

Passing the OSU Mathematical Skills Assessment B Test

(for the severely math averse)

OSU provides a set of sample math problems, assumedly indicative of what will be on the real math placement test, at [this web page](#), which I will go over in painstaking, mind-numbing detail. Along the way we will learn rules that can be used as pragmatic paths to the answers. This process will have the unfortunate side-effect of teaching you how to “do math”, which I apologize for in advance. I assure you this is not my purpose. As we find rules along the way, they will be kept in a running list, and referenced several times throughout the document.

Here is question 1, grabbed from the sample test page:

1. Perform the indicated operations and reduce the answer to lowest terms.

$$\text{a. } \left(\frac{x^3 + x}{x^2 + x} \right) \left(\frac{x^2 - 1}{x^2 + 1} \right) \quad \text{b. } \frac{x}{x+1} + \frac{1}{x-1} + 1 \quad \text{c. } \frac{3 - \frac{1}{x}}{9 - \frac{1}{x^2}}$$

This is a question in three parts (a, b, and c) that wants you to reduce fractional expressions containing variables. It's a doozy way to start the exercise, a sure indicator that OSU wants nothing more than for you to be disheartened and fail.

First, let's blow up problem 1a so we can see it:

$$\left(\frac{x^3 + x}{x^2 + x} \right) \left(\frac{x^2 - 1}{x^2 + 1} \right)$$

Next, let's start our list of rules with all the absolute basics we need to dissect this::

1. Letters such as “x” stand for unknown numbers.

Easy enough. The expression in question 1a has a lot of x's in it. Ultimately we don't care what number x stands for. If this expression were followed by an equals sign (=) and another expression on the other side, then it would be an equation... let's make that a rule:

2. If it doesn't have an equals sign, it's an expression, not an equation.

If the above question was an equation, we would probably be trying to “solve for x”, and find out its value. In this question, though, nobody cares. For this question we just want to make the expression itself look less complicated. We're going to do this by finding clever ways to erase parts of the expression that cancel out, like multiplying and dividing by the same number. We'll

get to that later.

3. Arithmetic functions (+ - $\cdot$ $\div$) work the same way they did in elementary school.

Self-explanatory, I hope. + is addition, - is subtraction, $\cdot$ or $\times$ is multiplication, and $\div$ is division. For this question, we won't be doing any arithmetic, though, since we don't know or care what x really stands for. What we'll be doing instead is manipulating unknowns using arithmetic operations, which is all Algebra really is. We'll find out, for example, why multiplying $(x + 1)$ by x gives us $(x^2 + x)$. Later, though. For now, let's identify some more rules.

4. A raised number beside a term means raising it to a power, or multiplying it by itself.

x^1 is just x . 4^1 would just be 4.

x^2 means $(x \cdot x)$, or "x squared", or "x times x". 4^2 would be $4 \cdot 4$, or 16.

x^3 means $(x \cdot x \cdot x)$, "x cubed", or "x times x times x". 4^3 would be $4 \cdot 4 \cdot 4$, or 64

Et cetera. For natural numbers (1, 2, 3,...) this is sort of intuitive. What happens when you raise a number to 0 or a negative number or a fraction is completely arbitrary madness that doesn't concern us right now.

5. Things enclosed in parentheses get evaluated separately from everything else.

Fairly simple. Parentheses just group things for us, which has a nice side-effect sometimes of showing us what chunks of an expression can be played with without affecting anything else. In the expression $(4 + 2) \cdot 6$, the $4 + 2$ gets evaluated first, and then that sum gets multiplied by 6. What's the answer to my example? Don't care. Moving on.

6. A term on top of another with a line between them is a fraction. Divide the top by the bottom.

That horizontal line between $x^3 + x$ and $x^2 + x$? It's called a vinculum. Partly because it's Latin for "tie" (like you're tying two things together), and partly because mathematicians are all freaks and like to make things sound scary. I prefer "dividing bar", which describes both its purpose, and what you do when you see one.

The top term is the numerator, the bottom is the denominator, which is useful to know because math books refer to them that way. Fractions are usually read "numerator over denominator".

The fraction $\frac{a}{b}$ would be read "a over b".

7. Two terms placed together with no explanation get multiplied together.

If you see xy in an expression, for example, you're multiplying x by y . If you see $2x$, then you're

doubling x .

OK, then. Here are the rules identified so far:

1. Letters such as " x " stand for unknown numbers.
2. If it doesn't have an equals sign, it's an expression, not an equation.
3. Arithmetic functions ($+$ $-$ $\cdot$ $\div$) work the same way they did in elementary school.
4. A raised number beside a term means raising it to a power, or multiplying it by itself.
5. Things enclosed in parentheses get evaluated separately from everything else.
6. A term on top of another with a line between them is a fraction. Divide the top by the bottom.
7. Two terms placed together with no explanation get multiplied together.

