# Binomial/Triangle Lesson

### Purpose

In this activity your team will succeed by engaging in *productive struggle* by

- exploring binomial coefficients and their relationships with each other,
- using a number triangle as an alternate visualization of binomial coefficients.

### Introductions & Teamwork Prep (5 minutes)

When you get with your team, each student shares the following:

- **Name:** Provide your preferred name and, if you like, pronoun.
- **Fact:** Share something that other people might not know about you.

After introductions, decide each member's role within
your team using the Roles Rules given by your instructor. The roles are described in more detail on the Team Roles handout:

- **Facilitator:** manages resources and keeps team on task
- **Documenter**: records team's work, creates *final* version for presentation with visual aids
- **Reporter**: refers to documented work and leads the team's presentation to the class
- **Questioner**: asks/nudges the group about ideas, asks questions of teacher/other teams

PRINT first and last names in the table below for who has which role:

| Facilitator | Documenter | Reporter | Questioner |
|---|---|---|---|
| | | | |

### Background (4 minutes)

a. Copy the following number triangle onto your shared workspace (e.g., whiteboard). Space out the entries so you can annotate them later. Suggestion: use no more than half of the team space for the triangle so there is still room for other work. **Do not erase the triangle.**
b. Continue the triangle pattern for two more rows.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
... ... ... ... ... ... ...
... ... ... ... ... ... ... ...

### Activity 1. Subsets with Puppies (23 minutes, including report-out)

**Note on "choose" representation:** there are many ways to symbolically represent the idea of *the number of ways to pick k unordered objects from a collection of n objects*: *nCk*, "n choose k", $C(n,k)$, $(n,k)$, and $\binom{n}{k}$ are all commonly used.

**Context:** We have five puppies at home: Retriever, Dachshund, Shepherd, Husky, Bulldog. Suppose we currently only have two leashes and can only take two dogs out on a walk.

1(a) In how many ways can we make pairs to take two dogs for a walk? Use one of the “choose” notations to represent how many ways we can make the pair.

1(b) Now, look at the Retriever in particular: how many ways can we choose two dogs to take out on a walk if the Retriever has to be one of the two? Express it in the “choose” notation.

1(c) How many different two-dog pairs can we take on a walk if we do **not** take the Retriever?

1(d) Notice that the answer to (a) equals the answer to (b) plus the answer to (c). **Why**?

1(e) Use the information from 1(d) to figure out what (9 choose 3) + (9 choose 4) is. How about the value of (105 choose 77) + (105 choose 78)? What is (n choose k) + (n choose k+1) in general? **Note:** The team needs to help the Reporter be ready to explain *why*.

### Activity 2. Making connections (17 minutes, including report-out)

2(a) Where on the number triangle do the values you got for 1(a), 1(b), and 1(c) appear?

2(b) What do you notice about the possible connection between the entries in the triangle and the “(a)=(b)+(c)” relationship from Activity 1? Identify a way to connect the numbers in the triangle with $\binom{n}{k}$ and label the triangle entries to explain the connection.

2(c) Now let’s talk about cake. Use the triangle to determine how many different versions of a 4-layered cake can be made from 8 popular layer flavors: vanilla, chocolate, strawberry, lemon, banana, coffee, carrot, and confetti. For this task, assume each four-layer cake has four different flavors (no repetitions).

### Activity 3. Making more connections (14 minutes, including report-out)

3(a) Count the number of ways to take one dog on a walk out of (Retriever, Dachshund, Shepherd, Husky, Bulldog). Then count the number of ways to take four dogs on a walk.

3(b) Count the number of ways to take two dogs on a walk out of (Retriever, Dachshund, Shepherd, Husky, Bulldog). Then count the number of ways to take three dogs on a walk.

3(c) What is the relationship between the two numbers you got in part (a)? What is the relationship between the two numbers you got in part (b)? How might you represent a generalization of this phenomenon? Explain why it works in general.

### Wrapping up (~5 minutes)

As you may already know, we can also calculate binomial coefficients directly with a formula: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Using this factorial formula, we could have (and you should do this for practice!) show that $\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$ (Pascal’s Formula).

Discussion (talk about at least one of these):

1. Imagine a lecture where Pascal’s Formula is proved using algebra. What are some pros and cons of understanding Pascal’s Formula algebraically versus what we did in this lesson?
2. Look back over the other ideas from this lesson: the relationship between the number triangle and $\binom{n}{k}$ from Activity 2 and the patterns found in Activity 3 (if you did Activity 3).

   What are some pros and cons of using the (a) number triangle and (b) factorial formula?

### Survey (~5 minutes)

Please share your feedback on this lesson through the link. The QR code is also for the same survey.
https://forms.gle/t98BA78VxqFRysoaA