

FREEHOLD REGIONAL HIGH SCHOOL DISTRICT OFFICE OF CURRICULUM AND INSTRUCTION MATHEMATICS DEPARTMENT CURRICULUM

QUANTITATIVE REASONING

Grade Level: 12

Credits: 5

BOARD OF EDUCATION ADOPTION DATE: August 26, 2021

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FREEHOLD REGIONAL HIGH SCHOOL DISTRICT

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Quantitative Reasoning

Course Description

In Quantitative Reasoning students develop quantitative literacy skills to draw conclusions that are relevant to their daily lives. Students interpret and draw inferences from mathematical models such as formulas, graphs, tables, and schematics. Emphasis is placed on problem-solving skills and developing the ability to communicate about mathematics in contextual situations. Students apply previously learned skills to solve non-routine and unfamiliar problems in the context of various disciplines, such as health, economics, politics, science, engineering, social science, and the arts. Students critique and evaluate quantitative arguments that utilize algebraic, statistical and quantitative information. This course will support student preparations for success on placement assessments to enroll in credit bearing math courses at two and four year colleges.

Course Sequence and Pacing

Unit Title	Section Focus	Suggested Pacing
Unit 1: Number Sense	1.1 Majority Rules? 1.2 Voting Schemes 1.3 Budget Priorities & US Budgets 1.4 Budget Analysis & Visual Budgets 1.5 Financing 1.6 The Value of a Dollar 1.7 Which Job Should You Accept? 1.8 The Prisoners and Proportions	26 Sessions
Unit 2: Statistical Representation	 2.1 Comparing Graphs and Distributions 2.2 Population vs. Sample 2.3 Mean and Standard Deviation of a Distribution 2.4 Why Use a Weighted Average? & Calculating an Expected Value 2.5 What are the Risks? Calculating Risk, Conditional Probability, and the Probability of Independent Events 2.6 Smoothing the Data 2.7 Explaining Weighted Moving Averages 	27 Sessions
Unit 3: Sampling	3.1 Classifying Variables 3.2 Observational vs. Experimental Studies 3.3 Simple Random Samples 3.4 Biased Studies 3.5 Types of Bias	12 Sessions
Unit 4: Linear Modeling	4.1 More Water Please 4.2 What's My Car Worth? 4.3 Depreciation 4.4 Tax Systems 4.5 How Much Should I Be Paid? 4.6 Investigating CLimate Change 4.7 Causation and Correlation	16 Sessions
Unit 5: Non-Linear Modeling	5.1 How Money Makes Money 5.2 Is It Exponential? 5.3 Oh, Deer! 5.4 Exploring Logistics Models 5.5 How Long Is a Day? 5.6 Exploring Sine	15 Sessions
Unit 6: Sampling II	6.1 Stratified Random Sampling 6.2 Correcting Bias 6.3 Correlation Vs. Causation 6.4 Experimental Design	19 Sessions

		6.5 Placebos and Blinding 6.6 Blocking 6.7 Inference	
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Support Resources

Supporting resources and appendices for this curriculum are available. These include a Resource Catalog of standards-aligned activities, common formative assessment and interdisciplinary items for performance expectations and objectives in this course.

- Appendix A: Accommodations and Modifications for Various Student Populations
- Appendix B: Assessment Evidence
- Appendix C: Interdisciplinary Connections
- Appendix D: Mathematics 2023 NJSLS Crosswalk
 - (*) in the front of NJSLS Indicates 2023 version.

Quantitative Reasoning
Unit 1: Number Sense
Section 1.1 Majority Rules?

Suggested Pacing: 4 Sessions

NJSLS-M Performance Expectations

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-1.1[1] Understand that different methods of voting can produce different winners of an election.

QR-1.1[2] Determine the winner of an election using different methods (Runoff, Instant Runoff, Majority)

- 1. Identify possible criteria to determine a winner between two people and three people in an election.
- 2. Use your own intuition to determine who should win an election given the results.
- 3. Determine the winner of an election using the majority method.
- 4. Identify the number of votes cast in an election and how many votes would be need to win using the majority method (Ted Cruz and David Dewhurst)
- 5. Determine the winner of the election using the runoff election (between Ted Cruz and David Dewhurst)
- 6. Calculate the winner of an election (between Alex, Blake, Charlie) using the Instant Runoff method
- A. Identify the number of voters on the interviewing committee.
- B. Determine the number of votes needed for a majority.
- C. Identify the number of 1st place votes that each candidate (Alex, Blake, Charlie) received.
- D. Determine which candidate received the fewest 1st place votes
- E. Determine the winner of the election
- 7. Identify the pros and cons of the Instant Runoff Method.
- 8. Discuss other potential voting methods and which, if any, methods would guarantee that the winner declared was fair to all candidates.

Quantitative Reasoning
Unit 1: Number Sense
Section 1.2 Voting Schemes

Suggested Pacing: 3 Sessions

NJSLS-M Performance Expectations

N-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-1.2[1] Calculate the winner of an election using the Borda Count Method

N-Q.A.2*[2] Determine appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3*[2] Determine the limitations of different measurement tools.

N-Q.A.3*[4] Identify important quantities in a problem or real-world context.

- 1. Given a committee's preference schedule (between Alex, Blake, Charlier) identify:
- A. The total number of people on the committee.
- B. The number of votes needed for a majority.
- C. The candidate with the majority (if one exists).
- 2. Calculate the winner using the Borda Count method from the example in #1.
- A. Apply the correct point values to each candidate based on their ranking from each voter.
- B. Calculate the appropriate points next to each applicant's name in Column 2 based on the ranking and number of voters.
- C. Determine the number of points awarded to each candidate (Alex, Blake, Charlie) in column 3 based on their rankings and number of voters.
- D. Determine the total points awarded to each candidate and therefore who should be offered the job.
- E. Determine if the second place finisher should be offered the position if the first place candidate is no longer interested in the position.
- F. Identify the number of points each voter awards to candidates and why this information is relevant.
- 3. Discuss the advantages and disadvantages of the Borda Count Method in comparison to the Instant Runoff Method.

Quantitative Reasoning

Unit 1: Number Sense

Section 1.3 Budget Priorities & US Budgets

NJSLS-M Performance Expectations

N-Q.A.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2* Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

N-Q.A.3*[4] Identify important quantities in a problem or real-world context.

- 1. Discuss:
- A. What is a budget?
- B. How can you use a budget?
- C. What do percentages of money spent for different items in a budget tell you about priorities?

N-Q.A.2*[2] Determine appropriate quantities for the purpose of descriptive modeling.

- 1. Based on federal budget categories provided:
- A. Use your own life experience to determine the three categories you would prioritize.
- B. Write down the percentage of the overall budget that each of your categories should represent and explain your reasoning.
- N-Q.A.1*[1] Recognize units given or needed to solve problems.
- N-Q.A.1*[2] Calculate unit conversions.

Suggested Pacing: 4 Sessions

- N-Q.A.2*[2] Determine appropriate quantities for the purpose of descriptive modeling.
- N-Q.A.3*[1] Identify appropriate units of measurement to report quantities.
- 1. Identify the top three budget categories in the given pie graph. (President's Proposed Total Spending FY 2014)
- 2. Compare your 3 categories selected with the actual top 3 categories represented in the given graph.
- 3. Calculate the amount of money budgeted for the top 3 categories knowing the total US federal budget for 2014.
- 4. Compare the values calculated:
- A. Write a ratio comparing the amount budgeted for the military with the amount budgeted for Social Security, Unemployments, and Labor Category.
- B. Simplify the ratio.
- C. Explain your numerical findings in a sentence.
- D. Convert your ratio from part A or part B to a percentage.
- E. Determine the meaning of the percentage in this situation.
- 5. Discuss your observations about the distribution of funds into the categories of the US federal budget.
- N-Q.A.1*[1] Recognize units given or needed to solve problems.
- N-Q.A.2*[2] Determine appropriate quantities for the purpose of descriptive modeling.
- N-Q.A.3*[4] Identify important quantities in a problem or real-world context.
- QR-4.15 [1] Create part-to-part and part-to-whole ratios.
- QR-4.15 [2] Calculate percentages from given ratios.
- QR-4.15 [3] Create a pie chart from ratios.
- 1. Create ratios from data provided in a table. (Federal Budget 2012)
- A. National Defense spending to Total federal budget
- B. National Defense spending to Human Resources spending.
- 2. Compare and contrast the ratios created.
- 3. Determine which of the ratios created are part-to-part ratios or part-to-whole ratios.
- 4. Determine what these ratios tell you about the federal budget priorities.
- 5. Determine how to make data more useful.
- 6. Determine which ratios, if converted to percentages, are most valuable.
- 7. Calculate percentages from the ratios previously calculated.
- 8. Calculate part-to-whole ratios and percentages for the remainder of the line items from the given chart.
- 9. Create a pie chart from percentages previously created and identify areas of the most and least spending.
- 10. Analyze the data in the pie chart created in the previous step.

