Prefixes and Scientific Notation

Introduction

Amateur Radio is a technical hobby. There is no getting away from this; communicating via electro-magnetic waves propagated through the atmosphere and/or the vacuum of space relies on fundamental principles of physics and applied mathematics.

A lengthy discussion about the need for someone who just wants to talk to their mate across town to be able to perform calculations relating to resonant frequencies of tuned circuits, or even something simple such as converting a frequency to a wavelength when there is a plethora of calculators available on-line could be had.

However, at its most basic, the requirement to perform mathematical calculations is enshrined in the syllabus and unless or until those who set and, more importantly, approve the parameters for demonstrating theoretical and practical competence change their opinions, mathematics is going to remain part of the examination process.

This document explains the types of mathematics capability that those setting the syllabus think is required at each level. It then goes on to address some of the areas that the South Bristol ARC have seen repeatedly cause confusion among a large proportion of our students over the years.

The Syllabus

The key words "*Recall*" and "*Understand*" are used to denote differing levels of comprehension.

Recall indicates the need to remember a fact and apply it fairly directly to a question or situation. A thorough understanding of why the fact is so and the full range of circumstances in which it is applicable is not required, but questions will expect a basic understanding of its meaning and implications.

Understand indicates the need for a more detailed knowledge of the subject, fully comprehending the reasons why the point is correct and the range of circumstances in which it is relevant and applicable. Typically, this will be where the candidates will find themselves having to make judgements or apply a practice to a wider range of circumstances.

These terms should also be read in the context of the level of examination concerned.

At Foundation level there are more '*recall*' syllabus items than '*understand*' whereas at Full level the majority are of an '*understand*' nature. That will expect the candidate to know the background to the topic and the implications of not adopting the accepted practice.

For example, at the Foundation level the syllabus requires knowledge of the formula:

$$P = I \times V$$

This knowledge extends to:

- What the letters stand for,
- The ability to perform a calculation given any two of the factors, and
- Be able to interpret the effect of, for example, doubling or halving one of the factors

At Full level any '*Understand*' question may use calculations if the context permits.

Formulae

At the Foundation level it is important that candidates understand the fundamental principles behind the theoretical topics discussed. For that reason no formula sheet is provided.

At the Intermediate level formulae are provided **but may need to be transposed**.

At Full level, all formulae are provided but are not titled or explained. Candidates will be expected to recognise which formula is appropriate and may need to transpose it depending on the parameter to be calculated.

Mathematics and the Foundation Exam

By the time the candidate sits the Foundation exam they should be confident about:

- Addition, subtraction, multiplication and division,
- Simple fractions and their decimal equivalents (e.g. ½ = 0.5, ¼ = 0.25 etc.),
- Multiple and sub-multiple units from micro to Giga,
- Conversion of numbers from 10^{-3} to 10^{9} to/from decimal, and

- Understanding of simple formulae, e.g. a = b x c, and re-arrange them to make any parameter the subject of the formula.

Mathematics and the Intermediate Exam

By the time the candidate sits the Intermediate exam they should, in addition to the attributes at the Foundation level, be confident about:

- All attributes at the Foundation level,
- Multiple and sub-multiple prefixes from pico to Giga,
- Calculations with quantities from 10^{-12} to 10^{12} recognising that interim stages may go outside those limits, and
- Use of simple formulae containing brackets, squared or square root operators, e.g.

$$I = \sqrt{\left(\frac{P}{R}\right)} \; or \; P = \frac{V^2}{R}$$

Mathematics and the Full Exam

By the time the candidate sits the Full exam they should, in addition to the attributes at the Foundation and Intermediate levels, be confident about:

- All attributes at the Foundation level,
- All attributes at the Intermediate level, and
- Use and transposition of more complex formulae for example:

$$f = \frac{1}{\left(2\pi\sqrt{LC}\right)}$$

Calculators

Calculators are permitted at all levels of the UK Amateur Radio Exam, however they must be silent and non-programmable. It is not permitted to use a calculator app on a mobile telephone or tablet and if you are taking the exam on-line our understanding is that the exam software will prevent you using other programs on your computer, so if you're going to bring a calculator to the exam please ensure it is a stand-alone silent non-programmable device.

