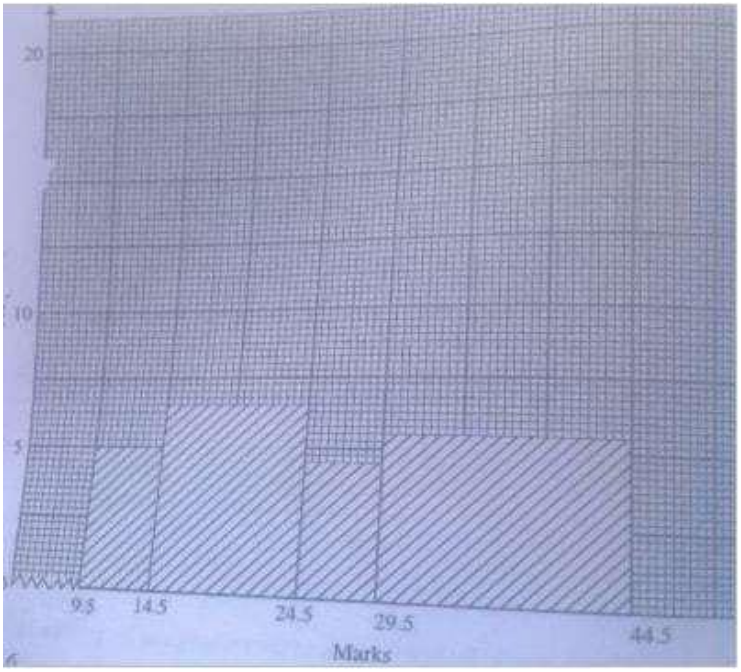
- The bars are joined together.
- The class boundaries are used to label the boundaries of the rectangular bars in the histogram



- Histograms can also be drawn when the class interval is not the same

The below information can be represented in a histogram as below

Marks	10-14	15- 24	25-29	30-44
No.of students	5	16	4	15



Note;

- When the class is doubled the frequency is halved

Frequency Polygon

- It is obtained by plotting the frequency against mid points.

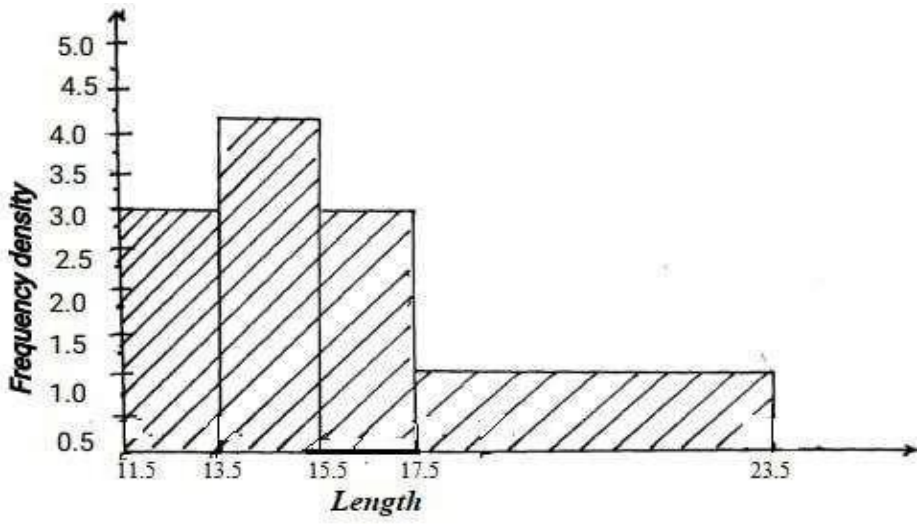
Questions

1. The height of 36 students in a class was recorded to the nearest centimeters as follows. 148 159 163 158 166 155 155 179 158 155 171 172 156 161 160 165 157 165 175 173 172 178 159 168 160 167
147 168 172 157 165 154 170 157 162 173

   Make a grouped table with 145.5 as lower class limit and class width of 5. (4mks)

   Use your table in (a) to draw a histogram to represent the data

2.