With these seven basic concepts, we can dissect question 1a and make some sense about what it's trying to express. The question was:

$$\left(\frac{x^3 + x}{x^2 + x} \right) \left(\frac{x^2 - 1}{x^2 + 1} \right)$$

Ultimately, this is multiplying two fractions together: x cubed plus x over x squared plus x , and x squared minus 1 over x squared plus 1. The mathematical notation above, though you may hate it, is a big improvement over the long sentence form, and allows you to manipulate the terms much more easily. For example, let's start with the term in blue here:

$$\left(\frac{x^3 + x}{x^2 + x} \right) \left(\frac{x^2 - 1}{x^2 + 1} \right)$$

Is there another way to express that? Both parts (x cubed, and x) contain x . Is that the same as x times something? Yes. It's $x \cdot (x^2 + 1)$. Why? Here's the dry rule::

$$8. a \cdot (b + c) = ab + ac$$

This is called the distributive law. It's trying to show us that multiplication by a number is the same as multiplying by the sum of its parts. Unclear? It may help if we plug in some real numbers, using my favorite number to use in examples: 1. Let's try a simple multiplication problem, $6 \cdot 2$, broken down into a different form:

$$6 \cdot (1 + 1)$$

In the parenthesis, 1 plus 1 is 2. Multiply that by 6, and you get 12. No-brainer. Using the distributive law, the problem can also be expressed as:

$$(6 \cdot 1) + (6 \cdot 1)$$

Since 6 times 1 is just 6, this expression just means 6 plus 6, or 12.

As a consequence of that law, whenever an expression has the same term in every piece, you can divide the term out of each part, throw everything in parentheses, and multiply it back outside of the parentheses. So, if we rewrite the expression in blue above as

$(x \cdot x^2) + (x \cdot 1)$, then it looks a lot like the right side of the distributive law. The left side, then, would be $x \cdot (x^2 + 1)$, making the following equation true:

$$x^3 + x = (x^2 + 1) \cdot x$$

If you're truly math-averse and not just reading this for giggles, that equation is going to feel uncomfortable, and even more uncomfortable is trying to deduce why I would think I needed to do it at all. Simply, I'm looking for things that cancel out, which we'll get to very soon. First, a new rule:

9. You can substitute equal expressions anywhere.

This is what let me substitute $(1+1)$ for 2 above. I can take the blue expression above, and replace it with the item we just found to be equal to it, giving us this:

$$\left(\frac{(x^2 + 1) \cdot x}{x^2 + x} \right) \left(\frac{x^2 - 1}{x^2 + 1} \right)$$

Now notice that the same term, shown in red below, appears in the top of one fraction, and the bottom of the other:

$$\left(\frac{(x^2 + 1) \cdot x}{x^2 + x} \right) \left(\frac{x^2 - 1}{x^2 + 1} \right)$$

A new rule will show us how this helps:

$$10. \frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}$$

I'm not sure if this law has an official math name, but it's how you multiply fractions. Multiplying two fractions just gives you a new fraction. Its numerator is the original two numerators multiplied together, and likewise its denominator is the original denominators multiplied together.

Once you get your head around that, think how the two red terms affect the final product: it's multiplied into the numerator and the denominator. This brings us to the eleventh rule, which

will help clear out a lot of chaff from the expression:

$$11. \frac{ab}{ac} = \frac{b}{c}$$

If the same number is multiplied in a fraction's numerator and denominator, it cancels out. Or, you can replace it with anything (except 0). Since we haven't formally multiplied the fractions together, I'm not going to just erase the red terms; instead, I'm going to replace them with 1:

$$\left(\frac{1 \cdot x}{x^2 + x} \right) \left(\frac{x^2 - 1}{1} \right)$$

I can simplify the $1 \cdot x$ to just x . Now, we can do a similar "distributive law" operation on the term in blue here:

$$\left(\frac{x}{x^2 + x} \right) \left(\frac{x^2 - 1}{1} \right)$$

Since x is in both terms, we can divide it out and multiply it back in thusly:

$$x^2 + x = (x + 1) \cdot x$$

...and substitute it into the expression:

$$\left(\frac{x}{(x + 1) \cdot x} \right) \left(\frac{x^2 - 1}{1} \right)$$

Now look at the red terms here:

$$\left(\frac{x}{(x + 1) \cdot x} \right) \left(\frac{x^2 - 1}{1} \right)$$

It should be a little more clear intuitively why those cancel out, but it's still based on rule 11. I'll replace both terms with 1:

$$\left(\frac{1}{(x + 1) \cdot 1} \right) \left(\frac{x^2 - 1}{1} \right)$$

...and simplify the left denominator, since $(x+1)$ times 1 is just $(x+1)$. This type of thing is called

an “identity operation”, by the way. We should probably make that a rule so I don’t keep explaining it.