Quantitative Reasoning

Unit 1: Number Sense

Section 1.4 Budget Analysis & Visual Budgets

Suggested Pacing: 5 Sessions

NJSLS-M Performance Expectations

N-Q.A.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2* Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

- N-Q.A.1*[2] Calculate unit conversions.
- N-Q.A.2*[2] Determine appropriate quantities for the purpose of descriptive modeling.
- N-Q.A.3*[4] Identify important quantities in a problem or real-world context.
- QR-4.16 [1] Calculate absolute change and relative change.
- 1. Identify government budget priorities that have increased or decreased over time.

- 2. Examine the spreadsheet data provided:
 - A. Convert the given data to actual numerical value by multiplying the values by a factor of 10.
 - B. Convert given numbers into millions of dollars.
- 3. Complete the given table with the appropriate budget amount.
- 4. Based on the Education budget:
 - A. Calculate the absolute change.
 - B. Calculate the relative change.
- C. Make a hypothesis about the federal government's priority for education and use your calculations and your graph to support your hypothesis.
- 5. Based on the National Defense or Net Interest:
 - A. Calculate the absolute change.
 - B. Calculate the relative change.
- C. Make a hypothesis about the federal government's priority for national Defense or Net Interest and use your calculations and your graph to support your hypothesis.
- N-Q.A.1*[2] Calculate unit conversions.
- N-Q.A.2*[2] Determine appropriate quantities for the purpose of descriptive modeling.
- N-Q.A.3*[4] Identify important quantities in a problem or real-world context.
- QR-4.16 [1] Calculate absolute change and relative change.
- QR-4.17 [1] Use spreadsheet to calculate percentages.
- QR-4.17 [2] Use spreadsheet to create pie charts.
- 1. Identify reasons the education budget may have increased in the previous 14 years.
- 2. Add information to the chart created in Lesson 7C by adding a new row for Total, Federal Outlays
 - A. Calculate the absolute change from 2000 to 2014 in Total, Federal Outlays
 - B. Calculate the relative change from 2000 to 2014 in Total, Federal Outlays
 - C. Compare your relative change in total with your relative change in education.
- 3. Insert a new row into the spreadsheet to calculate percentages of the total budget that is allocated to Education.
 - A. Determine the percentage of the federal budget that was allocated to Education in 2000 and 2014.
 - B. Create a new graph of Education as a Percentage of Total Federal Outlays.
- C. Identify interesting features of the graph and what does the graph tell you about US federal budget priorities. Use evidence from the graph, the data, and historical timeline to support your comments.
- 4. Insert a new row into the spreadsheet to calculate percentages of the total budget that is allocated to the National Defense category.
 - A. Determine the percentage of the federal budget that was allocated to defense in 2000 and 2014.
 - B. Create a new graph of national Defense as a Percentage of Total Federal Outlays.
- C. Identify interesting features of the graph and what does the graph tell you about US federal budget priorities. Use evidence from the graph, the data, and historical timeline to support your comments.

Quantitative Reasoning Unit 1: Number Sense Section 1.5 Financing

Suggested Pacing: 3 Sessions

NJSLS-M Performance Expectations

N-Q.A.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2* Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

N-Q.A.1*[2] Calculate unit conversions.

- N-Q.A.2*[2] Determine appropriate quantities for the purpose of descriptive modeling.
- N-Q.A.3*[4] Identify important quantities in a problem or real-world context.
- QR-4.18 [1] Calculate Front-End Debt-to-Income ratios and Back-End Debt-to-Income ratios
- QR-4.18 [2] Use online loan calculator.
- 1. Identify areas of spending that would affect your credit score in a positive way.
- 2. Discuss debt-to-income ratios.
- 3. Given specific criteria about your finances determine if you would qualify to purchase a home:
 - A. Calculate Front-End Debt-to-income (DTI) ratio.
 - B. Calculate Back-End DTI ratio.
 - C. Determine if the bank is likely to lend you the money to buy the house.
 - D. Determine what your total monthly payments would need to be to keep in line with the Back-End DTI limit provided.
 - E. Identify how much lower your house payment would need to be to stay within the limit provided.
- 4. Use online loan calculator to determine the length of time to pay off a loan.
- 5. Use online loan calculator to determine the required monthly payment to pay off a loan in a specified time frame.

Quantitative Reasoning

Unit 1: Number Sense

Section 1.6 The Value of a Dollar

NJSLS-M Performance Expectations

N-Q.A.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-1.8 [1] Understand that a mathematical tool is needed in order to compare monetary values by discussing that if you are given job offers in different cities, mathematical tools are needed in order to compare monetary values to help you make decisions.

N-Q.A.1*[2] Calculate unit conversions.

QR-1.8 [2] Understand that dollars have different values and purchasing power in different cities.

- 1. Use data to calculate the cost of living in Austin, Big Rapids, and Thousand Oaks.
- 2. Calculate unit rates for the annual cost of living for each pair of cities.
- 3. Use the unit rates to convert the buying power of one city to another.
- QR-1.8 [3] Recognize when converting units is needed.
- QR-1.8 [4] Use conversions to make comparisons.
- 1. Use unit rates to convert salaries in one city to the value of salary in another city.
- 2. Interpret the conversions between salaries in each city.
- 3. Use the conversions to compare the salaries in each city and make a decision.

Quantitative Reasoning

Unit 1: Number Sense

Section 1.7 Which Job Should You Accept?

NJSLS-M Performance Expectations

N-Q.A.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

N-Q.A.1*[2] Calculate unit conversions TO CONVERT PHENIX CITY SALARY TO EQUIVALENT SALARY BASED ON NATIONAL AVERAGE COST OF LIVING USING GIVEN CONVERSION RATE.

Suggested Pacing: 2 Sessions

Suggested Pacing: 2 Sessions

QR-1.9 [1] Understand the meaning of index numbers and perform calculations involving index numbers.

- 1. Define index numbers and their purpose.
- 2. Understand that the percentages calculated in the first activity of the lesson (the ratios of city's dollars to national dollars) are an example of the index numbers called the cost of living index.
- 3. Using the cost of living index, convert every city's salary given in the spreadsheet to national equivalent salary.

QR-1.9 [2] Make and justify decisions and evaluate claims using index numbers.

- 1. Use your calculations to decide which job you will take and justify your answer.
- 2. Discuss other applications of index numbers and what variables can affect them.

Quantitative Reasoning

Unit 1: Number Sense

Section 1.8 The Prisoners and Proportions

NJSLS-M Performance Expectations

N-Q.A.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2* Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3* Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

N-Q.A.1*[2] Calculate unit conversions.

N-Q.A.2*[2] Determine appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3*[4] Identify important quantities in a problem or real-world context.

