Quadratic Modeling Writing Task: Projectile Motion Analysis

In this assignment, you will apply your knowledge of quadratic equations to analyze a projectile motion scenario. You will develop and solve equations, explore different solution methods, and interpret the mathematical results in a real-world context.

Background

When an object is launched into the air, its height can be modeled using a quadratic function of the form:

$$h(t) = -16t^2 + v_0 t + h_0$$

Where:

- *h(t)* is the height (in feet) at time *t* (in seconds)
- v_0 is the initial velocity (in feet per second)
- h_0 is the initial height (in feet)
- -16 represents the effect of gravity (in feet per second squared)

The Assignment

A water rocket is launched from a platform at a community science fair. The rocket's height (in feet) *t* seconds after launch is modeled by the equation:

$$h(t) = -16t^2 + 96t + 6$$

Part 1: Equation Analysis & Multiple Solution Methods

1. Identify and explain the meaning of each coefficient in the given equation in the context of the rocket launch:
   - What is the initial height of the rocket?
   - What is the initial velocity?
   - What does the coefficient of t^2 represent?
2. Find when the rocket will hit the ground using three different methods:
   - Factoring
   - Completing the square
   - The quadratic formula
3. Show your work for each method and verify that they all give the same answer.

Part 2: Applications & Extensions

4. Determine the time(s) when the rocket will be exactly 70 feet above the ground.
5. If a tree is 50 feet tall and located in the path of the rocket, during what time interval will the rocket be above the tree?
6. Suppose a camera is set up to photograph the rocket, but it only works when the rocket is between 30 and 120 feet above the ground. Express as an inequality when the camera will be able to take pictures, and solve this inequality.

Part 3: Comparing Rocket Designs

7. A different rocket design has a height function of $h(t) = -16t^2 + 80t + 6$.
   a. Which rocket stays in the air longer?
   b. If both rockets are launched simultaneously, at what time(s) will they be at the same height?
   c. Graph both height functions on the same coordinate system and interpret what the intersection points represent.

Your submission should include:

- Clear, step-by-step solutions for all mathematical work
- Proper use of units throughout your calculations
- Accurate graphs where requested
- Thoughtful interpretations of what your mathematical results mean in context
- A well-organized presentation with sections clearly labeled

This assignment is worth 20 points. Your work will be assessed on mathematical accuracy, proper application of equation-solving techniques, clarity of explanations, and thoughtful interpretation of results in the context of the rocket scenario.

Rubric:

Criteria	Proficient	Developing	Not Evident	Points
Equation Analysis	Accurately identifies and correctly explains the meaning of all coefficients in the context of the rocket launch. Clearly connects mathematical components to their physical meaning (initial height, initial velocity, and gravity).	Identifies most coefficients with minor errors in explanation or contextual interpretation. Some connections between mathematical components and physical meaning are present but may lack clarity.	Significant errors in identifying coefficients or explaining their meaning. Limited or incorrect connection between mathematical components and physical meaning.	___/5
Multiple Solution Methods	Correctly applies all three solution methods (factoring, completing the square, and quadratic formula) to find when the rocket hits the ground. All work is clearly shown with proper mathematical notation. Accurately verifies that all methods yield the same answer.	Applies most solution methods correctly with minor computational errors. Work shown may lack some clarity or completeness. Attempt at verification is present but may contain minor errors.	Major errors in applying solution methods or missing one or more methods entirely. Work shown is disorganized or incomplete. No meaningful verification that methods yield the same result.	___/5

Applications & Extensions	Correctly determines when the rocket is at specific heights and time intervals. Accurately sets up and solves the inequality for the camera operation range. All answers include appropriate units and contextual interpretations.	Determines some specific heights and time intervals with minor errors. Inequality setup or solution contains minor errors. Some answers may lack units or clear contextual interpretation.	Multiple significant errors in determining specific heights, time intervals, or solving inequalities. Missing units or incorrect contextual interpretations throughout.	___/5
Comparing Rocket Designs	Thoroughly analyzes both rocket designs with correct comparisons of flight times. Accurately finds intersection times with clear explanations. Graph is precise with proper labeling and thorough interpretation of what intersection points represent in the physical scenario.	Compares rocket designs with some minor errors in analysis. Intersection times may contain minor computational errors. Graph is mostly accurate but may have minor labeling issues or limited interpretation of intersection points.	Major errors in comparison of rocket designs or finding intersection times. Graph is missing, significantly inaccurate, or lacks proper labeling. Little or no interpretation of what intersection points represent.	___/5
Total				___/20