Foundation

At Foundation level a calculator is not essential, although the South Bristol ARC would recommend having one on hand at the exam for the following reasons:

1. Just in case a question with awkward values has slipped through the net, and
2. Because it's too easy in the heat of the exam to make a stupid arithmetic error

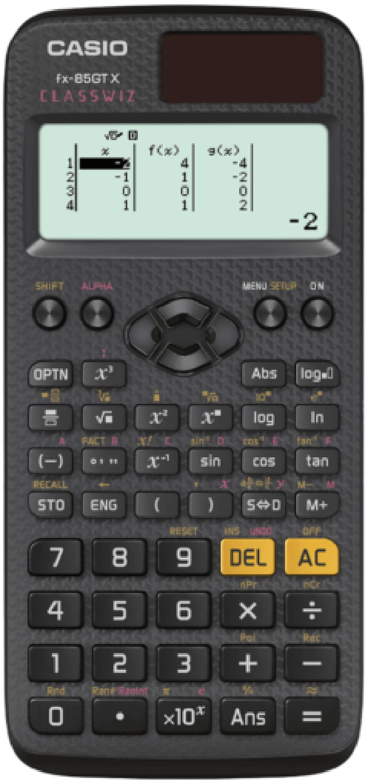
At the Foundation level the calculator need not be anything special, a simple 4 function calculator should suffice.

Intermediate and Full

At both the Intermediate and Full levels the South Bristol ARC recommends the use of a scientific calculator. The same device should be usable at both levels and need not be overly complex. Remember that it should be non-programmable, which is likely to rule out most of the graphing calculators and possibly also those that include pre-programmed conversions and scientific constants.

Essentially anything that is suitable for GCSE Maths ought to be suitable for the Intermediate and Full exams. The functions that are likely to be required are as follows:

- Basic arithmetic: addition, subtraction, multiplication and division
- An exponent function (x10^{x}) to be able to directly enter scientific notation
- Brackets to control operator precedence
- Squared and square roots
- Recipricols
- An internal constant for pi (3.1415927)
- Base 10 logs



Any calculator in this class is likely to offer additional functions, such as trigonometry, natural logs etc. but in the main these will be superfluous.

Quantities

Scientific and Engineering calculations are undertaken using the SI System (International System of Units). This is the modern metric system, the global standard for measurement, built on seven base units: second (time), meter (length), gramme (mass), ampere (electric current), kelvin (temperature), mole (amount of substance), and candela (luminous intensity). These units form a coherent, base-10 system used in science, trade, and daily life, with prefixes (like kilo-, milli-) creating decimal multiples for other units, ensuring universal understanding.

There are times when the amount of a given quantity results in a large or small number prefixes are adopted to simplify the presentation of numbers. This is not something new, there are clear examples of this in everyday life. Examples are:

- We don't ask for 0.005m bolts in Screwfix (other hardware stores are available), we ask for 5millimetre bolts. The term "milli" means thousandths, in this case thousandths of a metre. 0.005m is 5 thousandths of a metre or 5 millimetres.
- Similarly, although the SI unit of mass is the gramme, we rarely buy anything in "grammes" (medication is an exception) and things like bags of flour or sugar in supermarkets are often sold in kilogrammes. The term "kilo" means thousands, in this case thousands of grammes. 1 kilogramme is 1000 grammes.

There are other prefixes that we use in the hobby, again some of these will be being used already, possibly without recognising their significance:

- **Giga** - Probably the largest prefix likely to be encountered in Amateur Radio, a "giga" something is a billion (1,000,000,000) of those things. Whilst we might like to think about GigaWatts of power the most likely place to encounter Giga is in frequencies, for example the 23cm band is around 1.2GHz or 1,200,000,000Hz.
- **Mega** - A reasonably common prefix in Amateur Radio, at least with certain units, a "mega" something is a million (1,000,000) of those things. Although we are unlikely to have to deal with MegaWatts or MegaVolts, MegaHertz (millions of Hertz) and MegaOhms (millions of Ohms) are quite common, for example 470MΩ is 470,000,000Ω.
- **kilo** - Another common prefix in Amateur Radio with a range of units being encountered with this prefix, a "kilo" something is a thousand (1,000) of those things. Full licensees are now permitted 1kiloWatt (1,000W) on primary bands, kiloOhms (thousands of Ohms) of resistance are not uncommon and frequencies in the kiloHertz (thousands of Hertz) are almost lower than the available Amateur

Radio bands. 909kHz is 909,000Hz which is in the medium wave band with a wavelength of 330m.