- 1. Identify factors why the US has the most prisoners but does not have the largest population.
- 2. Calculate percentages given data (US citizens behind bars).
- 2. Compare and contrast ratios, rates, and percentages based on the data provided (US citizens behind bars)

Quantitative Reasoning

Unit 2: Statistical Representation

Section 2.1 Comparing Graphs and Distributions

NJSLS-M Performance Expectations

S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S-IC.B.6 Evaluate reports based on data.

S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

(*) S-IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed). (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-4.1[1] Explain which type of graph is best to represent data FOR THE THREE GRAPHICAL DISPLAYS IN THE PREVIEW ASSIGNMENT FOR 2.A

S-ID.A.2*[1] Choose the appropriate measure for center (mean, median) and spread (interquartile range, standard deviation) based on the shape of a data distribution FOR THE TREADMILL RATINGS EXAMPLE.

Suggested Pacing: 3 Sessions

Suggested Pacing: 6 Sessions

- S-ID.A.3*[2] Interpret differences in shape, center, and spread in the context of data sets FOR THE TREADMILL RATINGS EXAMPLE.
- S-IC.B.6 [3] Draw conclusions based on graphical and numerical summaries FOR THE TREADMILL RATINGS EXAMPLE. (+ for 2023)
- S-ID.A.1*[1] Represent data with plots on the real number line, using various display types by creating dot plots FOR THE DATA FROM LESSON 1.A
- S-ID.A.3*[2] Interpret differences in shape, center, and spread in the context of data sets FOR THE MINI-PROJECT ON MEDIAN HOUSEHOLD INCOMES AND POLITICAL PREFERENCE.
- S-IC.B.6 [3] Draw conclusions based on graphical and numerical summaries FOR THE MINI-PROJECT ON MEDIAN HOUSEHOLD INCOMES AND POLITICAL PREFERENCE. (+ for 2023)

Quantitative Reasoning
Unit 2: Statistical Representation
Section 2.2 Population vs. Sample

Suggested Pacing: 2 Sessions

Suggested Pacing: 5 Sessions

NJSLS-M Performance Expectations

S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-4.3[1] Explain why sampling is necessary in a statistical study.

QR-4.3[2] Explain how to use population proportions to decide the proportions that should be represented in a sample FOR THE DATA IN THIS CLASS.

QR-4.3[3] From a statistical study, describe the population in each scenario from which the sample was drawn FOR THE EXAMPLES OF FLORIDIANS WHO SUPPORT LEGALIZATION OF MARIJUANA, MICHIGAN TAX INCREASE SUPPORTERS AND NIELSEN TELEVISION RATINGS.

S-IC.A.1*[2] Explain that statistical inferences about population characteristics are based on random samples from that population FOR THE EXAMPLES OF FLORIDIANS WHO SUPPORT LEGALIZATION OF MARIJUANA, MICHIGAN TAX INCREASE SUPPORTERS AND NIELSEN TELEVISION RATINGS. (+ for 2023)

Quantitative Reasoning

Unit 2: Statistical Representation

Section 2.3 Mean and Standard Deviation of a Distribution

NJSLS-M Performance Expectations

- S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- (*) S-IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed). (+ for 2023)
- S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- S-ID.A.4* Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-ID.A.2*[1] Choose the appropriate measure for center (mean, median) based on the shape of a data distribution FOR THE WATER CONSUMPTION EXAMPLE.

QR-4.4[1] Determine the mean of a data set FOR THE WATER CONSUMPTION EXAMPLE.

S-IC.B.6 [3] Draw conclusions based on graphical and numerical summaries FOR THE WATER CONSUMPTION EXAMPLE. (+ for 2023)

- S-ID.A.2*[1] Choose the appropriate measure for spread (interquartile range, standard deviation) based on the shape of a data distribution FOR THE WATER CONSUMPTION EXAMPLE.
- QR-4.5[1] Calculate and interpret the standard deviation of a distribution FOR THE WATER CONSUMPTION EXAMPLE.
- S-ID.A.4*[1] Describe the characteristics of a normal distribution FOR THE WATER CONSUMPTION EXAMPLE.
- S-ID.A.4*[4] Use a normal distribution to estimate population percentages FOR THE WATER CONSUMPTION EXAMPLE.

Quantitative Reasoning

Unit 2: Statistical Representation

Section 2.4 Why Use a Weighted Average? & Calculating an Expected Value

NJSLS-M Performance Expectations

- (*) S-IC.A.1* Understand statistics as a process for making inferences about population parameters based on a random sample from that population. (+ for 2023)
- S-MD.A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

- QR-4.8 [1] Understand what a weighted average is and how it is used.
- 1. Use the data from polls A, B, and C to calculate the simple average of the results from three polls and analyze the results (how the President is doing).
- 2. Identify which of the three polls might be the least reliable and most reliable, why and explain how they might affect the average.
- 3. Define weighted average, its role in statistical reliability and steps to calculate it.
- QR-4.8 [2] Calculate weighted averages.
- 1. Calculate the weighted averages of Polls 1, 2, and 3 using the table of data.
- 2. Use the given spreadsheet with the results from 15 polls to calculate the weighted average.
- QR-4.8 [3] Use weighted averages to analyze data and draw conclusions about how the President is doing in the polls.
- S-IC.A.1*[1] Explain that statistics is a process for making inferences about population parameters or characteristics. BY DISCUSSING THAT THE WEIGHTED AVERAGE ALLOWS US TO MAKE INFERENCES ABOUT THE POPULATION'S ATTITUDE TOWARDS THE PRESIDENT. (+ for 2023)
- QR-4.9 [1] Understand that an expected value is a weighted average.
- 1. Use the given spreadsheet to calculate the simple overall average salary in ten cities.
- 2. Multiply the population of Phoenix by the average salary in Phoenix. Explain what it might represent.
- 3. Define expected value and compare it to the weighted average.
- S-MD.A.2[1] Calculate the expected value of a random variable. FOR THE AVERAGE SALARY OF TEN CITIES.

Quantitative Reasoning

Unit 2: Statistical Representation

Section 2.5 What are the Risks? Calculating Risk, Conditional Probability, and the Probability of Independent Events

Suggested Pacing: 5 Sessions

Suggested Pacing: 5 Sessions

NJSLS-M Performance Expectations

- (*) S-CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). (+ for 2023)
- (*) S-CP.B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) P(A and B), and interpret the answer in terms of the model. (+ for 2023)
- S-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
- (*) S-CP.A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (+ for 2023)
- S-CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
- (*)S-CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

- S-CP.A.1 [2] Describe a sample space and events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcome OF ROLLING A DIE AND FLIPPING A COIN. (+ for 2023)
- S-CP.A.2 [1] Categorize events as independent if the occurrence of one does not affect the probability of the occurrence of the other. (+ for 2023)
- S-CP.B.8 [1] Derive and apply the general Multiplication Rule in a uniform probability model for independent events P(A and B) = P(A) * P(B).
- 1. Determine the probability of rolling a 2. Determine the probability of flipping the head.
- 2. Determine the probability of rolling a 2 AND flipping the head.
- 3. Derive the Multiplication Rule P(A and B) = P(A) * P(B).
- 4. Apply the Multiplication Rule to calculate the risk that two women will develop breast cancer. Discuss cases where this is not an independent event.
- S-CP.B.7 [1] Derive and apply the general Addition Rule in a uniform probability model for independent events P(A or B) = P(A) +P(B) P(A and B).
- 1. Determine the probability of rolling a 2 OR flipping a head. Explain your reasoning.
- 2. Derive the Addition Rule P(A or B) = P(A) + P(B) P(A and B).
- (+ for 2023)
- S-CP.B.8 [1] Apply the Multiplication Rule P(A and B) = P(A) * P(B) TO CALCULATE THE PROBABILITY OF GETTING A 2 on TWO CONSECUTIVE ROLLS.
- S-CP.A.3 [1] Define the concept of dependent events and conditional probability. Define conditional probability P(A|B) as the probability of A given B. (+ for 2023)
- QR-4.7 [2] Use the data from Crimes Against Persons Offenses, Offense Category by Location, 2018, to compare probabilities of two events. Describe the events as independent or dependent and justify your answer.
- QR-4.7 [3] Use the data from Crimes Against Persons Offenses, Offense Category by Location, 2018, to identify conditional probability of given events.
- S-CP.B.8 [2] Apply the general Multiplication Rule in a uniform probability model P(A and B) = P(A) P(B|A) = P(B)P(A|B) TO CALCULATE CONDITIONAL PROBABILITY OF DEPENDENT "AND" EVENTS USING THE DATA FROM CRIMES AGAINST PERSONS OFFENSES, OFFENSE CATEGORY BY LOCATION 2018.
- S-CP.B.8 [3] Interpret the answer in terms of the model.