Use the histogram above to complete the frequency table below:

Length	Frequency
11 .5 ≤ x ≤ 13.5 13.5 ≤ x ≤ 15.5 15.5 ≤ x ≤ 17.5 17.5 ≤ x ≤ 23.5	

3. Kambui spent her salary as follows:

Food	40%
Transport	10%
Education	20%
Clothing	20%
Rent	10%

Draw a pie chart to represent the above information

4. The examination marks in a mathematics test for 60 students were as follows;-

60	54	34	83	52	74	61	27	65	22
70	71	47	60	63	59	58	46	39	35
69	42	53	74	92	27	39	41	49	54
25	51	71	59	68	73	90	88	93	85
46	82	58	85	61	69	24	40	88	34
30	26	1 7	1 5	80	90	65	55	69	89

Class	Tally	Frequency	Upper class limit
10-29			
30-39			
40-69			
70-74			
75-89			
90-99			

From the table;

a. State the modal class

b. On the grid provided , draw a histogram to represent the above information

5. The marks scored by 200 from 4 students of a school were recorded as in the table below.

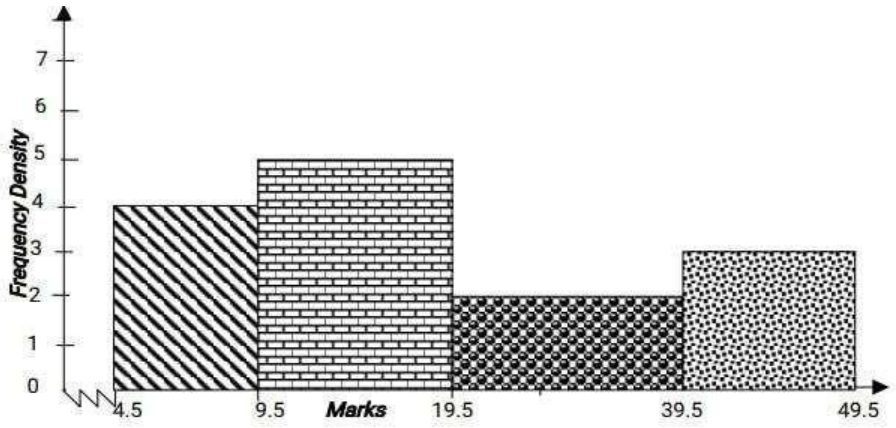
Marks	41–50	51–55	56–65	66–70	71–85
Frequency	21	62	55	50	1 2

a. On the graph paper provided, draw a histogram to represent this information.

b. On the same diagram, construct a frequency polygon.

c. Use your histogram to estimate the modal mark.

6. The diagram below shows a histogram representing the marks obtained in a certain test:-



a. If the frequency of the first class is 20, prepare a frequency distribution table for the data b. State the modal class

3. Estimate:

The mean mark

The median mark

Probability
Introduction

- The likelihood of an occurrence of an event or the numerical measure of chance is called probability.

Experimental Probability

- This is where probability is determined by experience or experiment. What is done or observed is the experiment. Each toss is called a trial and the result of a trial is the outcome.
- The experimental probability of a result is given by (the number of favorable outcomes)
  (the total number of trials)

Example

A boy had a fair die with faces marked 1 to6 .He threw this die up 50 times and each time he recorded the number on the top face. The result of his experiment is shown below.

face	1	2	3	4	5	6
Number of shown up	11	6	7	9	9	8

What is the experimental provability of getting?

49. 1

50.6

Solution

26. P(Event) = the number of favorable outcomes
    the total number of trials

P(1)= $^{11}/_{50}$

b. P(4)= $^{9}/_{50}$

Example

From the past records, out of the ten matches a school football team has played, it has won seven.How many possible games might the school win in thirty matches?

Solution

P (winning in one math) = $^{7}/_{10}$.

Therefore the number of possible wins in thirty matches = $^{7}/_{10}$ x 30 = 21 matches

Range of Probability Measure

- If P(A) is the probability of an event A happening and P(A') is the probability of an event A not happening, Then P(A')= 1 – P(A) and P(A') + P(A)= 1
- Probability are expressed as fractions, decimals or percentages.

Probability Space

- A list of all possible outcomes is **probability space** or **sample space.**
- The coin is such that the head or tail have equal chances of occurring. - The events head or tail are said to be equally likely or equiprobable.

