Advanced Composite Mechanics

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Submitted by

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Computation of Effective (Composite) Transversely Isotropic Engineering Constants Using Method of Cells (MOC) and Finite Element Analysis (FEA) in Abaqus

Abstract:

In this study, a composite is assumed to be made of carbon fiber T300 and 934-epoxy matrix. The typical properties for fiber and matrix are collected from the lecture of Dr. Binienda and the study of Ran et al. 2014. Method of cells (MOC) with the MATLAB application and Finite Element Analysis (FEA) using Abaqus are performed to predict the overall behavior of the composite from the known material properties of fiber and matrix. The computed effective transversely isotropic engineering constants obtained from the FEA analysis are validated, finally, by comparing with the result of analytical method (MOC).

Analytical method (*Method of Cell*)

The mathematical model named as Method of cell (MOC) is applied to derive equations for effective mechanical properties for thin composite made of fiber and matrix blocks. An assumption is made to clarify the material properties was known with fiber as orthotropic while matrix as isotropic. The plane stress conditions are considered with zero voids. The MOC is based upon the assumption that the composite can be approximated by a periodic array. In using this periodicity, it is possible to analyze a single representative volume element of the continuum rather than the whole continuum. The representative volume element (**Figure 1**) is then used as the building block from which the continuum is constructed.

	E_{LL} (GPa)	E_{TT} (GPa)	ν_{LT}	ν_{TT}	G_{LT} (GPa)	G_{TT} (GPa)	α_{LL} (10^{-6}/°C)	α_{TT} (10^{-6}/°C)
T300 Fiber	233	23	0.2	0.4	8.96	8.27	-0.54	10.08
934 Epoxy	4.34	4.34	0.37	0.37	1.59	1.59	41.4	41.4

Table 1: Element Properties of fiber and matrix

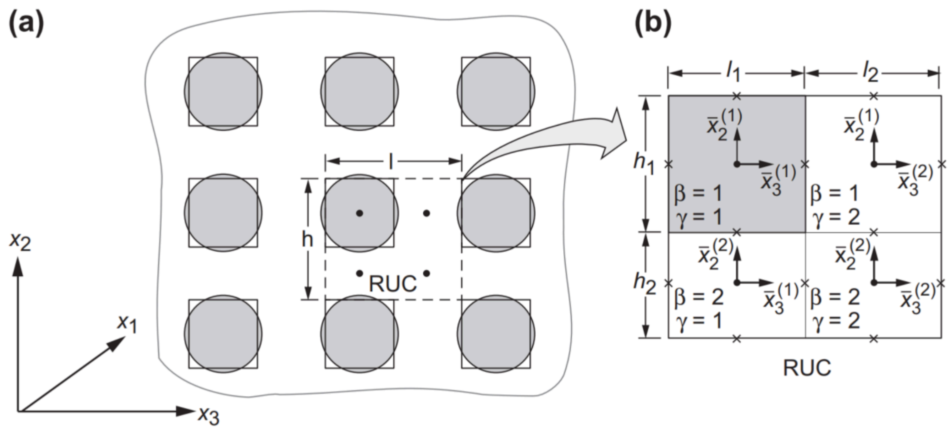

Figure 1: MOC model of continuously reinforced fiber composite. (a) A composite with a doubly periodic array of fibers extending in the x1-direction. (b) A repeating unit cell with four subcells (Aboudi et al. 2013)

In this project, the method of cell is initiated using MATLAB application to evaluate micromechanical analysis of composites. The composite is assumed to be unidirectional. The cell is sub-divided into four cells through which one of the representative cells (RVE) is comprised of fiber and the matrix material.

$$\mathbf{C}^{(\beta\gamma)} = \begin{bmatrix} c_{11}^{(\beta\gamma)} & c_{12}^{(\beta\gamma)} & c_{12}^{(\beta\gamma)} & 0 & 0 & 0 \\ & c_{22}^{(\beta\gamma)} & c_{23}^{(\beta\gamma)} & 0 & 0 & 0 \\ & & c_{22}^{(\beta\gamma)} & 0 & 0 & 0 \\ & & & c_{44}^{(\beta\gamma)} & 0 & 0 \\ \text{symmetric} & & & & c_{44}^{(\beta\gamma)} & 0 \\ & & & & & c_{66}^{(\beta\gamma)} \end{bmatrix}$$

$$c_{11} = E_A + 4\kappa\nu_A^2,$$
$$c_{12} = 2\kappa\nu_A,$$
$$c_{22} = \kappa + 0.5E_T/(1+\nu_T),$$
$$c_{23} = \kappa - 0.5E_T/(1+\nu_T),$$
$$c_{44} = G_A,$$
$$c_{66} = (c_{22} - c_{23})/2.$$

The micromechanical analysis is initiated on each sub-cell with different notations. There is interaction between the four cells generated to evaluate the overall behavior of composite properties. The equations used to compute the average behavior of the composite are provided below (Aboudi 1987, Aboudi et al. 2013):

Here E_A, ν_A are the axial Young's modulus and Poisson's ratio, G_A is the axial shear modulus, and E_T, ν_T are respectively, the transverse Young's modulus and Poisson's ratio of the material occupying the subcell $(\beta\gamma)$. κ is given by

$$\kappa = 0.25E_A/[0.5(1 - \nu_T)(E_A/E_T) - \nu_A^2].$$

$$\begin{aligned}
&A_1 = c_{22}^{(m)}(1 + h_2/h_1),\\
&A_2 = c_{23}^{(m)} l_1/l_2,\\
&A_3 = c_{23}^{(m)}, A_4 = c_{22}^{(m)} h_1/h_2 + c_{22}^{(f)},\\
&A_5 = c_{23}^{(f)}, A_6 = l_2 c_{23}^{(m)}/l_1, A_7 = c_{23}^{(f)},\\
&A_8 = h_2 c_{23}^{(m)}/h_1, A_9 = c_{22}^{(f)} + c_{22}^{(m)} l_1/l_2,\\
&A_{10} = h_1 c_{23}^{(m)}/h_2, A_{11} = A_3, A_{12} = c_{22}^{(m)}(l_2/l_1 + 1).
\end{aligned}$$

$$\begin{aligned}
DT_1 &= -(A_5A_8A_{12} + A_6A_9A_{11}),\\
DT_2 &= A_2A_8A_{12} + A_3A_9A_{11} - A_1A_9A_{12},\\
DT_3 &= A_1A_5A_{12} + A_2A_6A_{11} - A_3A_5A_{11},\\
DT_4 &= A_1A_6A_9 + A_8(A_3A_5 - A_2A_6),
\end{aligned}$$

