Dynamic Linear Programming

Disequilibrium- Known Life

Primal Algebra:

$$\text{Max} \quad \sum_t \sum_j (1+r)^{-t} C_{jt} X_{j,t} \quad + \quad (1+r)^{-T} \quad \sum_j \sum_{\substack{e \\ e<K_j}} F_{je} I_{je}$$

$$\begin{array}{llllll}
\text{s.t.} & \sum_j \sum_{e<K_j} A_{ije} X_{j,t-e} & & \leq & b_{it} & \text{for all i and t} \\
& X_{j,-e} & & = & X^*_{j,-e} & \text{for all } e>0 \text{ and } e<K_j \text{ and all j} \\
& -\; X_{j,T-e} \quad + & I_{je} & = & 0 & \text{for all } e<K_j \text{ and all j} \\
& X_{j,t}, & I_{je} & \geq & 0 & \text{for all j, t and } e<K_j
\end{array}$$

Where

$$C_{jt} = \sum_e (1+r)^{-(t+e)} h_{je}$$

$$\text{where } e \leq \{K_j - t, T - t\}$$

Inspecting the resource equation

$$\sum_{j} \sum_{e=0}^{e<K_j} A_{ije} X_{j,t-e} \leq b_{it} \text{ for year } t$$

Suppose K_j =3 and for we expand the sum over e

$$\sum_j (A_{ij0} X_{j,t-0} + A_{ij1} X_{j,t-1} + A_{ij2} X_{j,t-2}) \leq b_{it} \quad \text{This is for year } t$$

$A_{ij0} X_{j,t}$ is the $e{=}0$ case and gives resource use in year when activity is new ($e{=}0$) A_{ij0} times the amount started in year t-e or t-0 or t $X_{j,t}$

$A_{ij1} X_{j,t-1}$ is the e=1 case and gives resource use when activity is one year old (e=1) A_{ij1} times the amount started last year in year t-1 (t-e) $X_{j,t-1}$

$A_{ij2}X_{j,t-2}$ is the e=2 case and gives resource use when activity is two years old (e=2) A_{ij2} times the amount started two years ago in year t-2 $X_{j,t-2}$

So we add up resource use for new plus one year old plus two years old when we discontinue in start of year 3

Dynamic Linear Programming

Disequilibrium- Known Life / Example

The Strawberry Problem:

	Year 0	Year 1	Year 2
Cost/Acre	150	280	300
Yield/acre in tons	0	7	7
Water/acre in acre-feet	.8	4.5	4.5

The price per ton of strawberries is $140. The farm has 700 acres, 50 planted in 1 year old strawberries and 10 in 0 year old strawberries. Water available consists of 1200 acre ft. per year.

We need terminal conditions to value those items which are carried into the fifth and beyond years. Assume that the following values have been derived.

Product Terminal Value

New strawberries (0 years old the year after the model)	$160/acre
1 year old strawberries	$110/acre

Dynamic Linear Programming

Disequilibrium- Known Life / Example

Table 8.1. Disequilibrium Known Life Example

Rows	Year 1		Year 2		Year 3		Year 4		Year 5		Terminal Conditions Strawberries		RHS	
	Wheat	Straw-berries	Wheat	Straw-berries	Wheat	Straw-berries	Wheat	Straw-berries	Wheat	Straw-berries	Aje 0	Aje 1		
Objective	340	1230	340	1230	340	1230	340	550	340	-150	160	110		Max
Land Year 1	1	1											≤	640
Water Year 1	1	0.8											≤	930
Land Year 2		1	1	1									≤	690
Water Year 2		4.5	1	0.8									≤	1155
Land Year 3		1		1	1	1							≤	700
Water Year 3		4.5		4.5	1	0.8							≤	1200
Land Year 4				1		1	1	1					≤	700
Water Year 4				4.5		4.5	1	0.8					≤	1200
Land Year 5						1		1	1	1			≤	700
Water Year 5						4.5		4.5	1	0.8			≤	1200
Strawberry 0										-1	1		≤	0
Strawberry 1								-1				1	≤	0