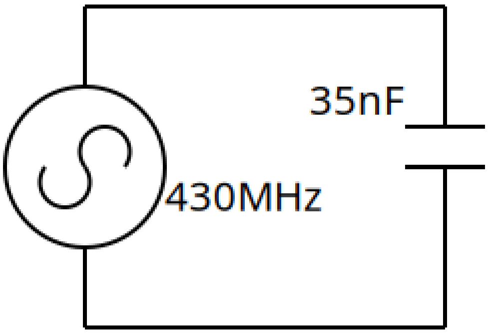
- **milli** - The first of the fractional prefixes. This prefix, and the ones that follow, are used to reduce the number of zeros before the significant digits of small numbers. A "milli" something is a thousandth (1 / 1,000) of those things. F1 timings are usually recorded and published to the nearest thousandth of a second, that is to 3 digits behind the decimal point; for example in 2020 Max Verstappen set a lap record at Silverstone completing a lap in 1 minute 27.097 seconds, the ".097" is 97 thousandths of a second or 97milliseconds. This is a particularly common prefix for Amateur Radio quantities such as current (milliAmps - mA) and power (milliWatts - mW)
- **micro** - This is a prefix that is unlikely to be regularly encountered in day-to-day life except possibly when taking supplements or medication. As an example the recommended adult daily intake of vitamin "D" is 10microgrammes (10μg - 0.000010g). A "micro" something is a millionth (1 / 1,000,000) of those things. In Amateur Radio this prefix is most likely to be associated with Inductance (milliHenries - μH) and Capacitance (milliFarads - μF) although received signal strength is based on 50μV into a 50Ω load being "S9".
- **nano** - A term that has impinged on the general public consciousness through Science Fiction and poorly informed journalists referring to "nano tech". In this case they are using the term to imply something that is extremely small, and in Science Fiction it is typically used in terms like "Nano Bots" implying a technology that is small enough to be part of the human body often conveyed via the blood stream. In the real world, and in the Amateur Radio world for that matter, a "nano" something is a billionth (1 / 1,000,000,000) of those things. It is most likely to be encountered in connection with capacitance with nanoFarads (nF) being common.
- **pico** - The smallest quantity that the Foundation, Intermediate and Full exams will consider, although there are terms for larger and smaller quantities that are outside the scope of this piece. A "pico" something is a trillionth (1 / 1,000,000,000,000) of those things and is most likely to be encountered in capacitance where picoFarads (pF) are viable values.

Prefixes, clearly, serve a useful purpose. With large quantities, instead of having to write 1,200,000,000Hz or say "1 billion 2 hundred million Hertz" it is possible to write 1.2GHz or say "one point two GigaHertz". Similarly, for small quantities instead of having to write

0.000000035F or say "zero point zero zero zero zero zero zero zero three five Farads" it is possible to write 35nF or say "thirty five nanoFarads".

However, prefixes also bring risks. Much like sharp chef's knives when they are used properly they are useful tools but used wrongly they can injure, well not in a literal sense in the case of prefixes; but they can certainly mutilate a calculation.

Consider the following Intermediate level calculation where a 430MHz signal is fed into a 35nF capacitor and the aim is to determine the capacitive reactance in the circuit. Don't worry about the technical background, this will be covered at the appropriate point but consider the mathematics of the situation.



$$X_C = \frac{1}{2\pi f C}$$

- f = 430MHz
- C = 35nF

$$X_c = \frac{1}{2\times3.141\times430\times35} = \frac{1}{94561.939} = 0.0000106\Omega$$

$$X_c = 10.6\mu\Omega$$

Sadly, this is not the correct answer, but why?

The reason is that although the frequency and the capacitance has been entered into the equation in the correct position the order of magnitude, or the size if you prefer, of the quantities is not being recognised.

To be able to perform calculations the frequency and the capacitance need to be converted back to base units, in other words the prefixes have to be removed. This results in:

$$X_C = \frac{1}{2\pi f C}$$

- f = 430MHz = 430,000,000Hz
- C = 35nF = 0.000000035F

$$X_c = \frac{1}{2\times3.141\times430000000\times0.000000035} = \frac{1}{94.562} = 0.0106\Omega$$

$$X_c = 10.6m\Omega$$

But, now all of the advantages of using prefixes have been lost. In fact things have got worse because now it is necessary to enter all of the extra zeros to get the order of magnitude correct. Surely there's a shorthand way of entering this and potentially avoiding transcription errors?