$$\begin{aligned}
DT_5 &= A_6A_9A_{10} + A_{12}(A_5A_7 - A_4A_9),\\
DT_6 &= -(A_2A_7A_{12} + A_3A_9A_{10}),\\
DT_7 &= A_3A_5A_{10} + A_2(A_4A_{12} - A_6A_{10}),\\
DT_8 &= A_2A_6A_7 + A_3(A_4A_9 - A_5A_7),\\
DT_9 &= A_4A_8A_{12} + A_6(A_7A_{11} - A_8A_{10}),\\
DT_{10} &= A_1A_7A_{12} + A_3(A_8A_{10} - A_7A_{11}),\\
DT_{11} &= A_3A_4A_{11} + A_1(A_6A_{10} - A_4A_{12}),\\
DT_{12} &= -(A_1A_6A_7 + A_3A_4A_8),\\
DT_{13} &= A_4A_9A_{11} + A_5(A_8A_{10} - A_7A_{11}),\\
DT_{14} &= A_1A_9A_{10} + A_2(A_7A_{11} - A_8A_{10}),\\
DT_{15} &= -(A_1A_5A_{10} + A_2A_4A_{11}),\\
DT_{16} &= A_1(A_5A_7 - A_4A_9) + A_2A_4A_8,
\end{aligned}$$

$$D = a_1[a_{12}(a_5a_7 - a_4a_9) + a_6a_9a_{10}]$$
$$+a_2[a_4a_8a_{12} + a_6(a_7a_{11} - a_8a_{10})]$$
$$+a_3[a_4a_9a_{11} + a_5(a_8a_{10} - a_7a_{11})]$$

$$Q_1 = v_{11}c_{12}^{(f)}(T_1 + T_9) - v_{12}c_{12}^{(m)}(h_2T_5/h_1 + l_1T_9/l_2) - v_{21}c_{12}^{(m)}(h_1T_1/h_2 + l_2T_{13}/l_1) + v_{22}c_{12}^{(m)}(T_5 + T_{13}),$$

Q_2 is obtained from Q_1 by replacing T_i in the latter by T_{i+1} (e.g. T_1 is replaced by T_2, T_9 by T_{10} etc....). Similarly, Q_3 and Q_4 are obtained from Q_1 by replacing T_i in the latter by T_{i+2} and T_{i+3}, respectively.

$$Q_1' = v_{11}(c_{22}^{(f)}T_1 + c_{23}^{(f)}T_9) - v_{12}(c_{22}^{(m)}h_2T_5/h_1 + c_{23}^{(m)}l_1T_9/l_2) - v_{21}(c_{22}^{(m)}h_1T_1/h_2 + c_{23}^{(m)}l_2T_{13}/l_1) + v_{22}(c_{22}^{(m)}T_5 + c_{23}^{(m)}T_{13})$$

$$\Delta_1 = (h_1l_2 + h_2l_1 + h_2l_2),$$

$$\Delta_2 = h_1l_1c_{66}^{(m)} + \Delta_1c_{66}^{(f)}.$$

$$V\,b_{11} = \mathrm{v}_{11}C_{11}^{(f)} + C_{11}^{(m)}(\mathrm{v}_{12} + \mathrm{v}_{21} + \mathrm{v}_{22}) + \left(C_{12}^{(m)} - C_{12}^{(f)}\right)\ (Q_2 + Q_3)$$

$$V\,b_{12} = \frac{h}{h_1}\left(C_{12}^{(m)}\mathrm{v}_{12} + Q_1C_{22}^{(m)} + Q_3C_{23}^{(m)}\right) + \frac{h}{h_2}\left(C_{12}^{(m)}\mathrm{v}_{21} + Q_2C_{22}^{(m)} + Q_4C_{23}^{(m)}\right)$$

$$V\,b_{13} = \frac{l}{l_1}\left(C_{12}^{(m)}\mathrm{v}_{21} + Q_2C_{23}^{(m)} + Q_4C_{22}^{(m)}\right) + \frac{l}{l_2}\left(C_{12}^{(m)}\mathrm{v}_{12} + Q_1C_{23}^{(m)} + Q_3C_{22}^{(m)}\right)$$

$$V\,b_{22} = \frac{h}{h_1}\left[C_{22}^{(m)}(\mathrm{v}_{12} + Q_1') + Q_3'C_{23}^{(m)}\right] + \frac{h}{h_2}\left[C_{22}^{(m)}(\mathrm{v}_{21} + Q_2') + Q_4'C_{23}^{(m)}\right]$$

$$V\,b_{23} = \frac{l}{l_1}\left[C_{23}^{(m)}(\mathrm{v}_{21} + Q_2') + Q_4'C_{22}^{(m)}\right] + \frac{l}{l_2}\left[C_{23}^{(m)}(\mathrm{v}_{12} + Q_1') + Q_3'C_{22}^{(m)}\right]$$

$$V\,b_{33} = \frac{l}{l_1}\left[C_{22}^{(m)}(\mathrm{v}_{21} + Q_4'') + Q_2''C_{23}^{(m)}\right] + \frac{l}{l_2}\left[C_{22}^{(m)}(\mathrm{v}_{12} + Q_3'') + Q_1''C_{23}^{(m)}\right]$$

$$b_{44} = C_{44}^{(f)}C_{44}^{(m)}\frac{hl}{\Delta_2}$$

$$b_{55} = \frac{C_{55}^{(m)} C_{55}^{(f)} \left[l\,(\mathrm{v}_{11} + \mathrm{v}_{12}) + l_2(\mathrm{v}_{21} + \mathrm{v}_{22}) \right] + C_{55}^{(m)} (\mathrm{v}_{21} + \mathrm{v}_{22})\ \ l_1}{hl\left(l_1 C_{55}^{(m)} + l_2 C_{55}^{(f)} \right)}$$

$$b_{66} = \frac{C_{66}^{(m)} C_{66}^{(f)} \left[h\,(\mathrm{v}_{11} + \mathrm{v}_{21}) + h_2\,(\mathrm{v}_{12} + \mathrm{v}_{22}) \right] + C_{66}^{(m)} (\mathrm{v}_{12} + \mathrm{v}_{22})\, h_1}{hl\left(h_1 C_{66}^{(m)} + h_2 C_{66}^{(f)} \right)}$$

Elastic stiffness matrix, B

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 & 0 \\ & b_{22} & b_{23} & 0 & 0 & 0 \\ & & b_{33} & 0 & 0 & 0 \\ \text{symmetric} & & & b_{44} & 0 & 0 \\ & & & & b_{55} & 0 \\ & & & & & b_{66} \end{bmatrix}$$

For square fibers at equal spacing the number of the effective elastic constants reduces to six since $b_{12} = b_{13}$, $b_{22} = b_{33}$ *and* $b_{44} = b_{55}$. It follows that the composite is not represented effectively by a transversely isotropic material. To obtain the requested representation, the following procedure can be applied.