Theoretical Probability

- This can be calculated without necessarily using any past experience or doing any experiment. - The probability of an event happening = $\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

Example

A basket contains 5 red balls, 4 green balls and 3 blue balls. If a ball is picked at random from the basket, find:

47. The probability of picking a blue ball
48. The probability of not picking a red ball

Solution

49. Total number of balls is 1 2
    The number of blue balls is 3
    therefore, P (a blue ball)
    = $^{3}/_{12}$

50. The number of balls which are not red is 7.

    Therefore P (not a red ball)= $^{7}/_{12}$

Example

A bag contains 6 black balls and some brown ones. If a ball is picked at random the probability that it is black is 0.25.

Find the number of brown balls.

Solution

Let the number of balls be x

Then the probability that a black ball is picked at random is $^{6}/_{x}$

Therefore $^{6}/_{x} = 0.25$

$x = 24$

The total number of balls is 24

Then the number of brown balls is $24 - 6 = 18$

Note:

- When all possible outcomes are countable, they are said to be **discrete**.

Types of Probability

Combined Events

- These are probability of two or more events occurring

Mutually Exclusive Events

- Occurrence of one excludes the occurrence of the other or the occurrence of one event depends on the occurrence of the other.
- If A and B are two mutually exclusive events, then (A or B) = P (A) + P (B). For example when a coin is tossed the result will either be a head or a tail.

Example

If a coin is tossed ;

P(head) + P(tail)

$= \frac{1}{2} + \frac{1}{2} = 1$

Note;

- If [OR] is used then we add

Independent Events

- Two events A and B are independent if the occurrence of A does not influence the occurrence of B and vice versa.
- If A and B are two independent events, the probability of them occurring together is the product of their individual probabilities .That is;
  P (A and B) = P (A) x P(B)

Note;

- When we use [AND] we multiply ,this is the multiplication law of probability.

Example

A coin is tosses twice. What is the probability of getting a tail in both tosses?

Solution

The outcome of the 2nd toss is independ of the outcome of the first .

Therefore;

P (T and T) = P(T) X P(T)

$77. \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Example

A boy throws fair coin and a regular tetrahedron with its four faces marked 1,2, 3 and 4. Find the probability that he gets a

3 on the tetrahedron and a head on the coin.

Solution

These are independent events.

P (H) = $^{1}/_{2}$, P(3) = $^{1}/_{4}$

Therefore;

P (H and 3) = P (H) x P (3)

78. $\frac{1}{2} \times \frac{1}{4}$

79. $^{1}/_{8}$

Example

A bag contains 8 black balls and 5 white ones. If two balls are drawn from the bag, one at a time,find the probability of drawing a black ball and a white ball.

80. Without replacement
81. With replacement

Solution

34. There are only two ways we can get a black and a white ball: either drawing a white then a black,or drawing a black then a white. We need to find the two probabilities;

    P(W followed by B) = P (W and B)

    $^{8}/_{13} \times {}^{5}/_{12} = {}^{10}/_{39}$

35. P(B followed by W) = P (B and W)

    $^{5}/_{13} \times {}^{8}/_{12} = {}^{10}/_{39}$

Note;

- The two events are mutually exclusive, therefore.

P (W followed by B) or (B followed by W)= P(W followed by B) + P (B followed by W)

= P (W and B) + P(B and W)

$= {}^{40}/_{156} + {}^{40}/_{156} = {}^{20}/_{39}$

Since we are replacing, the number of balls remains 13.

Therefore;

P (W and B) = $^{5}/_{13} \times {}^{8}/_{13} = {}^{40}/_{169}$

P (B and W) = $^{8}/_{13} \times {}^{5}/_{13} = {}^{40}/_{169}$

Therefore;

P [(W and B) or (B and W)] = P (W and B) + P (B and W)

$= {}^{40}/_{169} + {}^{40}/_{169} = {}^{80}/_{169}$

Example

Kamau ,Njoroge and Kariuki are practicing archery .The probability of Kamau hitting the target is 2/5,that of Njoroge hitting the target is $\frac{1}{4}$ and that of Kariuki hitting the target is $^{3}/_{7}$, Find the probability that in one attempt;