12. Multiplying or dividing a term by 1 gives you the same term.

I mean, you already knew that, right? Still, we should write it down. OK, now comes something a little more abstract. Can we do anything to reduce the term in blue here?

$$\left(\frac{1}{x+1}\right)\left(\frac{x^2-1}{1}\right)$$

Surprisingly, the answer is yes. Math “geeks” will recognize the term in blue as being equal to $(x+1) \cdot (x-1)$, because it comes up a lot. I didn’t recognize this, and on my first pass simplifying the expression I got the “wrong” answer... wrong because I didn’t simplify it as much as was possible.

So how does that work? There is a rule called FOIL (first, outer, inner, last) for multiplying two binomials together. OK, let’s take it a step at a time. First, what’s a binomial?

13. A binomial is the sum of two terms. $x^2 + 5$, $x - 1$, $x + y$, and $4 - 2$ are all binomials

Easy enough. A topic that comes up in math classes is finding the “binomial factors” of a more complicated expression, in particular finding “a” and “b” in an equation like:

$$(x + a) \cdot (x + b) = x^2 + 5x + 6$$

Hang on to that equation for a second, we’ll come back and solve for a and b. First, a couple of new rules.

14. A “Dot Product” is multiplying pairs of terms together, then adding up the products.

If I have two series of numbers (1,2,3,4) and (5,6,7,8), then its dot product would be:
 $(1 \cdot 5) + (2 \cdot 6) + (3 \cdot 7) + (4 \cdot 8)$

15. FOIL: A dot product of binomials using the First, Outer, Inner, and Last entries.

$$(x + a)(x + b) = x^2 + bx + ax + ab \text{ First..}$$

$$(x + a)(x + b) = x^2 + bx + ax + ab \text{ ...plus outer...}$$

$$(x + a)(x + b) = x^2 + bx + ax + ab \text{ ...plus inner...}$$

$$(x + a)(x + b) = x^2 + bx + ax + ab \text{ ...plus last.}$$

Why that works is an extension of the distributive law. Let's take the original distributive law example, $a \cdot (b + c) = ab + ac$, and change the variables to match the $(x + a) \cdot (x + b)$ expression that we're trying to explain:

$$(x + a) \cdot (x + b) = (x + a)x + (x + a)b$$

The right side of the equation can also be expanded with the distributive law:

$$(x + a) \cdot (x + b) = (x + a)x + (x + a)b = x^2 + ax + bx + ab$$

The final expression gives you the same thing that FOIL does. If we take the $ax + bx$ and use the distributive law to change it to $(a + b)x$, then we see an interesting thing:

$$(x + a) \cdot (x + b) = x^2 + (a + b)x + ab$$

Don't see it yet? That's alright. Look at it now with the equation we're trying to solve:

$$(x + a) \cdot (x + b) = x^2 + (a + b)x + ab = x^2 + 5x + 6$$

So, $a + b$ equals 5, and $a \cdot b = 6$. Are there any two numbers that, if added together give 5, and multiplied together give 6? Easy: 2 and 3!

$$(x + 2) \cdot (x + 3) = x^2 + 5x + 6$$

Let's make that a rule:

16. The equation $(x + a) \cdot (x + b) = x^2 + cx + d$ implies $a + b = c$ and $a \cdot b = d$

Using FOIL and that math trick, you can find binomial factors of similar equations. Now, back to the actual OSU math B test question we're REALLY trying to solve, let's look at where we left off:

$$\left(\frac{1}{x + 1}\right) \left(\frac{x^2 - 1}{1}\right)$$

If we rewrite the blue part above as $x^2 + 0x - 1$, what two numbers fit rule 16? The answer is

simple, but unexpected: -1 and 1 . $-1 + 1 = 0$ and $-1 \cdot 1 = -1$. So:

$$(x + 1) \cdot (x - 1) = x^2 - x + x - 1 = x^2 - 1$$

And we can substitute that in for $x^2 - 1$.

$$\left(\frac{1}{x + 1}\right) \left(\frac{(x + 1) \cdot (x - 1)}{1}\right)$$

Now we see that another term, shown in red below, can cancel out:

$$\left(\frac{1}{x + 1}\right) \left(\frac{(x + 1) \cdot (x - 1)}{1}\right)$$

This can be simplified to:

$$\left(\frac{1}{1}\right) \left(\frac{1 \cdot (x - 1)}{1}\right)$$

...and simplified more to:

$$1 \cdot \frac{(x - 1)}{1}$$

...and finally:

$$x - 1$$

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8. $a \cdot (b + c) = ab + ac$
9. You can substitute equal expressions anywhere.

10. $\frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}$

11. $\frac{ab}{ac} = \frac{b}{c}$

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