The elastic constants of the transversely isotropic solid can be determined from the elastic modulus matrix E = $[e_{ij}]$ given by

$$E = \frac{1}{\pi} \int_0^{\pi} B'(\xi) d\xi$$

The nonvanishing elements of the symmetric matrix E are expressed in terms of the elements of B in the form

$$e_{11} = b_{11},$$
$$e_{12} = e_{13} = (b_{12} + b_{13})/2,$$
$$e_{22} = e_{33} = 3(b_{22} + b_{33})/8 + b_{23}/4 + b_{66}/2,$$
$$e_{23} = (b_{22} + b_{33})/8 + 3b_{23}/4 - b_{66}/2,$$
$$e_{44} = e_{55} = (b_{44} + b_{55})/2,$$
$$e_{66} = (e_{22} - e_{23})/2,$$

which form altogether five independent constants, thus yielding a transversely isotropic material.

The effective elastic engineering constants of the composite can be computed by inverting E.

- ***Engineering effective constants***

The engineering effective constants can be readily determined from:

$$\mathbf{S}^* = \begin{bmatrix} 1/E_A^* & -\nu_A^*/E_A^* & -\nu_A^*/E_A^* & 0 & 0 & 0 \\ & 1/E_T^* & -\nu_T^*/E_T^* & 0 & 0 & 0 \\ & & 1/E_T^* & 0 & 0 & 0 \\ & & & 1/G_T^* & 0 & 0 \\ \text{symmetric} & & & & 1/G_A^* & 0 \\ & & & & & 1/G_A^* \end{bmatrix}$$

- ***Coefficient of Thermal Expansion (CTE)***

In addition, the effective thermal stresses are given by:

$$V\,\Gamma_1^* = (\Gamma_2^{(m)} - \Gamma_2^{(f)})\,(Q_2 + Q_3) + \mathrm{v}_{11}\Gamma_1^{(f)} + (\mathrm{v}_{12} + \mathrm{v}_{21} + \mathrm{v}_{22})\,\Gamma_1^{(m)}$$
$$V\,\Gamma_2^* = (\Gamma_2^{(m)} - \Gamma_2^{(f)})\,(Q_2' + Q_3') + \mathrm{v}_{11}\Gamma_2^{(f)} + (\mathrm{v}_{12} + \mathrm{v}_{21} + \mathrm{v}_{22})\,\Gamma_2^{(m)}$$
$$V\,\Gamma_3^* = (\Gamma_2^{(m)} - \Gamma_2^{(f)})\,(Q_2'' + Q_3'') + \mathrm{v}_{11}\Gamma_2^{(f)} + (\mathrm{v}_{12} + \mathrm{v}_{21} + \mathrm{v}_{22})\,\Gamma_2^{(m)}$$

$$\Gamma_1^{(\beta\gamma)} = C_{11}^{(\beta\gamma)}\alpha_1^{(\beta\gamma)} + 2C_{12}^{(\beta\gamma)}\alpha_2^{(\beta\gamma)}$$

$$\Gamma_2^{(\beta\gamma)} = \Gamma_3^{(\beta\gamma)} = C_{12}^{(\beta\gamma)}\alpha_1^{(\beta\gamma)} + \left(C_{22}^{(\beta\gamma)} + C_{23}^{(\beta\gamma)}\right)\ \alpha_2^{(\beta\gamma)}$$

From which the effective CTEs are given by:

$$\begin{bmatrix} \alpha_1^* \\ \alpha_2^* \\ \alpha_3^* \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_1^* \\ \Gamma_2^* \\ \Gamma_3^* \end{bmatrix}$$

The MATLAB code of the method of cell is attached in APPENDIX.

Finite Element Analysis

Abaqus is used to perform the finite element method on the transversely isotropic composite. Additionally, an Abaqus add on which is called the Micromechanics Plug-in is used to assist in the determination of the engineering constants. Initially, a representative volume element is modeled using a square array. The RVE is a 1x1x1 unit cell consisting of matrix with a fiber in the center. After creating the model, the materials are defined in Abaqus: one being the fiber and the other being the matrix. Material properties are then created and assigned to the appropriate sets. Since the fiber properties are not isotropic, it is important to define a local coordinate system that would determine the orientation of the fiber properties. The matrix, having isotropic properties, does not require this. Then, RVE has been meshed into around 5000 elements depending on the volume fraction. The element type of the mesh is a 10-node quadratic tetrahedron.

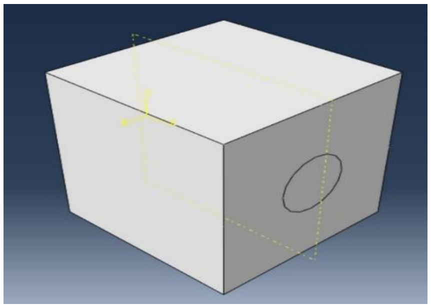

Figure 2

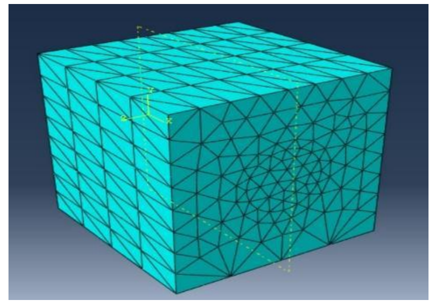

Figure 3

From here on, the Micromechanics plug-in has been used. A mechanical scenario has been selected and periodic boundary condition is also provided by a strain driven field. It is important that a periodic boundary condition needs to be used for this case. The plug-in then will apply the appropriate boundary conditions and solve for both the engineering constants and coefficients of thermal expansion. When the solver is finished the effective composite properties are saved as a new homogenized material. After the plug-in is run, the applied boundary conditions can be seen and checked. These BC's were similar to the ones used in *Prediction of Composite Properties from a Representative Volume Element* by Sun et al. 1996. The difference being that Sun et al. 1996 used a prescribed load while we used a prescribed displacement.

Boundary condition

● ***<u>Normal loading</u>***

In the determination of Axial Young's Modulus E_{LL} and Transverse Young's Modulus E_{TT}, a displacement of 1 was applied to the x=1 face and the x=0 face was fixed. Moreover, y=0, y=1, z=0, and z=1 faces were only allowed to displace in the x direction. All faces parallel to the x direction have a roller boundary condition so that only displacement parallel to the x direction is allowed. The BC's for E_{LL} is illustrated in Figure 4. Similarly, transverse young's modulus E_{TT} can be determined in z-direction and is shown in Figure-5.

