Computer Number Systems

Problems - Level 3

1. Solve for X_2:

$$X_2 = \frac{FFF_{16} + 1_8}{777_8 + 1_2}$$

2. Given the hex digits 4,6,B, and C, what is the largest base 16 number that can be written using all 4 digits only once and that is less than 46, 300_{10}.

3. Multiply and give the answer in hex: $777_8 * 1001_2$

4. Convert 12473 from octal to hex.

5. Express 2000(base 10) as an octal number.

6. Express $159AE6_{16} + A37D9F_{16}$ in hex.

7. How many hex numbers less than 100_{16} contain the string 101010 in their binary representation?

8. What is the base 16 value of $B265_{16} + 8AD7E_{16}$?

9. How many numbers between 300 and 500 inclusive, have exactly eight 1's in their binary representation?

10. What is the base 16 value of $FEDCBA_{16} – ABCDEF_{16}$?

Computer Number Systems

Solutions - Level 3

1. $FFF_{16} + 1_8 = (1000_{16} – 1_{16}) + 1_{16}$
   $777_8 + 1_2 = (1000_8 – 1_8) + 1_8$

   In base ten, that is $16^3/8^3 = 8$. IN base 2 is 1000_2.

2. $16^3<46300<16^4$ and 46300/4096=11.03…, so the first hex digit is B. Now, 46300-4096*11=1244 and 1244/256=4.85…, so the second digit is 4. 1244-4*256=220 and 220/16 =13.75, so the third digit is C. This leaves 6 as the final digit. $B4C6_{16}$ = 46728.

3. $777_8 * 1001_2 = 111\ 111\ 111_2 * 1001_2$
   $=1000111110111_2$
   $=1\ 0001\ 1111\ 0111_2$
   $= 1\ \ 1\ \ \ \ F\ \ \ \ 7_{16}$

4. Expand each octal digit into a triplet of binary bits, regroup by quadruplets from right to left, and then convert each quadruplet into a hex digit.
   $1\ 2\ 4\ 7\ 3_8 = 001\ 010\ 100\ 111\ 011_2$
   $=0001\ 0101\ 0011\ 1011_2$
   $=1\ 5\ 3\ B_{16}$

5. This Y2K problem is straightforward:
   $2000=3*512 + 7*64 + 2*8 + 0 = 3720_8$

6. You could convert both numbers to base 10, do the addition and convert the sum back to hex. However, its probably easier to do the addition directly in base 16:
   6 + F = 15
   carry + E + 9 = 18
   carry + A + D = 18
   carry + 9 + 7 = 11
   carry + 5 + B = 11
   carry + 5 + 3 = 9
   1 + A = B

   $B91885_{16}$

7. The numbers less than 100_{16} contains two hex digits, or 8 binary bits. The string 101010 must appear in one of the following positions:
   101010xx
   x101010x
   xx101010

   For each of these positions, there are 4 possible binary numbers, as shown in the following chart. Of the 12 numbers, 10101010 appears twice.

101010xx	**x101010x**	**xx101010**
10101000	01010100	00101010
10101001	01010101	01101010
10101010	11010100	*10101010*

10101011	11010101	11101010

8. Word in hex, from the right to the left, carrying as needed:

   5 + E = 13
   carry + 6 +7 = E
   2 + D = F
   B + D = 15
   carry + 8 = 9

   $95FE3_{16}$

9. There are 4. Convert 300 and 500 to binary:
   300 = 100101100
   500 = 111110100

   There are 9 bits in both of these numbers, so all the numbers between 300 and 500 also have 9 bits. The 9-bit long binary numbers in this range with eight 1's are as follows:

   383 = 101111111
   447 = 110111111
   479 = 111011111
   495 = 111101111

10. Work from the right to the left, borrowing as needed:

    1A – F = B
    1(B-borrow) – E = C
    1(C-borrow) – D = E
    1(D-borrow) – C = 0
    E – B = 3
    F – A = 5

    $530ECB_{16}$