Dynamic Linear Programming

Disequilibrium- Known Life / Example

Solutions:

Objective = 1224296

Variables	Value	Reduced Cost	Equation	Slack	Shadow Price
Wheat Year 1	506.2	0	Land Year 1	0	340.0
Strawberries Year 1	133.8	0	Water Year 1	316.76	0
Wheat Year 2	539.9	0	Land Year 2	0	283.1
Strawberries Year 2	16.3	0	Water Year 2	0	56.9
Wheat Year 3	423.4	0	Land Year 3	0	336.9
Strawberries Year 3	126.6	0	Water Year 3	0	3.1
Wheat Year 4	557.1	0	Land Year 4	0	279.8
Strawberries Year 4	0	0	Water Year 4	0	60.2
Wheat Year 5	573.4	0	Land Year 5	0	340.0
Strawberries Year5	0	0	Water Year 5	57.05	0
Term Straw -0	0	0	Strawberry 0	0	-160
Term Straw -1	0	-8	Strawberry 1	0	-118

Dynamic Linear Programming
Disequilibrium- Unknown Life

Primal Algebra

$$
\begin{array}{llllll}
\text{Max} & \sum_{t}\sum_{j}\sum_{e \le K_j}(1+r)^{-t} C_{je} X_{j,t,e} + (1+r)^{-T} \sum_{j}\sum_{e<K_j} F_{je} I_{je} & & & \\
\text{s.t.} & \sum_{j}\sum_{e \le K_j} A_{ije} X_{j,t,e} & \le & b_{it} & \text{for all i and t} \\
 & X_{j,0,e} & = & X^{*}_{j,0,e} & \text{for all j and } e < K_j \\
 & -X_{j,T,e} + I_{je} & = & 0 & \text{for all j and } e < K_j \\
 & -X_{j,t-1,e-1} + X_{j,t,e} & \le & 0 & \text{for all t, j and } e > 0 \text{ and } e \le K_j \\
 & X_{j,t,e} \;,\; I_{je} & \ge & 0 & \text{for all j, t, e}
\end{array}
$$

Dynamic Linear Programming

Disequilibrium- Unknown Life / Example

	Year 0	Year 1	Year 2
Cost/Acre	150	280	300
Yield/acre in tons	0	7	7
Water/acre in acre-feet	.8	4.5	4.5

For this example, we use the data above plus longer retention of strawberries. Assume they may be kept up to 4 years and that in the fourth year (Year 3) the planting costs $400 per acre with the yield being 5, and water use being 5.7 acre feet. We will also assume that the terminal value of 3-year old strawberries is $20/acre.