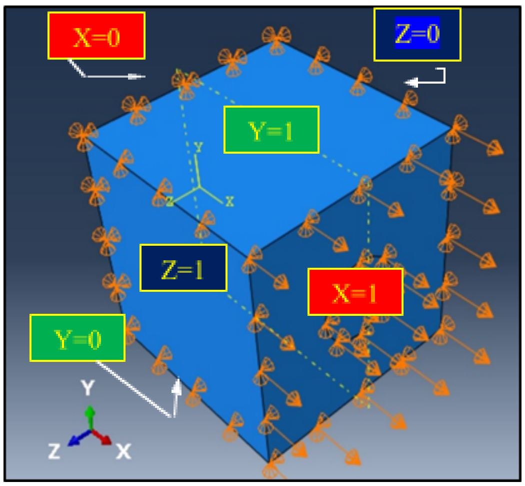

Figure 4 Longitudinal Normal

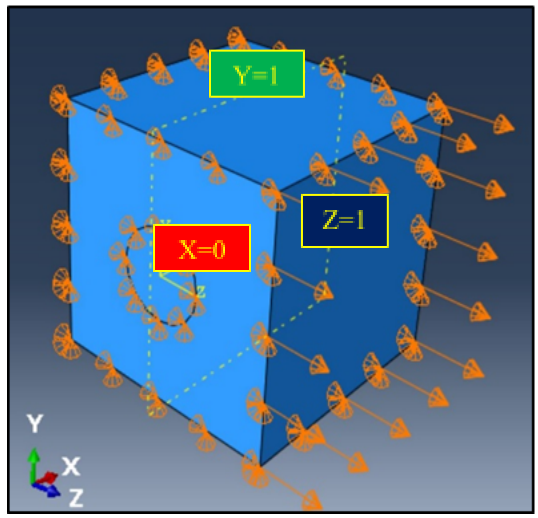

Figure 5 Transverse Normal

- ***Transverse and longitudinal shear loading***

The transverse and longitudinal shear loading methods were also constrained according to Sun et al. 1996. A prescribed displacement was used rather than a load.

For the transverse shear case, a 0.5-unit displacement was applied to the y=0 and y=1 faces in the transverse direction. These two displacements were applied in opposing directions (in the positive and negative z directions). The BC's for the transverse shear case is demonstrated in Figure 6.

For the longitudinal shear case, a 0.5-unit displacement was applied to the y=0 and y=1 faces in the longitudinal direction. These two displacements were applied in opposing directions (in the positive and negative x directions). The BC's for the longitudinal shear case is shown in Figure 7.

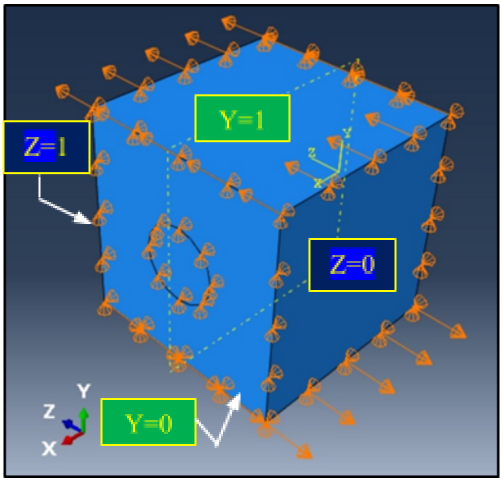

Figure 6 Transverse Shear

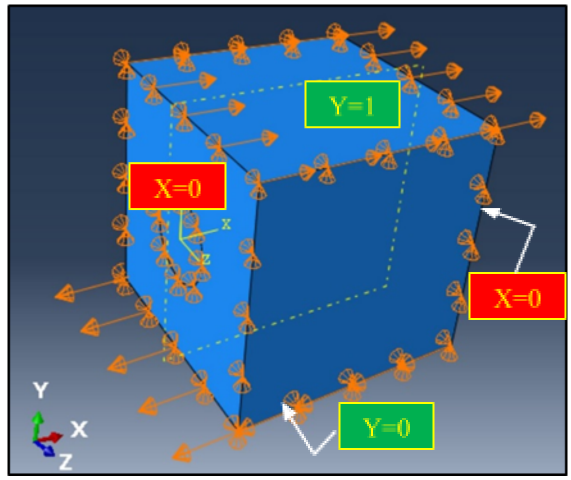

Figure 7 Longitudinal Shear

Results and Discussion

V_f= Fiber Volume Ration

E_{LL} = Axial Young's Modulus

E_{TT} = Transverse Young's Modulus

ν_{LT} = Axial Poisson Ratio

ν_{TT} = Transvers Poisson Ratio

G_{LT} = Axial Shear Modulus

G_{TT} = Transverse Shear Modulus

α_{LL} = Axial Coefficient of Thermal Expansion

α_{TT} = Transverse Coefficient of Thermal Expansion

Method of Cell (MOC)

V_f	E_{LL} (GPa)	E_{TT} (GPa)	ν_{LT}	ν_{TT}	G_{LT} (GPa)	G_{TT} (GPa)	α_{LL} (10^{-6}/°C)	α_{TT} (10^{-6}/°C)
0.01	6.88	4.87	0.36	0.42	1.71	1.37	27.21	46.08
0.10	27.45	5.69	0.34	0.52	1.88	1.52	5.85	49.34
0.25	61.72	6.85	0.32	0.52	2.25	1.86	1.89	43.19
0.40	95.99	8.35	0.29	0.50	2.78	2.36	0.75	36.17
0.50	118.83	9.62	0.27	0.49	3.23	2.80	0.35	31.52
0.60	141.67	11.16	0.26	0.47	3.79	3.38	0.07	26.96
0.75	175.92	14.18	0.24	0.45	4.89	4.62	-0.22	20.37
0.90	210.17	18.59	0.21	0.42	6.53	6.68	-0.43	14.10
0.99	230.72	22.55	0.20	0.40	8.04	8.68	-0.53	10.49

Table 3: Composite Properties from MOC

FEA

V_f	E_{LL} (GPa)	E_{TT} (GPa)	ν_{LT}	ν_{TT}	G_{LT} (GPa)	G_{TT} (GPa)	α_{LL} (10^{-6}/°C)	α_{TT} (10^{-6}/°C)
0.00	4.62	4.62	0.36	0.36	1.36	1.36	41.40	41.40
0.10	27.27	5.80	0.34	0.51	1.94	1.89	6.03	49.05
0.25	61.51	7.18	0.32	0.50	2.39	2.21	1.97	42.80
0.50	118.35	10.54	0.27	0.47	3.46	2.93	0.38	31.20
0.60	141.35	12.36	0.26	0.45	4.11	3.40	0.09	26.60
0.75	175.46	15.61	0.23	0.43	5.62	4.69	-0.19	19.50
0.90	201.50	18.66	0.22	0.41	7.29	6.54	-0.35	14.50
1.00	233.00	23.10	0.20	0.40	8.96	8.27	-0.54	10.10