(xxi) Only one hits the target

(xxii) All three hit the target

(xxiii) None of them hits the target

(xxiv) Two hit the target

(xxv) At least one hits the target

Solution

a. P(only one hits the target)

=P (only Kamau hits and other two miss) = $^{2}/_{5} \times {}^{3}/_{5} \times {}^{4}/_{7}$

$= {}^{6}/_{35}$

P (only Njoroge hits and other two miss) = $^{1}/_{4} \times {}^{3}/_{5} \times {}^{4}/_{7}$

$= {}^{3}/_{35}$

P (only Kariuki hits and other two miss) = $^{3}/_{7} \times {}^{3}/_{5} \times \frac{3}{4}$

$= {}^{27}/_{140}$

P (only one hits) = P (Kamau hits or Njoroge hits or Kariuki hits)

$^{6}/_{35} + {}^{3}/_{35} + {}^{27}/_{140}$

$^{9}/_{20}$

22. P (all three hit) = $^{2}/_{5} \times {}^{1}/_{4} \times {}^{3}/_{7}$

$^{3}/_{70}$

23. P (none hits) = $^{3}/_{5} \times {}^{3}/_{4} \times {}^{4}/_{7}$

$^{9}/_{35}$

24.P (two hit the target) is the probability of ;

Kamau and Njoroge hit the target and Kariuki misses = $^{2}/_{5} \times {}^{3}/_{7} \times {}^{4}/_{7}$

Njoroge and Kariuki hit the target and Kamau misses = $^{1}/_{4} \times {}^{3}/_{7} \times {}^{3}/_{5}$

Or

Kamau and Kariuki hit the target and Njoroge misses = $^{2}/_{5} \times {}^{3}/_{7} \times {}^{3}/_{4}$

Therefore P (two hit target) = $(^{2}/_{5} \times {}^{1}/_{4} \times {}^{4}/_{7}) + (^{1}/_{4} \times {}^{3}/_{7} \times {}^{3}/_{5}) + (^{2}/_{5} \times {}^{3}/_{7} \times {}^{3}/_{4})$

$^{8}/_{140} + {}^{9}/_{140} + {}^{18}/_{140}$

$\frac{1}{4}$

25. P (at least one hits the target) = 1 – P (none hits the target)

$1 - {}^{9}/_{35}$

$^{26}/_{35}$

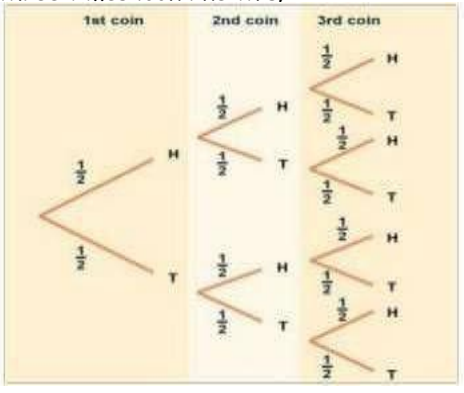
9. $^{26}/_{35}$

Note;

- P (one hits the target) is different from P (at least one hits the target)

Tree Diagram

- Tree diagrams allows us to see all the possible outcomes of an event and calculate their probality.
- Each branch in a tree diagram represents a possible outcome .A tree diagram which represent a coin being tossed three times look like this;



- From the tree diagram, we can see that there are eight possible outcomes. To find out the probability of a particular outcome, we need to look at all the available paths (set of branches).
- The sum of the probabilities for any set of branches is always 1 .
- Also note that in a tree diagram to find a probability of an outcome we multiply along the branches and add vertically.
  - The probability of three heads is:

$$P(HHH) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

P (2 Heads and a Tail) = P (H H T) + P (H T H) + P (T H H)

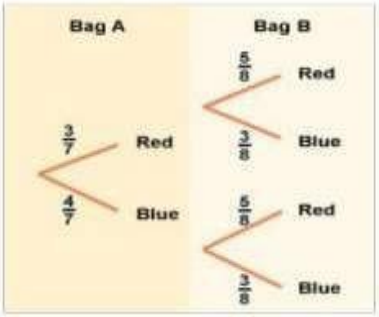
81. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

82. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

83. $\frac{3}{8}$

Example

Bag A contains three red marbles and four blue marbles.Bag B contains 5 red marbles and three blue marbles. A marble is taken from each bag in turn.