Disequilibrium- Unknown Life / Example

Table 8.3 Disequilibrium Unknown Life Sample Problem

	Year 1					Year 2					Year 3					Year 4					Year 5					Terminal Conditions			
		Strawberries					Strawberries					Strawberries					Strawberries					Strawberries				Strawberries			RHS
Rows	Wheat	0	1	2	3	Wheat	0	1	2	3	Wheat	0	1	2	3	Wheat	0	1	2	3	Wheat	0	1	2	3	0	1	2	
Objective	340	-150	700	680	300	340	-150	700	680	300	340	-150	700	680	300	340	-150	700	680	300	340	-150	700	680	300	160	110	20	Max
Land Year 1	1	1	1	1	1																								≤ 700
Water Year 1	1	0.8	4.5	4.5	5.7																								≤ 1200
Init Straw 0			1																										≤ 10
Init Straw 1				1																									≤ 50
Init Straw 2					1																								≤ 0
Straw 0-1 Year 1		-1						1																					≤ 0
Straw 1-2 Year 1			-1						1																				≤ 0
Straw 2-3 Year 1				-1						1																			≤ 0
Land Year 2						1	1	1	1	1																			≤ 700
Water Year 2						1	0.8	4.5	4.5	5.7																			≤ 1200
Straw 0-1 Year 2							-1						1																≤ 0
Straw 1-2 Year 2								-1						1															≤ 0
Straw 2-3 Year 2									-1						1														≤ 0
Land Year 3											1	1	1	1	1														≤ 700
Water Year 3											1	0.8	4.5	4.5	5.7														≤ 1200
Straw 0-1 Year 3												-1						1											≤ 0
Straw 1-2 Year 3													-1						1										≤ 0
Straw 2-3 Year 3														-1						1									≤ 0
Land Year 4																1	1	1	1	1									≤ 700
Water Year 4																1	0.8	4.5	4.5	5.7									≤ 1200
Straw 0-1 Year 4																	-1						1						≤ 0
Straw 1-2 Year 4																		-1						1					≤ 0
Straw 2-3 Year 4																			-1						1				≤ 0
Land Year 5																					1	1	1	1	1				≤ 700
Water Year 5																					1	0.8	4.5	4.5	5.7				≤ 1200
Term Straw 0																						-1				1			≤ 0
Term Straw 1																							-1				1		≤ 0
Term Straw 2																								-1				1	≤ 0

Dynamic Linear Programming

Disequilibrium- Unknown Life / Example

Solutions:

Table 8.4. Disequilibrium Unknown Life Example Model Solution

Objective = 1280757.0

Variable	Value	Reduced Cost	Equation	Slack	Shadow Price
Wheat year 1	547.1	0	Land Year 1	0	340
Straw 0 year old year 1	92.9	0	Water Year 1	308.57	0
Straw 1 year old year 1	50.0	0	Init Straw 0	0	490
Straw 2 year old year 1	10.0	0	Init Straw 1	0	340
Straw 3 year old year 1	0	-40	Init Straw 2	0	0
Wheat year 2	557.1	0	Straw 0-1 Year 1	0	-490
Straw 0 year old year 2	0	-8	Straw 1-2 Year 1	0	-130
Straw 1 year old year 2	92.9	0	Straw 2-3 Year 1	10	0
Straw 2 year old year 2	50.0	0	Land Year 2	0	280
Straw 3 year old year 2	0	-322	Water Year 2	0	60
Wheat year 3	464.3	0	Straw 0-1 Year 2	0	-470
Straw 0 year old year 3	142.9	0	Straw 1-2 Year 2	0	-340
Straw 1 year old year 3	0	0	Straw 2-3 Year 2	50	0
Straw 2 year old year 3	92.9	0	Land Year 3	0	340
Straw 3 year old year 3	0	-40	Water Year 3	203.57	0
Wheat year 4	557.1	0	Straw 0-1 Year 3	0	-490
Straw 0 year old year 4	0	0	Straw 1-2 Year 3	0	-110
Straw 1 year old year 4	142.9	0	Straw 2-3 Year 3	92.857	0
Straw 2 year old year 4	0	0	Land Year 4	0	274
Straw 3 year old year 4	0	-349	Water Year 4	0	65.7
Wheat year 5	557.1	0	Straw 0-1 Year 4	0	-470
Straw 0 year old year 5	0	-330	Straw 1-2 Year 4	0	-360
Straw 1 year old year 5	0	0	Straw 2-3 Year 4	0	0
Straw 2 year old year 5	142.9	0	Land Year 5	0	340
Straw 3 year old year 5	0	-40	Water Year 5	0	0
Term Straw 0		0	Term Straw 0	0	-160
Term Straw 1		0	Term Straw 1	0	-110
Term Straw 2		0	Term Straw 2	142.86	-20

Dynamic Linear Programming
Equilibrium- Known Life

Equilibrium models are developed as follows: assume we have a variable with a life of 4 periods and resource use, yield, etc., equal to A_e, where e is the elapsed age of the activity (0-3). Let us (assuming we start with zero initial activity) portray the resource use over several periods.