Quantitative Reasoning
Unit 2:Statistical Representation
Section 2.6 Smoothing the Data

Suggested Pacing: 2 Sessions

Suggested Pacing: 2 Sessions

NJSLS-M Performance Expectations

(*) S-IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed). (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-IC.B.6 [3] Draw conclusions based on graphical and numerical summaries. (+ for 2023)

QR-4.10[1] Calculate weighted moving averages using spreadsheets

QR-4.10 [2] Describe the difference between simple moving averages and weighted moving averages.

- 1. Use the given graph ("Louis' Scores") with Simple Moving Averages to make conclusions.
- 2. Calculate the weighted moving averages ("Louis' Scores") in a spreadsheet using the given step by step instructions.
- 3. Describe the difference between a simple moving average and a weighted moving average.
- 4. Explain how to modify the method provided in part 2 if you have 4 data values.

Quantitative Reasoning

Unit 2: Statistical Representation

Section 2.7 Explaining Weighted Moving Averages

NJSLS-M Performance Expectations

(*) S-IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed). (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-IC.B.6 [3] Draw conclusions based on graphical and numerical summaries.

QR-4.11[1] Calculate weighted moving averages using spreadsheets

QR-4.11 [2] Create line graphs using spreadsheets

- 1. Use the given graph ("Mean Household Income") with Weighted Moving Averages to make conclusions.
- 2. Use the provided spreadsheet and the given instructions to complete:
- A. Calculate weighted moving average
- B. Create 4 line graphs
- 3. Draw conclusions using your graphs about the four categories.
- 4. Explain what a median is and how it relates to this data.

Quantitative Reasoning

Unit 3: Sampling

Section 3.1 Classifying Variables

NJSLS-M Performance Expectations

(*) S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

Suggested Pacing: 3 Sessions

- QR-5.1[1] Explain what a sample is in contrast to a population and explain why we use samples to represent a population BY EXPLORING INCOME DATA FOR THE ENTIRE WORLD.
- QR-5.1[2] Describe how to gather a representative sample TO BE REPRESENTATIVE OF DIFFERENT LEVELS OF INCOME FOR THE POPULATION OF THE WORLD.
- QR-5.1[3] Understand that a statistic is a numerical summary from a sample and correctly identify the most appropriate statistics to compute in a statistical study THAT EXPLORES THE INCOME GAP AROUND THE WORLD.
- QR-5.1[4] Distinguish between explanatory and response variables, and between quantitative and categorical variables IN THE STUDY OF INCOME GAP AROUND THE WORLD.
- S-IC.A.1*[2] Explain using a viably constructed argument that statistical inferences about population characteristics are based on random samples from that population IN THE STUDY OF INCOME GAP AROUND THE WORLD. (+ for 2023)

Quantitative Reasoning

Unit 3: Sampling

Section 3.2 Observational vs. Experimental Studies

NJSLS-M Performance Expectations

- (*) S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (+ for 2023)
- (*) S-IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

- S-IC.B.3 [2] Recognize the differences among surveys, experiments, and observational studies in making statistical inferences, explain how randomization relates to each of these methods of data collection. (+ for 2023)
- QR-5.2[1] Compare two studies RELATED TO THE MEDITERRANEAN DIET to determine which is observational and which is experimental.
- QR-5.2[2] Identify and classify variables as quantitative or categorical and explanatory or response from two studies RELATED TO THE MEDITERRANEAN DIET
- QR-5.2[3] For the experiment RELATED TO THE MEDITERRANEAN DIET, identify the treatment
- S-IC.B.5 [2] Use data from a randomized experiment RELATED TO THE MEDITERRANEAN DIET to compare two treatments. (+ for 2023)

Quantitative Reasoning

Unit 3: Sampling

Section 3.3 Simple Random Samples

NJSLS-M Performance Expectations

(*) S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

- S-IC.B.3 [1] Recognize the purpose of surveys experiments, and observational studies in making statistical inferences and justifying conclusions, and explain how randomization relates to each of these methods of data collection. (+ for 2023)
- QR-5.3[1] Describe a process of collecting data FOR THE STUDY OF A STUDENT POPULATION'S BMI and explain why it is appropriate compared to other approaches.
- QR-5.3[2] Explain what a simple random sample is and how to select a simple random sample using a random number generator FOR THE STUDY OF A STUDENT POPULATION'S BMI.
- QR-5.3[3] Explain the difference between a statistic and a parameter FOR THE STUDY OF A STUDENT POPULATION'S BMI.
- QR-5.3[4] Explain when you can use a statistic to estimate a parameter and when it is not appropriate to do so FOR THE STUDY OF A STUDENT POPULATION'S BMI.

Suggested Pacing: 2 Sessions

Suggested Pacing: 3 Session

QR-5.3[5] Quantify the error of a statistic FOR THE STUDY OF A STUDENT POPULATION'S BMI and explain the cause of this error.

Quantitative Reasoning
Unit 3: Sampling
Section 3.4 Biased Studies

Suggested Pacing: 2 Sessions

NJSLS-M Performance Expectations

(*) S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-5.5[1] Find the mean of a sample distribution OF BLOOD PRESSURES from a graph

S-IC.B.3 [1] Explain how randomization relates to each of these methods of data collection. (+ for 2023)

QR-5.5[2] Calculate and interpret in context the error of the sample distribution OF BLOOD PRESSURES given the parameter.

QR-5.5[3] Explain the difference between sampling error and non-sampling error in the sample distribution OF BLOOD PRESSURES.

QR-5.5[4] Discuss methods to correct for non-sampling errors such as bias in the sample distribution OF BLOOD PRESSURES.

Quantitative Reasoning Unit 3: Sampling Section 3.5 Types of Bias **Suggested Pacing: 2 Sessions**

NJSLS-M Performance Expectations

(*) S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-5.6[1] Calculate and interpret a sample proportion from the CARRYING WEAPONS example

S-IC.B.3 [1] Explain how randomization relates to each of these methods of data collection. (+ for 2023)

QR-5.6[2] Explain the effect of non-response bias in the CARRYING WEAPONS example including the direction of the bias.

QR-5.6[3] Explain how to create random proportional samples in the GENDER VOTING example and discuss the reason for doing so.

QR-5.6[4] Explain how wording of questions bias may lead to biased results in the GUN VIOLENCE example.

Quantitative Reasoning
Unit 4: Linear Modeling
Section 4.1 More Water Please

Suggested Pacing: 3 Sessions

NJSLS-M Performance Expectations

F-IF.B.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.B.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-LE.A.2* Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.A.1.b* Distinguish between situations that can be modeled with linear functions and with exponential functions: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

S-ID.B.6a* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

F-LE.A.1.b*[1] Recognize situations in which one quantity changes at a constant rate per unit (equal differences), relative to another to solve mathematical and real-world problems.

QR-2.1 [1] Understand that a linear relationship is characterized by a constant rate of change.

- a. Determine how many weeks it will take to reach 80 ounces of daily water consumption based on a given initial value and rate of change.
- b. Discuss different student approaches for finding how many weeks it will take to reach 80 ounces of daily water consumption.
- c. Discuss that a linear relationship is characterized by a constant rate of change.
- QR-2.1 [2] Understand that mathematical relationships can be represented in different ways: A VERBAL DESCRIPTION, A TABLE OF DATA, A GRAPH, AND AN EQUATION.
- QR-2.1 [3] Create representations to describe mathematical relationships by completing a table of data for the relationship between weeks passing and daily water intake, based on the information given in part 1.