18. What is the probability of getting a blue bead followed by a red
19. What is the probability of getting a bead of each color

Solution

a. Multiply the probabilities together

P(blue and red) = $^{4}/_{7} \times {}^{5}/_{8} = {}^{20}/_{56}$

$= {}^{5}/_{14}$

b. P(blue and red or red and blue) = P(blue and red) + P (red and blue)

$= {}^{4}/_{7} \times {}^{5}/_{8} + {}^{3}/_{7} \times {}^{3}/_{8}$

$= {}^{20}/_{56} + {}^{9}/_{56}$

$= {}^{29}/_{56}$

Example

The probability that Omweri goes to Nakuru is $\frac{1}{4}$ .If he goes to Nakuru, the probability that he will see flamingo is 19. If he does not go to Nakuru, the probability that he will see flamingo is $^{1}/_{3}$. Find the probability that;

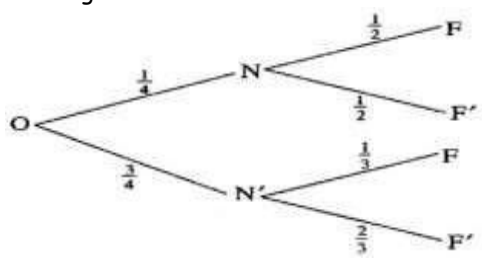
Omweri will go to Nakuru and see a flamingo.

Omweri will not go to Nakuru yet he will see a flamingo

Omweri will see a flamingo

Solution

Let N stand for going to Nakuru ,N' stand for not going to Nakuru, F stand for seeing a flamingo and F' stand for not seeing a flamingo.



115. P (He goes to Nakuru and sees a flamingo) = P(N and F) P(N) × P(F)

$\frac{1}{4} \times \frac{1}{2}$

$1/8$

116. P(He does not go to Nakuru and yet sees a flamingo) =P(N') × P(F) P (N' and F)

$^{3}/_{4} \times {}^{1}/_{3}$

$\frac{1}{4}$

117. P (He sees a flamingo) = P(N and F) or P (N' and F) P (N and F) + P (N' and F)

18. $^{1}/_{8} + {}^{1}/_{4}$
19. $^{3}/_{8}$

Questions

19. The probabilities that a husband and wife will be alive 25 years from now are 0.7 and 0.9 respectively.
    Find the probability that in 25 years time,
        Both will be alive
        Neither will be alive
        One will be alive
        At least one will be alive
20. A bag contains blue, green and red pens of the same type in the ratio 8:2:5 respectively. A pen is picked at random without replacement and its colour noted
        Determine the probability that the first pen picked is
            Blue
            Either green or red
        Using a tree diagram, determine the probability that
            The first two pens picked are both green
            Only one of the first two pens picked is red.
21. A science club is made up of boys and girls. The club has 3 officials. Using a tree diagram or otherwise find the probability that:
        The club officials are all boys
        Two of the officials are girls
22. Two baskets A and B each contain a mixture of oranges and limes, all of the same size. Basket A contains 26 oranges and 13 limes. Basket B contains 18 oranges and 15 limes. A child selected a basket at random and picked a fruit at a random from it.
        Illustrate this information by a probabilities tree diagram
        Find the probability that the fruit picked was an orange.
23. In a class there are 22 girls and boys. The probability of a girl completing the secondary education course is 3 whereas that of a boy is 2/3
        A student is picked at random from class. Find the possibility that,
            The student picked is a boy and will complete the course
            The student picked will complete the course
        Two students are picked at random. Find the possibility that they are a boy and a girl and that both will not complete the course.
24. Three representatives are to be selected randomly from a group of 7 girls and 8 boys. Calculate the probability of selecting two girls and one boy.
25. A poultry farmer vaccinated 540 of his 720 chickens against a disease. Two months later, 5% of the vaccinated and 80% of the unvaccinated chicken, contracted the disease. Calculate the probability that a chicken chosen random contacted the disease.
26. The probability of three darts players Akinyi, Kamau, and Juma hitting the bulls eye are 0.2, 0.3 and 1 .5 respectively.