	Begin Activity in Period						
	1	2	3	4	5	6	7
Period 1 Resource Usage	A_0						
Period 2 Resource Usage	A_1	A_0					
Period 3 Resource Usage	A_2	A_1	A_0				
Period 4 Resource Usage	A_3	A_2	A_1	A_0			
Period 5 Resource Usage		A_3	A_2	A_1	A_0		
Period 6 Resource Usage			A_3	A_2	A_1	A_0	
Period 7 Resource Usage				A_3	A_2	A_1	A_0

$$A_3X_{t-3}+ A_2X_{t-2}+ A_1X_{t-1}+ A_0X_t$$
$$X_{t-3}=X_{t-2}= X_{t-1}=X_t=X$$
$$A_3X+ A_2X+ A_1X+ A_0X$$
$$(A_3+ A_2+ A_1+ A_0)X$$

Algebraically

$$\begin{array}{llcll} \text{Max} & \sum_j C_j X_j & & & \\ \text{s.t.} & \sum_j A_{ij} X_j & \leq & b_i & \text{for all j} \\ & X_j & \geq & 0 & \text{for all j} \end{array}$$

Dynamic Linear Programming
Equilibrium- Known Life
Example 1a

Table 8.5. Equilibrium Known Life Example Formulation

	Wheat	Strawberries		
Objective	340	1230		
Land	1	3	$\leq$	700
Water	1	9.8	$\leq$	1200

Table 8.6. Equilibrium Known Life Example Solution

Objective = 253441

Variables	Value	Reduced Cost	Equation	Slack	Shadow Price
Wheat	479	0	Land	0	309
Strawberries	74	0	Water	0	31

Dynamic Linear Programming
Equilibrium- Known Life
Example 1b

The above model can also be re-expressed in terms of average resource use. This is done in Table 8.7, where an average of one acre of land is used every year generating an average of $410 and the usage of 3.27 acre feet of water. The solution for this model is essentially identical to the solution for the previous model, but the strawberry variable equals 221. This indicates that the equilibrium solution averages 221 acres of strawberries. Thus, in the strawberry rotation, one-third of the 221 acres (or 74 as in the earlier model) would be first year, one-third second year and one-third third year.

Table 8.7. Equilibrium Known Life Example Formulation with Average Activities

	Wheat	Strawberries		
Objective	340	410		
Land	1	1	$\leq$	700
Water	1	3.27	$\leq$	1200

Dynamic Linear Programming
Equilibrium- Known Life

Toward a crop rotation

Suppose we can grow 2 crops
Suppose their yields depend on what proceeds them on the land
(i.e. corn after corn has a different yield and cost structure than corn after soybeans. and soybeans after soybeans has a different performance than soybeans after corn)

Let's look at modeling land precedence first in a one-year model

		------------- Year1 ------------						------------- Year2 ------------						
		Plow after corn	Plow after Soy	Plant corn after corn	Plant corn after soy	Plant soy after corn	Plant soy after Soy	Plow after corn	Plow after Soy	Plant corn after corn	Plant corn after soy	Plant soy after corn	Plant soy after Soy	
-Year1-	**Land after corn**	**1**												**≤ last yr corn**
	Land after soybeans		**1**											**≤ last yr soy**
	Land Plowed aft corn	-1		1		1								≤ 0
	Land Plowed aft soy		-1		1		1							≤ 0

- Year 2 -	**Land after corn next yr**			**-1**	**-1**									≤ 0
	Land aft soybeans nxt yr					**-1**	**-1**							≤ 0

Dynamic Linear Programming

Equilibrium- Known Life

Toward a crop rotation

Now let's look at the land precedence in a two year model

		------------- Year1 ------------						------------- Year2 ------------						
		Plow after corn	Plow after Soy	Plant corn after corn	Plant corn after soy	Plant soy after corn	Plant soy after Soy	Plow after corn	Plow after Soy	Plant corn after corn	Plant corn after soy	Plant soy after corn	Plant soy after Soy	
- Year 1 -	Land after corn	1												≤ last yr corn
	Land after soybeans		1											≤ last yr soy
	Plowed aft corn	-1		1		1								≤ 0
	Plowed aft soy		-1		1		1							≤ 0
- Year 2 -	**Land after corn**			**-1**	**-1**			**1**						**≤ 0**
	Land after soybeans					**-1**	**-1**		**1**					**≤ 0**
	Plowed aft corn							-1		1		1		≤ 0
	Plowed aft soy								-1		1		1	≤ 0