F-IF.B.6*[2] Calculate the average rate of change FOR THE DAILY WATER INTAKE OVER TIME (as a table) over a specified interval

S-ID.B.6a*[1] Represent data on a scatter plot (2 quantitative variables) OF WEEKS PASSING AND DAILY WATER INTAKE.

S-ID.B.6a*[4] Determine which function best models scatter plot data represented on the coordinate plane, and describe how the two quantitative variables are related. BY EXAMINING THE GRAPH OF WATER CONSUMPTION.

F-IF.B.5*[2] Identify an appropriate domain based on the unit, quantity, and type of function it describes.

F-IF.B.5*[4] Explain why a domain is appropriate for a given situation.

- a. Identify whether the domain of weeks passing is discrete or continuous.
- b. Discuss why the domain is appropriate for the situation.
- F-LE.A.2*[3] Construct linear functions, including arithmetic sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table) GIVEN A SLOPE AND A Y-INTERCEPT.
- a. Write a linear function to describe the relationship between weeks and ounces of water.
- b. Use the function to calculate the number of weeks it would take to reach an intake of 64 oz.
- QR-2.1 [4] Investigate and compare mathematical relationships using a variety of representations: A VERBAL DESCRIPTION, A TABLE OF DATA, A GRAPH, AND AN EQUATION.
- a. Identify the various representations of the mathematical relationship between weeks and ounces of water consumed per day.
- b. Use each representation to determine how many weeks it will take to reach a daily water consumption of 80 oz.
- c. List the advantages and disadvantages of each representation.

Quantitative Reasoning
Unit 4: Linear Modeling
Section 4.2 What's My Car Worth?

Suggested Pacing: 2 Sessions

NJSLS-M Performance Expectations

F-LE.A.2* Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-IF.B.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F-IF.B.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-LE.A.1.b* Distinguish between situations that can be modeled with linear functions and with exponential functions: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

S-ID.B.6a* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

F-LE.A.1.b*[1] Recognize situations in which one quantity changes at a constant rate per unit (equal differences), relative to another to solve mathematical and real-world problems.

QR-2.2 [1] Understand that all proportional relationships are linear, but not all linear relationships are proportional. Explain the difference between proportional and linear relationships.

- a. Discuss why cars depreciate over time.
- b. Discuss the meaning of proportional quantities.
- c. Discuss the similarities and differences between proportional and linear relationships.
- d. Recognize the relationship between the value of a car and the years of ownership as linear, but not proportional.

F-IF.B.6*[2] Calculate the average rate of change of a function (presented as a table) over a specified interval. BY COMPUTING THE RATE OF CHANGE OF THE VALUE OF A CAR OVER 8 YEARS.

S-ID.B.6a*[1] Represent the data on a scatter plot (2 quantitative variables) FOR THE VALUE OF THE CAR OVER TIME.

F-IF.B.5*[2] Identify an appropriate domain based on the unit, quantity, and type of function it describes FOR THE YEARS OF OWNERSHIP OF THE CAR BY DECIDING IF IT MAKES SENSE TO CONNECT THE DATA POINTS ON THE SCATTER PLOT FOR THE VALUE OF THE CAR OVER TIME.

F-LE.A.2*[3] Construct linear functions, given a description of a relationship. FOR THE VALUE OF THE CAR IN TERMS OF THE YEAR.

F-IF.B.6*[4] Interpret the average rate of change of the function RELATING YEARS OF OWNERSHIP AND VALUE OF THE CAR.

QR-2.2 [2] Compare and contrast linear and proportional relationships by comparing two linear functions and determining which is also proportional.

Quantitative Reasoning Unit 4: Linear Modeling Section 4.3 Depreciation **Suggested Pacing: 2 Sessions**

NJSLS-M Performance Expectations

S-ID.B.6a* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

A-CED.A.4* Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-2.6 [1] Interpolate and extrapolate using a graphical representation of the relationship between two variables.

- 1. Use a scatter plot to estimate the value of a lawn mower 3 years after purchase.
- 2. Find the basis of the mower, the class life, the salvage value, the depreciable basis, and the depreciation for the first year.
- 3. Create a table of values for the data in the given scatter plot.
- 4. Define interpolation.
- 5. Discuss if it would be appropriate to use interpolation to find the value of the mower at different years.
- 6. Use the graph to estimate the value of the mower at Year 6. Define extrapolation.
- 7. Discuss if it would be appropriate to use extrapolation to find the value of the mower at different years, keeping in mind the lifespan of the mower.

S-ID.B.6a*[5] Use functions fitted to data to solve problems in the context of the data.

QR-2.6 [2] Use a symbolic model to find the exact value of one variable, given the value of the other variable, and relate those values to the context of the problem.

- 1. Use the given function of value of the mower depending on years owned to find the value of the mower at the end of four years
- 2. Decide if this is interpolation or extrapolation.
- 3. Use the given function to find the value of the mower at the end of seven years.
- 4. Decide if this is interpolation or extrapolation.
- 5. Use the given function to determine how long it would take for the mower to depreciate to a value of \$2,300.

QR-2.6 [3] Understand that finding the output value from a graph of a model, given an input value, is an estimate and discuss how accurate the estimations are based on the size of the data set.

A-CED.A.4*[2] Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. BY REARRANGING THE FUNCTION FOR VALUE OF THE MOWER IN TERMS OF YEARS SO THAT THE INPUT IS ISOLATED INSTEAD OF THE OUTPUT.

Quantitative Reasoning Unit 4: Linear Modeling Section 4.4 Tax Systems

Suggested Pacing: 3 Sessions

NJSLS-M Performance Expectations

F-IF.C.7.a* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases: Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.C.7.b* Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases: Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-BF.A.1.a* Write a function that describes a relationship between two quantities: Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE.B.5* Interpret the parameters in a linear or exponential function in terms of a context.

F-LE.A.2* Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-2.5 [1] Understand the meaning of a progressive income tax system and a flat tax system.

- 1. Define "income tax" and discuss the purpose of taxes.
- 2. Discuss what it means for a tax system to be "progressive".
- 3. Look at a basic example of what it means to use a progressive tax system by calculating the annual income tax you would pay with two different salaries in both a simplified and more realistic progressive tax system
- 4. Use these calculations to make a decision about which salary you would rather earn and how much more you would actually earn after taxes are taken out.
- 5. Calculate the taxes owed on various incomes in a progressive tax system.
- 6. Summarize the steps for calculating progressive income tax depending on the magnitude of the gross income.

F-BF.A.1.a*[2] Write a function that describes a relationship between two quantities by determining steps for calculation from a context, FOR THE RELATIONSHIP BETWEEN TAXES OWED AND GROSS INCOME FOR EACH TAX BRACKET.

F-LE.B.5*[3] Interpret the parameters in a linear function in terms of a context. BY INTERPRETING THE SLOPE AND VERTICAL INTERCEPT FOR THE FUNCTION OF TAXES OWED DEPENDING ON GROSS INCOME FOR EACH TAX BRACKET.

F-IF.C.7.b*[1] Graph piecewise-defined functions by hand. BY GRAPHING THE FUNCTION OF TAXES OWED DEPENDING ON GROSS INCOME FOR EACH TAX BRACKET.

F-LE.A.2*[3] Construct linear functions given-input-output pairs (including reading these from a table).

- 1. Make a conjecture about the meaning of a flat tax rate.
- 2. Complete a table of values comparing gross income and tax amount in a flat tax system.
- 3. Write a linear function that describes the relationship between gross income and tax amount in a flat tax system.

F-IF.C.7.a*[1] Graph linear functions by hand.

QR-2.5 [2] Compare a progressive income tax system to a flat tax system and identify different outcomes. Understand that tax policies frequently impact high and low income citizens differently.