        Draw a probability tree diagram to show the possible outcomes
        Find the probability that:
            All hit the bull's eye

Only one of them hit the bull's eye

At most one missed the bull's eye

9.

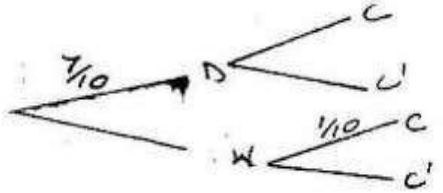
169. An unbiased coin with two faces, head (H) and tail (T), is tossed three times, list all the possible outcomes.

Hence determine the probability of getting:

At least two heads

Only one tail

21. During a certain motor rally it is predicted that the weather will be either dry (D) or wet (W). The probability that the weather will be dry is estimated to be 7/10. The probability for a driver to complete (C) the rally during the dry weather is estimated to be 5/6. The probability for a driver to complete the rally during wet weather is estimated to be 1/10. Complete the probability tree diagram given below.



What is the probability that:

The driver completes the rally?

The weather was wet and the driver did not complete the rally?

2 There are three cars A, B and C in a race. A is twice as likely to win as B while B is twice as likely to win as c. Find the probability that.

a. A wins the race
b. Either B or C wins the race.

3 In the year 2003, the population of a certain district was 1.8 million. Thirty per cent of the population was in the age group 15 – 40 years. In the same year, 1 20,000 people in the district visited the Voluntary Counseling and Testing (VCT) centre for an HIV test. If a person was selected at random from the district in this year. Find the probability that the person visited a VCT centre and was in the age group 1 5 – 40 years.

12.

a Two integers x and y are selected at random from the integers 1 to 8. If the same integer may be selected twice, find the probability that

i $|x - y| = 2$
ii $|x - y|$ is 5 or more
iii $x>y$

b A die is biased so that when tossed, the probability of a number r showing up, is given by $p(r)= Kr$ where K is a constant and r = 1 , 2,3,4,5 and 6 (the number on the faces of the die

i Find the value of K
ii If the die is tossed twice, calculate the probability that the total score is 11

(%) Two bags A and B contain identical balls except for the colours. Bag A contains 4 red balls and 2 yellow balls. Bag B contains 2 red balls and 3 yellow balls.

If a ball is drawn at random from each bag, find the probability that both balls are of the same colour.

If two balls are drawn at random from each bag, one at a time without replacement, find the probability that:

The two balls drawn from bag A or bag B are red

All the four balls drawn are red

(%) During inter – school competitions, football and volleyball teams from Mokagu high school took part. The probability that their football and volleyball teams would win were 3/8 and 4/7 respectively. Find the probability that

Both their football and volleyball teams

At least one of their teams won

(%) A science club is made up of 5 boys and 7 girls. The club has 3 officials. Using a tree diagram or otherwise find the probability that:

The club officials are all boys

Two of the officials are girls

(%) Chicks on Onyango's farm were noted to have either brown feathers brown or black tail feathers. Of those with black feathers $^{2}/3$ were female while 2/5 of those with brown feathers were male. Otieno bought two chicks from Onyango. One had black tail feathers while the other had brown find the probability that Otieno's chicks were not of the same gender was

(%) Three representatives are to be selected randomly from a group of 7 girls and 8 boys. Calculate the probability of selecting two girls and one boy

(%) The probability that a man wins a game is $\frac{3}{4}$. He plays the game until he wins. Determine the probability that he wins in the fifth round.

10. The probability that Kamau will be selected for his school's basketball team is $\frac{1}{4}$. If he is selected for the basketball team. Then the probability that he will be selected for football is 1/3 if he is not selected for basketball then the probability that he is selected for football is 4/5. What is the probability that Kamau is selected for at least one of the two games?
11. Two baskets A and B each contains a mixture of oranges and lemons. Baskets A contains 26 oranges and 13 lemons. Baskets B contains 1 8 oranges and 15 lemons. A child selected a basket at random and picked at random a fruit from it. Determine the probability that the fruit picked an orange.