Yes, there is. Effectively now is the time to introduce **Standard Index Notation**, but there's a bit of explanation required.

- 430MHz = 430,000,000Hz

This can be considered as the product (result of multiplication) of two numbers:

- 430,000,000 = 430 x 1,000,000

1,000,000 can be considered as the product of 10's:

- 1,000,000 = 10 x 10 x 10 x 10 x 10 x 10
- 10 x 10 x 10 x 10 x 10 x 10 = 10^6

So:

- **430MHz = 430 x 10^6Hz**

The same logic can be applied to the small numbers except that instead of multiplying by powers of 10 it is necessary to divide by powers of 10.

- 35nF = 0.000000035F

This can be considered as the quotient (result of division) of two numbers:

- $0.000000035 = \frac{35}{1000000000}$

1,000,000,000 can be considered as the product of 10's:

- 1,000,000,000 = 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10
- 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 = 10^9

So:

- $\frac{35}{1000000000} = \frac{35}{10^9} = 35 \times 10^{-9}$
- **35nF = $35x10^{-9}$F**

In essence the standard index notation records how many times the decimal point has been moved to the left (+ve making big numbers smaller by removing trailing zeros) or the right (-ve making small numbers bigger by removing leading zeros).

This makes it possible to simplify the previous calculation as follows:

$$X_C = \frac{1}{2\pi f C}$$

- f = 430MHz = $430x10^6$Hz
- C = 35nF = $35x10^{-9}$F

$$X_c = \frac{1}{2\times3.141\times430\times10^6\times35\times10^{-9}} = \frac{1}{94.562} = 0.0106\Omega$$

$$X_c = 10.6 \times 10^{-3}\ or\ 10.6m\Omega$$

The table below indicates the relationship for a range of different prefixes.

Table of Prefixes and Standard Index Notation

Billions		100's of Millions	10's of Millions	Millions		100's of Thousands	10's of Thousands	Thousands		Hundreds	Tens	Units		Tenths	Hundreths	Thousandths	10 Thousandths	100 Thousandths	Millionths	10 Millionths	100 Millionths	Billionths	10 Billionths	100 Billionths	Trillionths
Giga				Mega				kilo		hecto	deco			deci	centi	milli			micro			nano			pico
G				M				k		h	da			d	c	m			µ			n			p
x10^{9}		x10^{8}	x10^{7}	x10^{6}		x10^{5}	x10^{4}	x10^{3}		x10^{2}	x10^{1}	x10^{0}		x10^{-1}	x10^{-2}	x10^{-3}	x10^{-4}	x10^{-5}	x10^{-6}	x10^{-7}	x10^{-8}	x10^{-9}	x10^{-10}	x10^{-11}	x10^{-12}
1	,	0	0	0	,	0	0	0	,	0	0	0													
				1	,	0	0	0	,	0	0	0													
								1	,	0	0	0													
												1	.	0											
												0	.	0	0	1									

Billions		100's of Millions	10's of Millions	Millions		100's of Thousands	10's of Thousands	Thousands		Hundreds	Tens	Units		Tenths	Hundreths	Thousandths	10 Thousandths	100 Thousandths	Millionths	10 Millionths	100 Millionths	Billionths	10 Billionths	100 Billionths	Trillionths
Giga				Mega				kilo		hecto	deco			deci	centi	milli			micro			nano			pico
G				M				k		h	da			d	c	m			μ			n			p
$\times10^{9}$		$\times10^{8}$	$\times10^{7}$	$\times10^{6}$		$\times10^{5}$	$\times10^{4}$	$\times10^{3}$		$\times10^{2}$	$\times10^{1}$	$\times10^{0}$		$\times10^{-1}$	$\times10^{-2}$	$\times10^{-3}$	$\times10^{-4}$	$\times10^{-5}$	$\times10^{-6}$	$\times10^{-7}$	$\times10^{-8}$	$\times10^{-9}$	$\times10^{-10}$	$\times10^{-11}$	$\times10^{-12}$
												0	.	0	0	0	0	0	1						
												0	.	0	0	0	0	0	0	0	0	1			
												0	.	0	0	0	0	0	0	0	0	0	0	0	1