- 1. Graph the function for the tax amount depending on gross income with a flat tax rate of 15%.
- 2. Compare the progressive and flat tax systems by looking at the graphs.
- 3. Discuss who ends up paying more in taxes in flat vs. progressive tax systems.

A-REI.C.6[1] Solve systems of linear equations by any method.

QR-2.5 [3] Explain advantages and disadvantages of different income tax systems.

- 1. Determine the boundary income by solving the system of equations using the substitution method.
- 2. Discuss the advantages and disadvantages of flat tax systems.

QR-2.5 [4] Understand the basics of current U.S. income tax system and rate structure.

- 1. Look up the current rates for single taxpayers in the United States.
- 2. Discuss information learned about the basics of the current U.S. income tax system and rate structure. State any remaining questions to bring up in class.

Quantitative Reasoning
Unit 4: Linear Modeling
Section 4.5 How Much Should I be Paid?

Suggested Pacing: 2 Sessions

NJSLS-M Performance Expectations

S-ID.A.1* Represent data with plots on the real number line (dot plots, histograms, and box plots).

S-ID.A.3* Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S-ID.B.6a* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

S-ID.C.9* Distinguish between correlation and causation.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-2.7 [1] Understand that visual displays, such as scatter plots and histograms, can be used to represent univariate and bivariate data.

- 1. Discuss important financial questions to consider when considering a job.
- 2. Discuss negotiation strategies to increase starting salary and why that is important.
- 3. Discuss how likely it is to receive an average 5% annual pay increase and why that may or may not be realistic.
- 4. Examine different statistics related to annual income and decide if the statistics are univariate or bivariate.
- 5. Define "quantitative" data.
- 6. Discuss various ways to analyze and visualize univariate quantitative data.

S-ID.A.1*[1] Represent data with plots on the real number line, using various display types by creating LINE GRAPHS FOR ANNUAL INCOME OF ENGINEERS WITH A B.S. DEGREE AND 5-9 YEARS OF EXPERIENCE.

S-ID.A.3*[2] Interpret differences in shape, center, and spread in the context of data sets. BY COMPARING AND CONTRASTING THE TWO NUMBER LINE GRAPHS FOR ANNUAL INCOME OF ENGINEERS WITH A B.S. DEGREE AND 5-9 YEARS OF EXPERIENCE.

S-ID.C.9*[1] Define positive, negative, and no correlation.

S-ID.B.6a*[1] Represent data on a scatter plot (2 quantitative variables).

- 1. Discuss independent vs. dependent variables.
- 2. Define the two variables in the table of data about the annual income of engineers with a B.S. degree.
- 3. Determine which variable is independent and which is dependent.
- 4. Construct a scatterplot of the data.
- QR-2.7 [2] Determine, informally, the correlation between bivariate data.
- QR-2.7 [3] Analyze data and related graphs and describe the trend of the data.
- 1. Use the scatter plot to decide if the correlation is positive or negative.
- 2. Discuss other ways to display the data.
- 3. Discuss other variables that could affect the relationship of the data.

Quantitative Reasoning Unit 4: Linear Modeling

Section 4.6 Investigating Climate Change

NJSLS-M Performance Expectations

S-ID.B.6a* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

S-ID.B.6c* Fit a linear function for a scatter plot that suggests a linear association.

S-ID.C.7* Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID.C.8* Compute (using technology) and interpret the correlation coefficient of a linear fit.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-ID.B.6a*[1] Represent data on a scatter plot USING TECHNOLOGY (2 quantitative variables). USE SPREADSHEET OF TOTAL ENERGY CO2 EMISSIONS (INDEPENDENT VARIABLE) AND GLOBAL TEMPERATURE ANOMALY (DEPENDENT VARIABLE)

QR-2.4 [1] Understand that linear and other models can be used to approximately fit data. Understand that data may suggest a nonlinear relationship.

- 1. Define the terms "regression", "statistical noise", and "trendline".
- 2. Discuss that different mathematical models can be used to fit different data, including linear and exponential.

S-ID.B.6c*[1] Fit a linear function for a scatter plot that suggests a linear association. BY CREATING A LINEAR TRENDLINE ON THE SCATTER PLOT USING TECHNOLOGY.

QR-2.4 [2] Use technology to estimate the parameters of the line of best fit.

S-ID.C.7*[1] Interpret the slope (rate of change) of a linear model in the context of the data. BY INTERPRETING THE SLOPE OF THE LINE OF FIT FOR THE TRENDLINE FROM PART 3.

Quantitative Reasoning Unit 4: Linear Modeling

Section 4.7 Causation and Correlation

Suggested Pacing: 2 Sessions

Suggested Pacing: 2 Sessions

NJSLS-M Performance Expectations

S-ID.B.6a* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

S-ID.C.7* Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S-ID.C.8* Compute (using technology) and interpret the correlation coefficient of a linear fit.

S-ID.C.9* Distinguish between correlation and causation.

F-LE.A.2* Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-ID.C.9*[1] Define positive, negative, and no correlation and explain why correlation does not imply causation.

S-ID.C.9*[2] Define causation.

- 1. Discuss how mistaking correlation for causation can have dire consequences.
- 2. Discuss other instances where there may be correlation but no causation.

S-ID.C.9*[3] Distinguish between correlation and causation. BY EXAMINING STATEMENTS AND MAKING AN EDUCATED GUESS IF THERE IS SIMPLY CORRELATION OR ALSO CAUSATION. JUSTIFY CHOICES.

S-ID.C.8*[2] Define the correlation coefficient.

QR-2.8 [1] Explain why, even if there is a strong correlation, a change in one variable may not cause a change in the other by discussing why eating ice cream does not cause drowning. Discuss why there is correlation between ice cream sales and drownings. Discuss what other variables may be at play.

S-ID.B.6a*[1] Represent data on a scatter plot (2 quantitative variables). OF THE RELATIONSHIP BETWEEN WEIGHT OF A CAR AND GAS MILEAGE.

S-ID.B.6a*[3] Using given scatter plot data OF GAS MILEAGE DEPENDING ON CAR WEIGHT represented on the coordinate plane, informally describe how the two quantitative variables are related. BY DISCUSSING IF THERE IS POSITIVE OR NEGATIVE CORRELATION AND IF IT APPEARS TO BE WEAK OR STRONG.

F-LE.A.2*[3] Construct a linear function given two input-output pairs (include reading these from a table). FOR THE RELATIONSHIP BETWEEN GAS MILEAGE AND CAR WEIGHT, BASED ON TWO DATA POINTS IN THE GIVEN TABLE.

S-ID.B.6a*[5] Use functions fitted to data to solve problems in the context of the data.

- 1. Use the function created in part D to find the MPG for a car that weighs 2,269 pounds.
- 2. Determine if this is interpolation or extrapolation.
- 3. Compare the interpolation to the actual data of a 2011 Mazda2.
- 4. Discuss how close the estimation is to the actual data and why that may be.

S-ID.C.7*[1] Interpret the slope (rate of change) and the intercept (constant term) of a linear model FOR THE GAS MILEAGE OF A CAR DEPENDING ON ITS WEIGHT in the context of the data.

Quantitative Reasoning
Unit 5: Non- Linear Modeling
Section 5.1 How Money Makes Money

Suggested Pacing: 4 Sessions

NJSLS-M Performance Expectations

F-IF.B.6* Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F-LE.A.1.a* Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F-LE.A.1.b* Distinguish between situations that can be modeled with linear functions and with exponential functions: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE.A.2* Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE.A.3* Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-2.3 [1] Understand the difference between simple and compound interest.

- a. Introduce the idea of interest and define the term principal. Discuss the purpose of interest.
- b. Discuss the difference between simple and compound interest.
- F-LE.A.1.b*[1] Recognize situations in which one quantity changes at a constant rate per unit (equal differences), relative to another to solve mathematical and real-world problems.
- a. Complete a table of values for the relationship between the total balance of an account after a number of years, earning simple interest.
- b. Recognize the situation as linear to help fill out the table.