Dynamic Linear Programming

Equilibrium- Known Life

Toward a crop rotation

But what if we go equilibrium - Lets look at the land precedence
Since next years plowing equals this years

		------------- Year1 ------------						~~------------ Year2 ------------~~						
		Plow after corn	Plow after Soy	Plant corn after corn	Plant corn after soy	Plant soy after corn	Plant soy after Soy	~~Plow after corn~~	~~Plow after Soy~~	~~Plant corn after corn~~	~~Plant corn after soy~~	~~Plant soy after corn~~	~~Plant soy after Soy~~	
-Year1-	Land after corn	1		-1	-1									≤ 0
	Land after soybeans		1			-1	-1							≤ 0
	Plowed aft corn	-1		1		1								≤ 0
	Plowed aft soy		-1		1		1							≤ 0
~~-Year2-~~	~~Land after corn~~			~~-1~~	~~-1~~			~~1~~						~~≤ 0~~
	~~Land after soybeans~~					~~-1~~	~~-1~~		~~1~~					~~≤ 0~~
	~~Plowed aft corn~~							~~-1~~		~~1~~		~~1~~		~~≤ 0~~
	~~Plowed aft soy~~								~~-1~~		~~1~~		~~1~~	~~≤ 0~~

	-------- Typical Year -------						
	Plow after corn	Plow after Soy	Plant corn after corn	Plant corn after soy	Plant soy after corn	Plant soy after Soy	
Land after corn	1		-1	-1			≤ 0
Land after soybeans		1			-1	-1	≤ 0
Plowed aft corn	-1		1		1		≤ 0
Plowed aft soy		-1		1		1	≤ 0

Dynamic Linear Programming

Equilibrium- Known Life
Example 2

Technical Data

	Corn	Soybeans
Prices/Unit	$2.50	$6.50
Production Cost/ Acre	100	50
Labor Use in hrs/acre		
Plowing	.4	.3
Planting	.15	.15
Harvesting	.35	.17

Yields:

		Corn after Corn Planting Period		Corn After Soybeans Planting Period		Soybeans (after Anything) Planting Period	
		Pd2	Pd3	Pd2	Pd3	Pd2	Pd3
Harvest	Pd4	130	120	145	133	35	45
Period	Pd5	125	110	137	129	33	42

Labor Availability (hrs)