Table 3: Composite Properties from FEA

Comparison between the MOC and FEA results

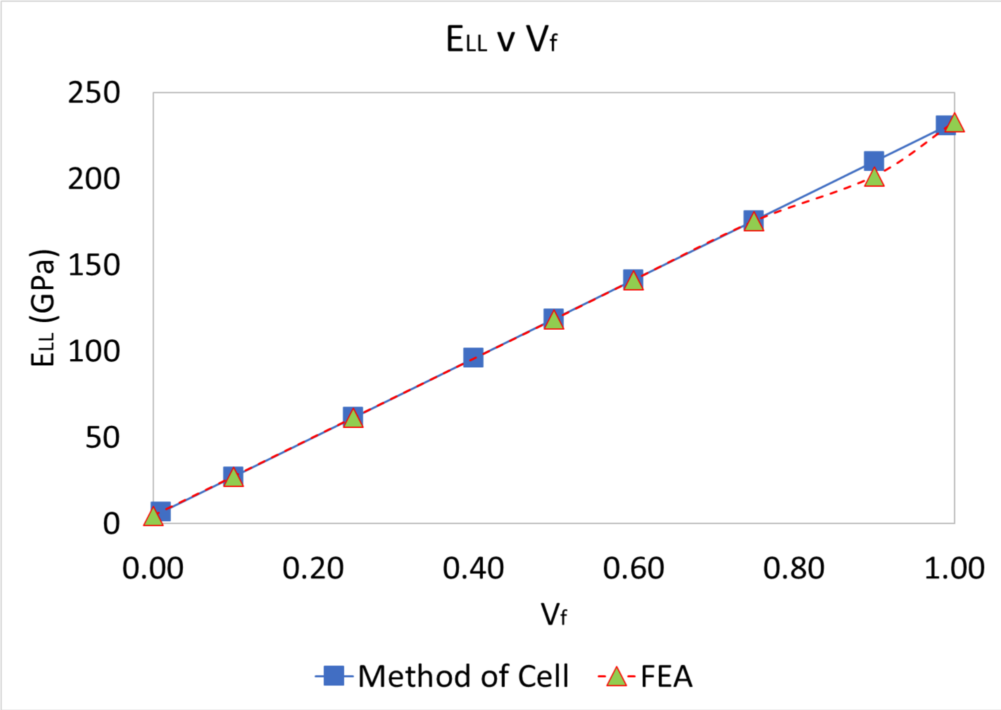

Figure 8. Axial Young's Modulus vs Fiber volume ratio

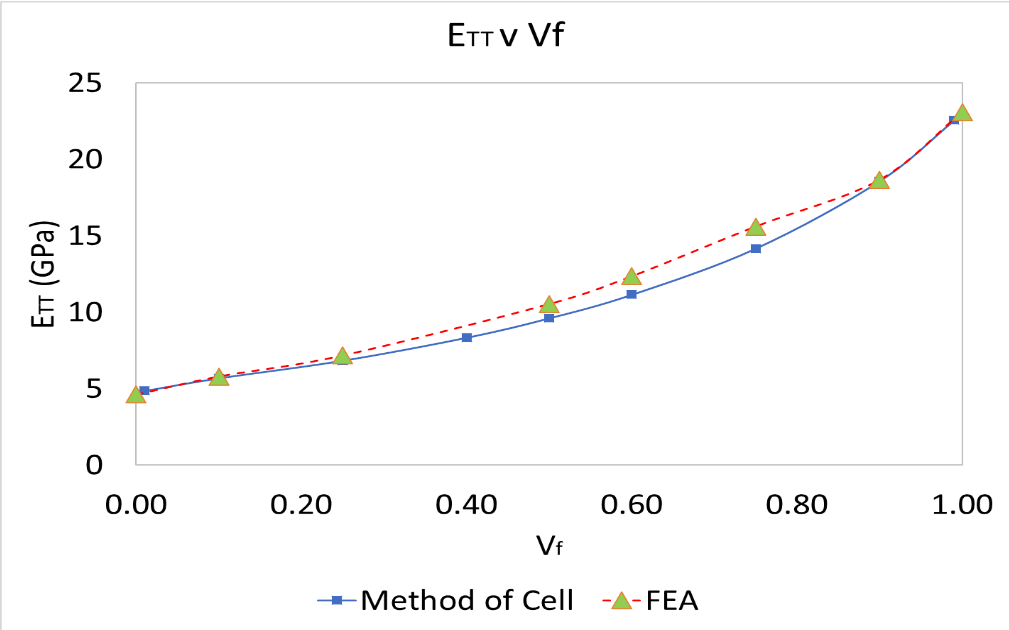

Figure 9. Transverse Young's Modulus vs Fiber volume ratio

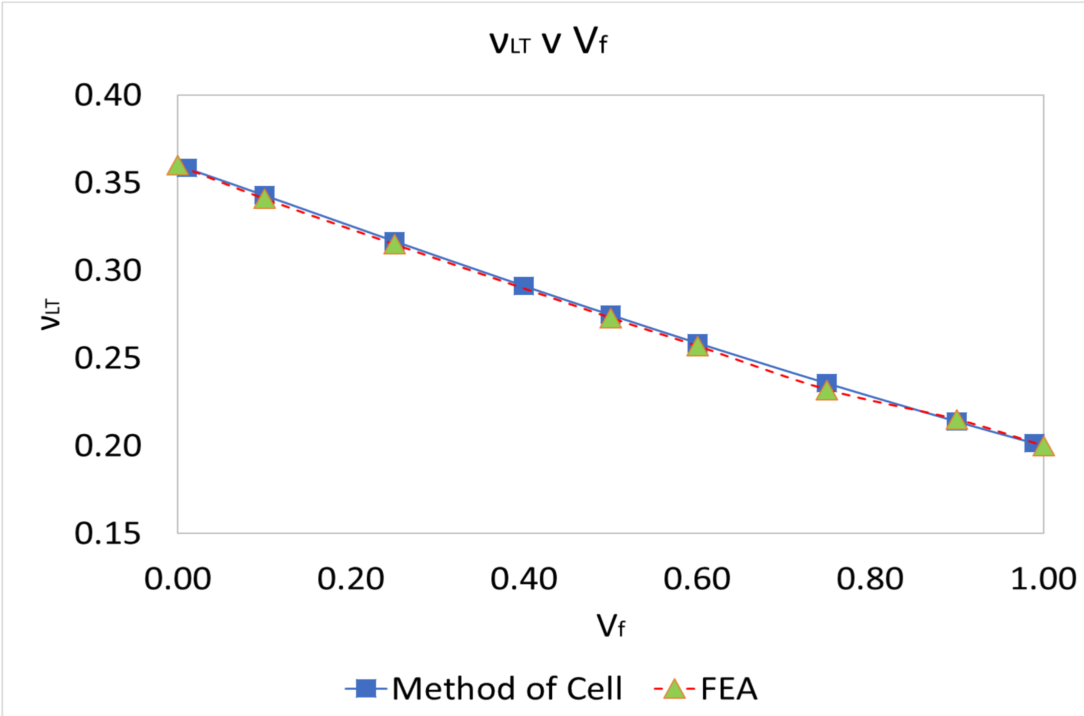

Figure 10. Axial Poisson's ratio vs Fiber volume ratio

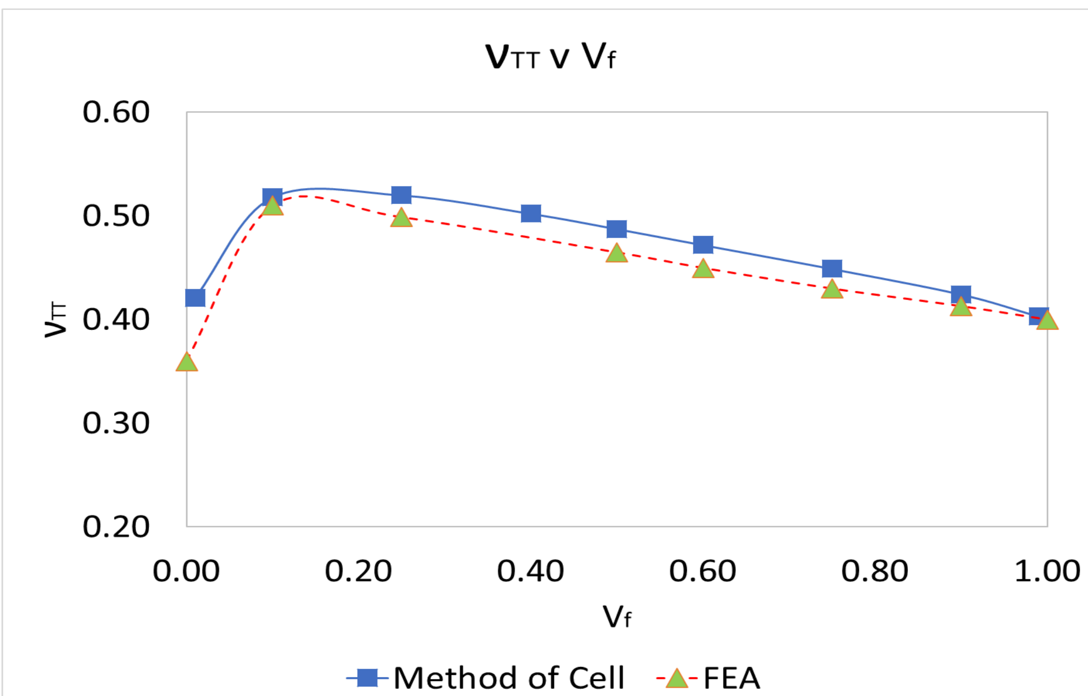

Figure 11. Transverse Poisson's ratio vs Fiber volume ratio