F-IF.B.6*[2] Calculate the average rate of change of a function (presented as a table) over a specified interval. BY CALCULATING THE RATE OF CHANGE OF THE TOTAL BALANCE IN THE SAVINGS ACCOUNT OVER 5 YEARS.

F-LE.A.2*[3] Construct linear functions GIVEN A SLOPE AND Y-INTERCEPT.

- 1. Write a linear function to describe the relationship between year and the total amount in the savings account.
- 2. Use the function to calculate the amount in the account after 30 years.
- F-LE.A.2*[5] Determine when a graph, a description of a relationship, or two input- output pairs (include reading these from a table) represents a linear or exponential function in order to solve problems.
- 1. Determine that simple interest is a linear model.
- 2. Draw a rough sketch of the graph of the data relating the Number of Years and the Interest in the savings account.
- 3. Discuss that the graph of the data is a linear shape.
- F-LE.A.2*[4] Construct exponential functions, given two input-output pairs (include reading these from a table).
- 1. Write an exponential function relating the number of years and total balance in the account using the data from the table.
- 2. Use the function to determine the amount of money in the account after 30 years.
- QR-2.3 [2] Understand how interest relates to linear and exponential mathematical models by discussing that simple interest is a linear model and compound interest is an exponential model.
- F-LE.A.3*[2] Compare tables and graphs of exponential and linear functions to observe that a quantity, increasing exponentially, exceeds a linear function to solve mathematical and real-world problems.
- 1. Sketch a graph of the compound interest model and describe the shape of the graph.
- 2. Compare the graphs of the compound and simple interest models.
- 3. Compare and contrast simple and compound interests.
- 4. Use the data and observations about simple and compound interest in the two savings accounts to discuss that a quantity, increasing exponentially, exceeds a linear function over time.

Quantitative Reasoning
Unit 5: Non-Linear Modeling
Section 5.2 Is it Exponential?

Suggested Pacing: 2 Sessions

NJSLS-M Performance Expectations

F-LE.A.1.a* Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

- F-LE.A.1.a*[1] Recognize that linear functions grow by equal differences over equal intervals.
- F-LE.A.1.a*[2] Recognize that exponential functions grow by equal factors over equal intervals.
- F-LE.A.1.a*[3] Distinguish between situations that can be modeled with linear functions and exponential functions to solve mathematical and real-world problems.
- 1. Compare population data and sources and discuss which would be most credible or accurate and why.

- 2. Examine data of the world population, compare calculations of first differences and average rate of growth.
- 3. Determine if a linear or exponential model would best fit the data.

QR-3.2 [1] Understand that models are mathematical simplifications of real-world data and phenomena by discussing why the average rate of growth is not constant for the data.

Quantitative Reasoning
Unit 5: Non-Linear Modeling
Section 5.3 Oh, Dear!

Suggested Pacing: 3 Sessions

NJSLS-M Performance Expectations

N-Q.A.1* Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

F-IF.B.5* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

S-ID.B.6a* Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

F-LE.A.1.a*[1] Recognize that linear functions grow by equal differences over equal intervals.

N-Q.A.1*[2] Calculate unit conversions.

F-IF.B.5*[2] Identify an appropriate domain based on the unit, quantity, and type of function it describes.

S-ID.B.6a*[1] Represent data on a scatter plot (2 quantitative variables).

S-ID.B.6a*[3] Using given scatter plot data represented on the coordinate plane, informally describe how the two quantitative variables are related.

N-Q.A.1*[1] Recognize units given or needed to solve problems.

N-Q.A.1*[2] Calculate unit conversions.

- 1. Discuss the limits on exponential growth in real populations.
- 2. Use the density of deer on Smith Island and the size of Smith Island to calculate the total deer population in that year.

S-ID.B.6a*[1] Represent data on a scatter plot (2 quantitative variables) FOR THE POPULATION OF DEER ON SMITH ISLAND OVER TIME IN YEARS.

F-IF.B.5*[2] Identify an appropriate domain based on the unit, quantity, and type of function it describes FOR THE YEARS BETWEEN 1980-2002 ON SMITH ISLAND BY DECIDING IF IT MAKES SENSE TO CONNECT THE DATA POINTS ON THE SCATTER PLOT FOR THE POPULATION OF DEER OVER TIME.

S-ID.B.6a*[3] Using given scatter plot data represented on the coordinate plane, informally describe how the two quantitative variables OF DEER POPULATION AND YEARS are related.

QR-3.3 [1] Identify behavior in a graph, draw conclusions about the behavior, and predict future outcomes.

- 1. Use the graph to discuss how the deer population on Smith Island was growing rapidly.
- 2. Use the graph to make a prediction about what would happen to the deer population on Bald Head Island between 2002 and 2010 without any human intervention.

QR-3.3 [2] Sketch a model for a population that increases at an increasing rate using the deer population on Smith Island between 1980 and 2020.

QR-3.3 [3] Sketch a model for a population that increases at a decreasing rate using the deer population in Smith Island after culling takes place.

Quantitative Reasoning

Suggested Pacing: 1 Sessions

Unit 5: Non-Linear Modeling

Section 5.4 Exploring Logistics Models

NJSLS-M Performance Expectations

F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

F-BF.B.3[7] Experiment with cases and illustrate an explanation of the effects on the graph, using technology. BY ADJUSTING PARAMETERS OF THE LOGISTIC MODEL AND OBSERVING EFFECTS ON THE SHAPE OF THE GRAPH.

QR-3.4 [2] Understand that initial population, growth rate, and carrying capacity are the parameters of the logistic model in this lesson by relating the logistic model to the population of deer on Smith Island.

Quantitative Reasoning
Unit 5: Non-Linear Modeling
Section 5.5 How Long Is a Day?

Suggested Pacing: 3 Sessions

NJSLS-M Performance Expectations

(*) F-TF.B.5* Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. (+ for 2023)

F-IF.C.7.e Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

(*) F-IF.C.7.f (+) Graph trigonometric functions, showing period, midline, and amplitude.

N-Q.A.1*Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

F-TF.B.5*[3] Explain why real-world or mathematical phenomena exhibit characteristics of periodicity. BY DISCUSSING HOW THE BEHAVIOR OF A PERIODIC GRAPH RELATES TO THE TIDES. (+ for 2023)

F-TF.B.5*[1] Define and recognize period, amplitude, and midline parameters in a symbolic trigonometric function. (+ for 2023)

- 1. Define period and amplitude.
- 2. Determine the period and amplitude of the graph of the high and low tides from the Preview Assignment.
- 3. Use the midline of the graph ""Length of a New Orleans Day"" to estimate the length of the average day in New Orleans.
- 4. Discuss multiple approaches to finding the amplitude of the function using the graph ""Length of a New Orleans Day"". Find the amplitude.
- 5. Identify the period of the function ""Length of a New Orleans Day.

N-Q.A.1*[2] Calculate unit conversions. TO DETERMINE HOW MANY MINUTES PASS BETWEEN TWO CONSECUTIVE SUNRISES.

F-TF.B.5*[2] Interpret the parameters of a trigonometric function in the context of real-world situations. BY INTERPRETING THE PERIOD OF THE GRAPH "LENGTH OF A NEW ORLEANS DAY".

F-IF.C.7.e*[2] Graph trigonometric functions. Show: period, and amplitude.

- 1. Research the lengths of the longest and shortest days of 2016 in Seattle, Washington, and when they occur.
- 2. Use the data and consider how it relates to the period and amplitude of the periodic function to sketch a graph of the length of a day in Seattle on the same axes as the New Orleans graph.
- 3. Identify the period and amplitude of the Seattle graph.
- 4. Research the lengths of the longest and shortest days of 2016 in Santiago, Chile, and when they occur.

5. Use the data and consider how it relates to the period and amplitude of the periodic function to sketch a graph of the length of a day in Santiago on the same axes as the New Orleans and Seattle graph.