Period	Available Labor
Post harvest/Preplant (Pd1)	80

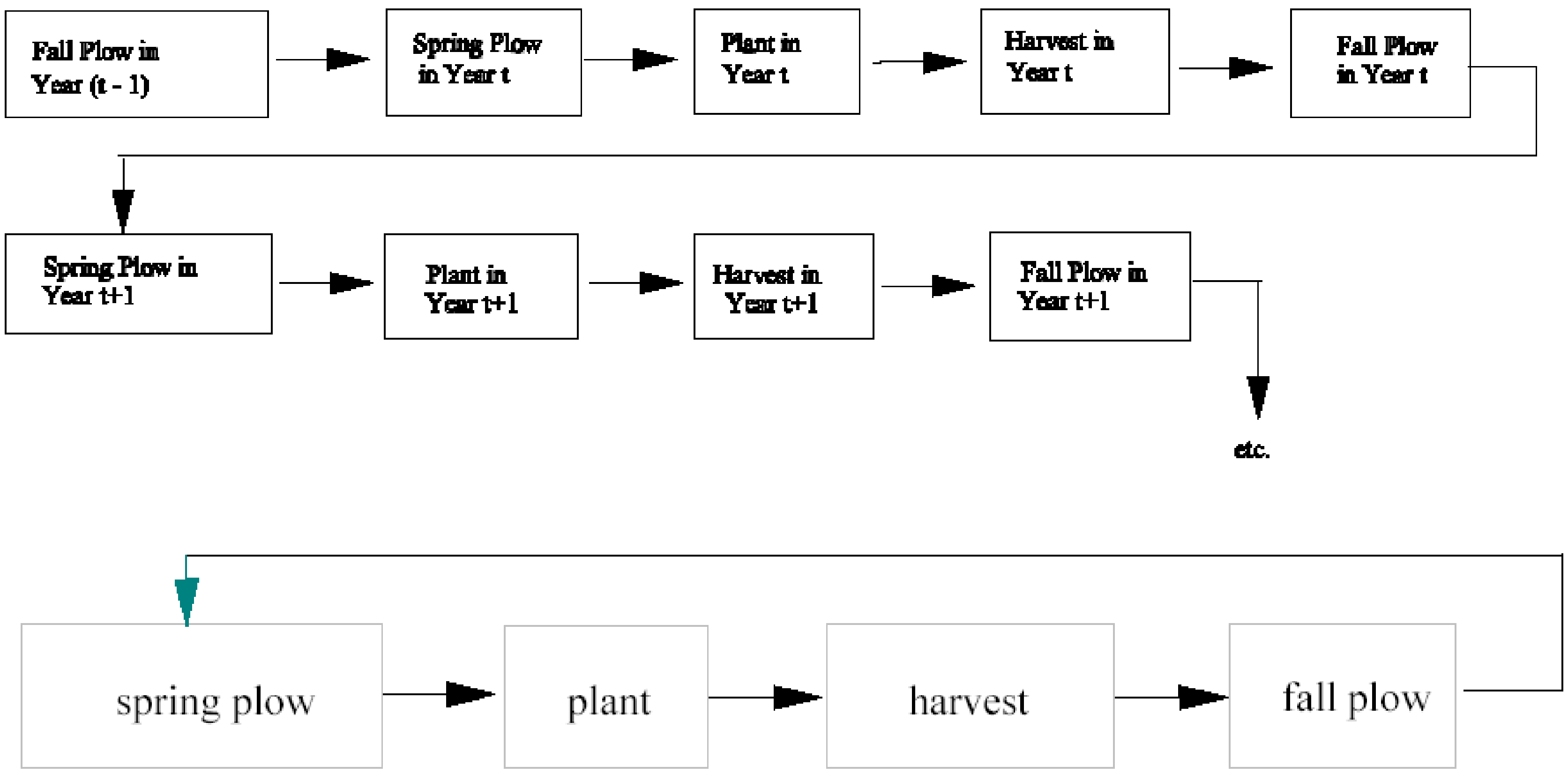
Plant (Pd2)	65
Plant (Pd3)	75
Harvest (Pd4)	100
Harvest (Pd5)	80