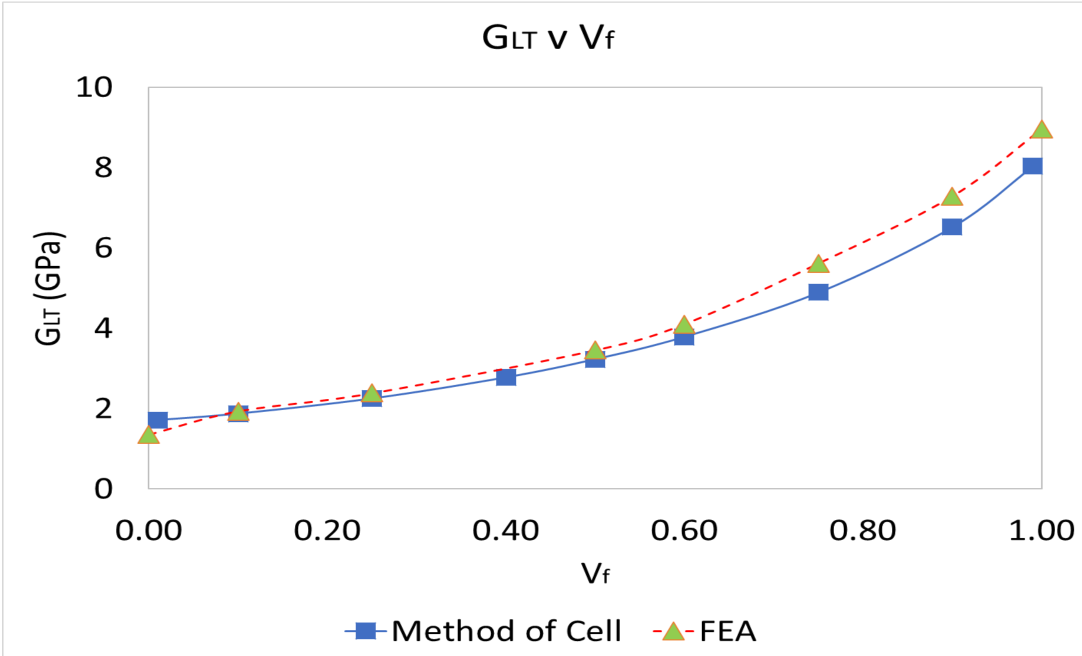

Figure 12. Axial Shear Modulus vs Fiber volume ratio

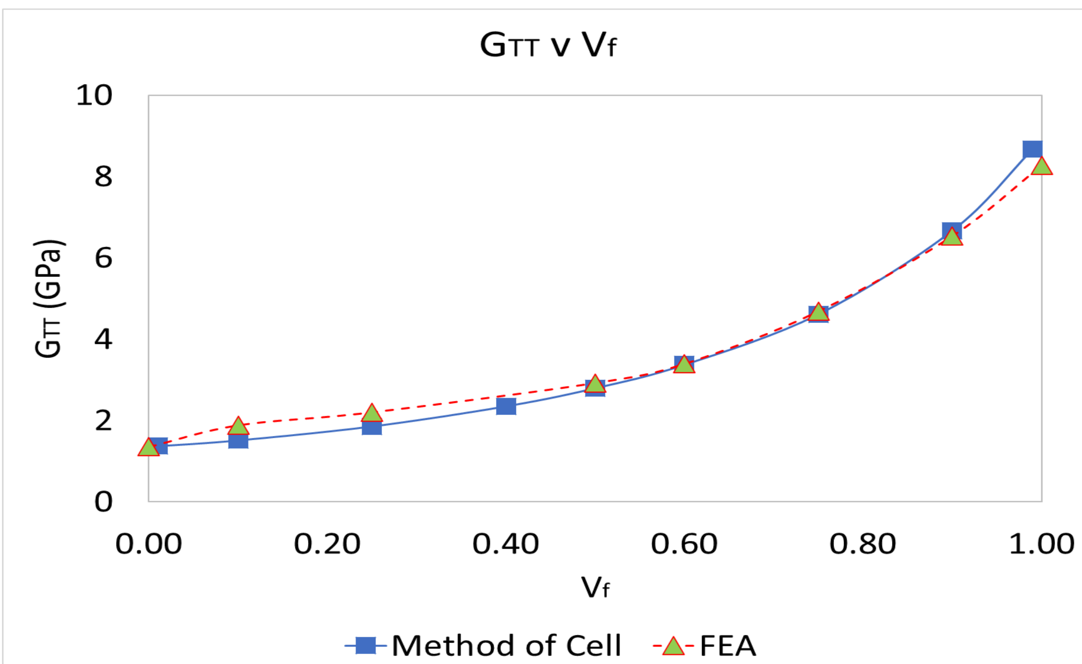

Figure 13 Transverse Shear Modulus vs Fiber volume ratio

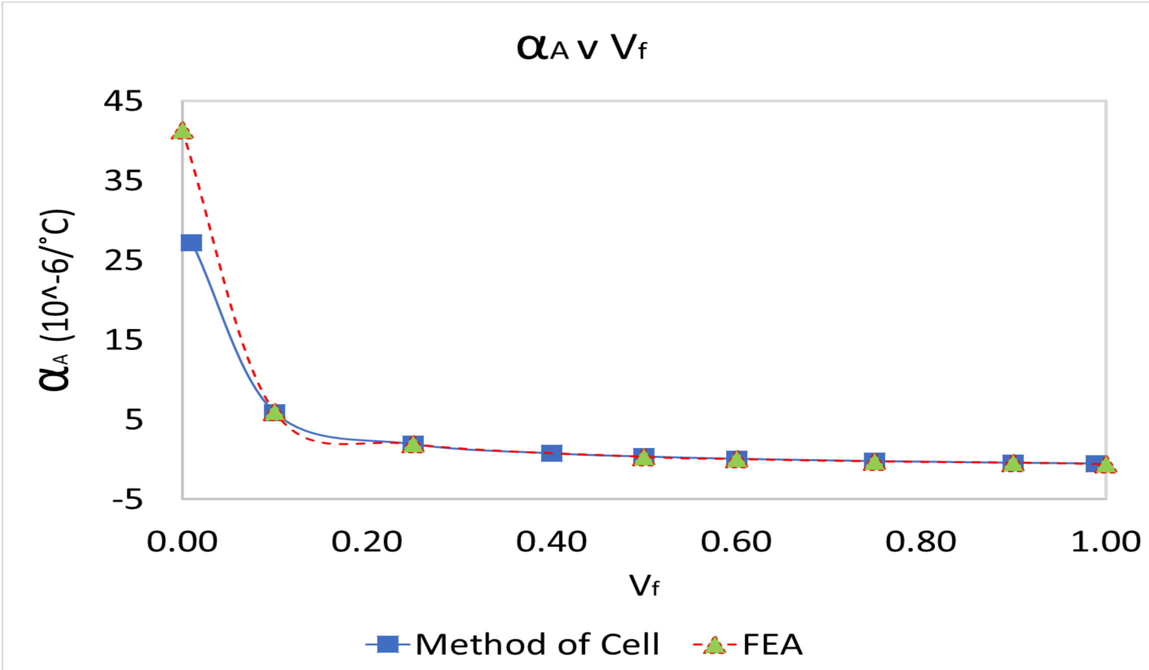

Figure 14. Axial Coefficient vs Fiber volume ratio

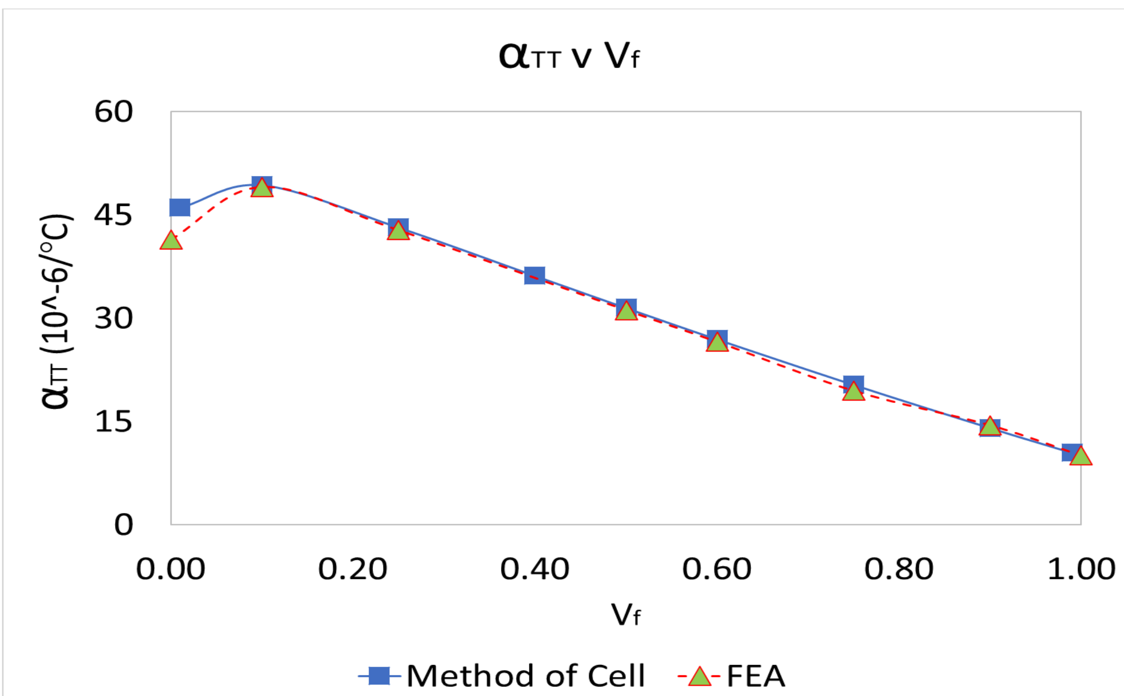

Figure 15 Transverse Coefficient vs Fiber volume ratio

Discussion

The comparison of the results (8 to 15) between MOC and FEA shows that the analytical solutions are reasonably in well agreement with ABAQUS FE analysis.

Errors between the FEM results and the MOC predictions still exist to some extent. A possible reason is due to the assumptions and simplifications of the MOC method. Sometimes, discrepancies were found at lower fiber volume ratio (V_f <0.15) . This is probably due to the inability of the analytical model to find properties at low fiber volume ratio.