QR-3.5 [1] Compare and contrast the graphs of different periodic models for the length of day in minutes of the cities New Orleans, Seattle, and Santiago.

Quantitative Reasoning
Unit 5: Non-Linear Modeling
Section 5.6 Exploring Sine

Suggested Pacing: 2 Sessions

NJSLS-M Performance Expectations

F-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F-LE.A.1.a* Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

(*) F-TF.B.5* Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

F-TF.B.5*[3] Explain why real-world or mathematical phenomena exhibit characteristics of periodicity. BY DISCUSSING HOW THE MOTION OF FERRIS WHEEL IS RELATED TO A PERIODIC MODEL. (+ for 2023)

QR-3.6 [1] Introduce the sine function. Understand that the sine function is periodic.

F-BF.B.3[7] Experiment with cases and illustrate an explanation of the effects on the graph, using technology. BY ADJUSTING PARAMETERS OF THE SINE FUNCTION AND OBSERVING EFFECTS ON THE SHAPE OF THE GRAPH.

QR-3.6 [2] Change the parameters of the sine curve to match given criteria by determining the values of the parameters necessary to create the parent function $y = \sin(x)$.

F-LE.A.1.a*[3] Distinguish between situations that can be modeled with linear functions and exponential functions.

Quantitative Reasoning Unit 6: Sampling II

Section 6.1 Stratified Random Sampling

Suggested Pacing: 4 Sessions

NJSLS-M Performance Expectations

(*) S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-IC.B.3 [2] Recognize the differences among surveys, experiments, and observational studies in making statistical inferences and justifying conclusions, explain how randomization relates to each of these methods of data collection. (+ for 2023)

QR-5.4[1] Explain how differences in the sample can affect the statistics collected FOR THE STUDY ON GENDER AND BMI.

QR-5.4[2] Explain what a stratified random sample is and explain how to collect it FOR THE STUDY ON GENDER AND BMI.

QR-5.4[3] Collect data FOR THE STUDY ON GENDER AND BMI using a stratified random sample.

- a. Decide how many men and women to include in the sample.
- b. Select the sample using a random number generator.
- c. Create a table of BMI values for the students selected for the sample.
- d. Calculate the statistic for the mean BMI.

QR-5.4[4] Calculate and comment on the absolute error found using the stratified random sampling compared to the simple random sample.

Quantitative Reasoning
Unit 6: Sampling II
Section 6.2 Correcting Bias

Suggested Pacing: 2 Sessions

Suggested Pacing: 3 Sessions

Suggested Pacing: 3 Sessions

NJSLS-M Performance Expectations

(*) S-IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed). (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-5.7[1] Identify the sample and population in the IMPROVING LIBRARY FACILITIES EXAMPLE

S-IC.B.6[3] Draw conclusions based on graphical and numerical summaries in the IMPROVING LIBRARY FACILITIES EXAMPLE, STUDENT STUDYING EXAMPLE and in the SMOKING MARIJUANA AND IQ EXAMPLE. (+ for 2023)

S-IC.B.6[3] Draw conclusions based on graphical and numerical summaries in the IMPROVING LIBRARY FACILITIES EXAMPLE, STUDENT STUDYING EXAMPLE and in the SMOKING MARIJUANA AND IQ EXAMPLE.

S-IC.B.6 [1] Define the characteristics of experimental design (control, randomization, and replication). (+ for 2023)

Quantitative Reasoning

Unit 6: Sampling II

Section 6.3 Correlation vs. Causation

NJSLS-M Performance Expectations

S-ID.C.9 Distinguish between correlation and causation.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-5.8 [1] Identify and classify variables in the SNACKING HABITS AND AFTER SCHOOL ACTIVITIES example.

S-ID.C.9*[3] Distinguish between correlation and causation in the SNACKING HABITS AND AFTER SCHOOL ACTIVITIES example.

QR-5.8[2] Classify studies as observational or experimental and explain which types of studies can be used to establish causation.

QR-5.8[3] Rewrite an observational study as an experiment in order to be able to assess causation for the SNACKING HABITS AND AFTER SCHOOL ACTIVITIES example.

QR-5.8[4] Identify the treatment in an experimental study and explain how to randomly allocate subjects into treatment groups for the SNACKING HABITS AND AFTER SCHOOL ACTIVITIES example

Quantitative Reasoning

Unit 6: Sampling II

Section 6.4 Experimental Design

NJSLS-M Performance Expectations

S-ID.C.9 Distinguish between correlation and causation.

(*) S-IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed). (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-ID.C.9*[3] Distinguish between correlation and causation.

QR-5.9[1] Explain whether or not causation can be established for the CHOCOLATE CONSUMPTION example and if causation cannot be established, give an explanation of why not.

QR-5.9[2] Explain the process and rationale for using a placebo as part of an experimental study for the CHOCOLATE CONSUMPTION example.

QR-5.9[3] Explain the process and rationale for using random allocation of subjects into treatment and control groups for the CHOCOLATE CONSUMPTION example.

S-IC.B.6 [2] Evaluate experimental study design, how data was gathered, and what analysis (numerical or graphical) was used for the CHOCOLATE CONSUMPTION example (+ for 2023)

Quantitative Reasoning Unit 6: Sampling II

Section 6.5 Placebos and Blinding

NJSLS-M Performance Expectations

S-ID.C.9 Distinguish between correlation and causation.

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-ID.C.9*[3] Distinguish between correlation and causation.

QR-5.10[1] Explain what the placebo effect is and how it affects statistical studies for the CHRONIC MIGRAINE HEADACHE example.

QR-5.10[2] Explain the process and benefits of blinding on test subjects and how it helps to lessen the effect of confounding variables for the CHRONIC MIGRAINE HEADACHE example.

QR-5.10[3] Explain the process and benefits of using a double blind study and how it helps to lessen the effects of confounding variables for the CHRONIC MIGRAINE HEADACHE example.

Quantitative Reasoning Unit 6: Sampling II Section 6.6 Blocking

NJSLS-M Performance Expectations

(*) S-IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed). (+ for 2023)

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

QR-5.11[1] Explain the effect on an experiment if the subjects are volunteers as opposed to being randomly selected from a population for the COLD SORES example

QR-5.11[2] Explain the process and benefits of using blocking in an experimental study for the COLD SORES example.

S-IC.B.6 [4] Support with graphical and numerical summaries how appropriate the report of data was for the COLD SCORES example (+ for 2023)

S-IC.B.6 [3] Draw conclusions based on graphical and numerical summaries for the COLD SORES example (+ for 2023)

Quantitative Reasoning Unit 6: Sampling II Section 6.7 Inference

NJSLS-M Performance Expectations

(*) S-IC.B.6 Evaluate reports based on data (e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed). (+ for 2023)

Suggested Pacing: 3 Sessions

Suggested Pacing: 2 Sessions

Suggested Pacing: 2 Sessions

Standards-Aligned Objectives. Instruction and assessment will align to the following objectives:

S-IC.B.6 [2]Evaluate experimental study design, how data was gathered, and what analysis (numerical or graphical) was used. (+ for 2023)

QR-5.12[1] Identify problems in studies that prevent researchers from making inferences to populations or treatments for the GASTRIC BYPASS SURGERY example.

QR-5.12 [2] Make appropriate conclusions from observational studies and from experimental studies for the CORONARY ARTERY RISK DEVELOPMENT example.

QR-5.12 [2] Explain when it is possible to use the results of a statistical study to make inferences about a population for the CORONARY ARTERY RISK DEVELOPMENT example.

Quantitative Reasoning				
NJSLS Career Awareness, Exploration, Preparation, and Training, and Life Literacies and Key Skills		Section		
9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.	1.4, 1.5, 1.8, 2.1, 2.2, 2.3, 2.6, 2.8, 4.5, 4.8, 5.4		

^{*} ID 9.2.12.CAP.11 duplicated in NJDOE NJSLS file page 1 and 2