Dynamic Linear Programming

Equilibrium- Known Life

Example 2

The Dynamic Process



Dynamic Linear Programming

Equilibrium- Known Life / Example 2

Table 8.8 Rotation Example in Equilibrium Known Life Context

											Plant Corn								Plant Soybeans								RHS
	Plow after corn					Plow after soybeans					after corn				after soybeans				after corn				after soybeans				
Rows											pl2	pl2	pl3	pl3	pl2	pl2	pl3	pl3	pl2	pl2	pl3	pl3	pl2	pl2	pl3	pl3	
	pd1	pd2	pd3	pd4	pd5	pd1	pd2	pd3	pd4	pd5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	hr4	hr5	
Objective											225	213	200	175	263	243	233	223	178	165	243	223	178	165	243	223	Max
Land	1	1	1	1	1	1	1	1	1	1																	≤ 400
Labor pd1	.4					.3																					≤ 80
Labor pd2		.4					.3				.2	.2			.2	.2			.2	.2			.2	.2			≤ 65
Labor pd3			.4					.3					.2	.2			.2	.2			.2	.2			.2	.2	≤ 75
Labor pd4				.4					.3		.4		.4		.4		.4		.3		.3		.3		.3		≤ 100
Labor pd5					.4					.3		.4		.4		.4		.4		.3		.3		.3		.3	≤ 80
plow after corn pd4				1							-1		-1		-1		-1										≤ 0
plow after corn pd5	1	1	1	1	1						-1	-1	-1	-1	-1	-1	-1	-1									≤ 0
plow after soyb pd4									1										-1		-1		-1		-1		≤ 0
plow after soyb pd5						1	1	1	1	1									-1	-1	-1	-1	-1	-1	-1	-1	≤ 0
plnt aft plow corn pd2	-1	-1		-1	-1						1	1							1	1							≤ 0
plnt aft plow corn pd3	-1	-1	-1	-1	-1						1	1	1	1					1	1	1	1					≤ 0
plnt aft plow soy pd2						-1	-1		-1	-1					1	1							1	1			≤ 0
plnt aft plow soy pd3						-1	-1	-1	-1	-1					1	1	1	1					1	1	1	1	≤ 0

Dynamic Linear Programming

Equilibrium- Known Life / Example 2

Table 8.9. Solution to Rotation Model

Objective= 100777

Variable	Value	Reduced Cost
Plow After Corn		
pd1	50	0
pd2	87.5	0
pd3	72.5	0
pd4	0	-22
pd5	0	0
Plow After Soybeans		
pd1	0	0
pd2	0	0
pd3	0	0
pd4	0	-17
pd5	200	0
Plant corn after corn		
pl2 hr4	0	-33
pl2 hr5	0	-26
pl3 hr4	0	-58
pl3 hr5	0	-63
Plant Corn After Soybeans		
pl2 hr4	200	0
pl2 hr5	0	-11
pl3 hr4	0	-30
pl3 hr5	0	-31
Plant Soybeans After Corn		
pl2 hr4	0	-70
pl2 hr5	0	-59
pl3 hr4	189	0
pl3 hr5	11	0
Plant Soybeans After Soybeans		
pl2 hr4	0	-70
pl2 hr5	0	-74
pl3 hr4	0	-5
pl3 hr5	0	-15

Equation	Slack	Shadow Price
Land	0	238
Labor pd1	80	0
Labor pd2	0	0
Labor pd3	16	0
Labor pd4	0	56
Labor pd5	0	0
Plant After Plow		
corn pd2	128	0
corn pd3	0	238
soybeans pd2	0	0
soybeans pd3	0	253
Plow After		
corn pd4	200	0
corn pd5	0	0
soyb pd4	189	0
soyb pd5	0	15

Dynamic Linear Programming

Equilibrium- Unknown Life

$$
\begin{array}{llcccccl}
\text{Max} & \sum_j \sum_e & & & C_{je}X_{je} & & & \\
\text{s.t.} & \sum_j \sum_e & & & A_{ije}X_{je} & \leq & b_i & \text{for all } i \\
& & -X_{j,e-1} & + & X_{je} & \leq & 0 & \text{for all } j \text{ and } e > 0 \\
& & & & X_{je} & \geq & 0 & \text{for all } j \text{ and } e
\end{array}
$$

where: X_{je} is the quantity of the jth activity produced and kept until it is e periods old;

C_{je} is the per unit returns to X_{je};

A_{ije} is the per unit usage of resource i by X_{je};

b_i is the amount of resource i available;

Objective: The model maximizes profits subject to constraints on resource use and age sequencing.

Constraints: The age sequencing constraints state that the amount of enterprise j of age e must be less than or equal to the amount of that enterprise that was kept until age e-1.

Dynamic Linear Programming

Equilibrium- Unknown Life / Example

Table 8.10. Equilibrium Unknown Life Example Formulation

		Strawberries				RHS	
Rows	Wheat	1	2	3	4	MAX	
Objective	340	-150	700	680	300		
Land	1	1	1	1	1	≤	700
Water	1	0.8	4.5	4.5	5.7	≤	1200
Straw 1-2		-1	1			≤	0
Straw 2-3			-1	1		≤	0
Straw 3-4				-1	1	≤	0

Dynamic Linear Programming

Equilibrium- Unknown Life / Example

Table 8.11. Equilibrium Unknown Life Example Solution

Objective = 253441

Variables	Value	Reduced Cost	Equation	Slack	Shadow Price
Wheat	479	0	Land	0	309
Strawberries 1 year old	74	0	Water	0	31
Strawberries 2 year old	74	0	Straw 1-2	0	483
Strawberries 3 year old	74	0	Straw 2-3	0	232
Strawberries 4 year old	0	185	Straw 3-4	74	0

Table 8.12. Alternative Formulation of Equilibrium Unknown Life

Rows	Wheat	Keep Strawberries 1	2	3	4	RHS	
Objective	340	-150	550	1230	1530	MAX	
Land	1	1	2	3	4	$\leq$	700
Water	1	0.8	5.3	9.8	15.5	$\leq$	1200

Dynamic Linear Programming

Recursive Programming

$$
\begin{array}{llcccccl}
\text{Max for period t} & Z_t = & \sum_j C_{jt} X_{jt} & + & \sum_k d_k Y_{kt} & & & \\
\text{s.t.} & & \sum_j A_{ijt} X_{jt} & + & \sum_k E_{ik} Y_{kt} & \leq & b_{it} & \text{for all i} \\
 & & X_{jt} & , & Y_{kt} & \geq & 0 & \text{for all j and k}
\end{array}
$$

Where: C_{jt} are objective function parameters functionally dependent upon the previous objective function parameters ($C_{j,t-1}$), lagged optimal decision variables (X_{jt-1}, Y_{kt-1}), and exogenous events;

X_{jt} are the values of the decision variables at time t;

A_{ijt} are the resource i usages by X_{jt} functionally dependent upon lagged values, d_k are objective function coefficients which are stable over time;

Y_{kt} are the optimal values of the kth Y variable in time period t; E_{ik} are the usages of resource i which do not change over time;

b_{it} are the resource i limits, functionally dependent upon lagged phenomena.

Dynamic Linear Programming

Recursive Programming / Example

$$
\begin{array}{lllllllll}
\text{Max} & Z_t & = & (130\ P_{1t} - 135) & X_{1t} & + & (45\ P_{2t} - 85) & X_{2t} & & \\
 & & & & X_{1t} & + & & X_{2t} & \leq & 600 \\
 & & & & X_{1t} & & & & \leq & 1.02\, X_{1t-1} \\
 & & & & X_{1t} & & & & \geq & .98\, X_{1t-1} \\
 & & & & & & & X_{2t} & \leq & 1.02\, X_{2t-1} \\
 & & & & & & & X_{2t} & \leq & .98\, X_{2t-1} \\
 & & & & X_{1t'} & & & X_{2t} & \geq & 0
\end{array}
$$

where: $P_{1t+1} = 20 - 0.00045\ Q_{it}$

$Q_{1t} = 130\ X_{1t}$

$P_{2t+1} = 10.9 - 0.00045\ Q_{2t}$

$Q_{2t} = 45\ X_{2t}$

Dynamic Linear Programming

Recursive Programming / Example

Solution

Time Period	X_{1t}	X_{2t}	P_{1t+1}	P_{2t+1}	Z_t
0	300.000	300.000	2.450	4.825	----
1	306.000	294.000	2.099	4.703	94995.750
2	311.880	288.120	1.755	4.584	79491.454
3	306.118	293.882	2.092	4.701	64163.394
4	311.995	288.005	1.748	4.582	79182.851
5	306.235	293.765	2.085	4.699	63860.831

6	312.110	287.890	1.742	4.580	78874.189