Moreover, errors were also more prominent in transverse direction than axial or longitudinal direction. It is probably due to higher number of different mesh shapes and sizes in fiber at transverse direction than in axial direction.

References

- Aboudi, J. (1987). Closed form constitutive equations for metal matrix composites. International Journal of Engineering Science, 25(9), 1229-1240.
- Aboudi, J., Arnold, S. M., & Bednarcyk, B. A. (2013). Chapter 4 - The Method of Cells Micromechanics, Micromechanics of composite materials: a generalized multiscale analysis approach. Butterworth-Heinemann.

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- Sun, C. T., & Vaidya, R. S. (1996). Prediction of composite properties from a representative volume element. Composites Science and Technology, 56(2), 171-179.

APPENDIX

MATLAB code

```
% Addvanced Composite Mechanics
% Midterm, Calculate Ea,Et,ua,Ga,Gt,alphaA,alphaT w.r.t. Vf using method of
cells
Fp=[233, 0.2, 23.1, 0.4, 8.96,-0.539,10.1] ; % Fiber properties
% Fp=[ Ea, nua, Et, nut, Ga,gammaA,gammaT]
Mp=[4.6, 0.36, 4.62, 0.36, 1.36,41.4,41.4] ; % Matrix properties
%Mp=[Ea, nua, Et, nut, Ga,gammaA,gammaT]
xf= (0.25*Fp(1,1))/(0.5*(1-Fp(1,4))*(Fp(1,1)/Fp(1,3))-(Fp(1,2))^2);
% xf= Fiber volume ratio
xm= (0.25*Mp(1,1))/(0.5*(1-Mp(1,4))*(Mp(1,1)/Mp(1,3))-(Mp(1,2)^2));
% xm= matrix

% Kf= fiber volume ratio= 0%
Kf=0;
S=1 ;                 %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;          %volume of fiber
Vc = S^2 ;            %volume of composite
Vm = Vc-Vf;           %volume of matrix
h2=S-h1;              % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
```

```
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
```

```
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea0=1/(Sstar(1,1));
ua0=-(Sstar(1,2))*Ea0;
Et0=1/(Sstar(2,2));
ut0=-(Sstar(2,3))*Et0;
Ga0=1/(Sstar(6,6));
Gt0=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_0=inv(Z)*[Gamma];
```

```
% Kf= fiber volume ratio= 1%
Kf=0.01;
S=1 ;                 %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;          %volume of fiber
Vc = S^2 ;            %volume of composite
Vm = Vc-Vf;           %volume of matrix
h2=S-h1;              % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
```

```
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
```

```
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea1=1/(Sstar(1,1));
ua1=-(Sstar(1,2))*Ea1;
Et1=1/(Sstar(2,2));
ut1=-(Sstar(2,3))*Et1;
Ga1=1/(Sstar(6,6));
Gt1=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_1=inv(Z)*[Gamma];

% Kf= fiber volume ratio= 10%
Kf=0.1;
S=1 ;                 %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;          %volume of fiber
Vc = S^2 ;            %volume of composite
Vm = Vc-Vf;           %volume of matrix
h2=S-h1;              % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
```

```
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
```

```
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea10=1/(Sstar(1,1));
ua10=-(Sstar(1,2))*Ea10;
Et10=1/(Sstar(2,2));
ut10=-(Sstar(2,3))*Et10;
Ga10=1/(Sstar(6,6));
Gt10=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
```

```
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_10=inv(Z)*[Gamma];



% Kf= fiber volume ratio= 25%
Kf=0.25;
S=1 ;                 %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;          %volume of fiber
Vc = S^2 ;            %volume of composite
Vm = Vc-Vf;           %volume of matrix
h2=S-h1;              % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));
```

```
 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
```

```
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea25=1/(Sstar(1,1));
ua25=-(Sstar(1,2))*Ea25;
Et25=1/(Sstar(2,2));
ut25=-(Sstar(2,3))*Et25;
Ga25=1/(Sstar(6,6));
Gt25=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_25=inv(Z)*[Gamma];

% Kf= fiber volume ratio= 40%
Kf=0.40;
S=1 ;                %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;         %volume of fiber
Vc = S^2 ;           %volume of composite
Vm = Vc-Vf;          %volume of matrix
h2=S-h1;             % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;
```

```
% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);
```

```
% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea40=1/(Sstar(1,1));
ua40=-(Sstar(1,2))*Ea40;
Et40=1/(Sstar(2,2));
ut40=-(Sstar(2,3))*Et40;
Ga40=1/(Sstar(6,6));
Gt40=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
```

```
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_40=inv(Z)*[Gamma];


% Kf= fiber volume ratio= 50%
Kf=0.5;
S=1 ;                %composite model size
h1= sqrt(Kf.*S^2) ;  % width and length of fiber
Vf = h1.^2 ;         %volume of fiber
Vc = S^2 ;           %volume of composite
Vm = Vc-Vf;          %volume of matrix
h2=S-h1;             % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D
```

```
D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;
```

```
% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea50=1/(Sstar(1,1));
ua50=-(Sstar(1,2))*Ea50;
Et50=1/(Sstar(2,2));
ut50=-(Sstar(2,3))*Et50;
Ga50=1/(Sstar(6,6));
Gt50=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_50=inv(Z)*[Gamma];

% Kf= fiber volume ratio= 60%
Kf=0.6;
S=1 ;                 %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;          %volume of fiber
Vc = S^2 ;            %volume of composite
Vm = Vc-Vf;           %volume of matrix
h2=S-h1;              % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
```

```
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
```

```
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea60=1/(Sstar(1,1));
ua60=-(Sstar(1,2))*Ea60;
Et60=1/(Sstar(2,2));
ut60=-(Sstar(2,3))*Et60;
Ga60=1/(Sstar(6,6));
Gt60=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];
```

```
Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_60=inv(Z)*[Gamma];


% Kf= fiber volume ratio= 75%
Kf=0.75;
S=1 ;                 %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;          %volume of fiber
Vc = S^2 ;            %volume of composite
Vm = Vc-Vf;           %volume of matrix
h2=S-h1;              % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;
```

```
 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
```

```
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea75=1/(Sstar(1,1));
ua75=-(Sstar(1,2))*Ea75;
Et75=1/(Sstar(2,2));
ut75=-(Sstar(2,3))*Et75;
Ga75=1/(Sstar(6,6));
Gt75=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_75=inv(Z)*[Gamma];


% Kf= fiber volume ratio= 90%
Kf=.90;
S=1 ;                %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;         %volume of fiber
Vc = S^2 ;           %volume of composite
Vm = Vc-Vf;          %volume of matrix
h2=S-h1;             % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
```

```
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
```

```
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea90=1/(Sstar(1,1));
ua90=-(Sstar(1,2))*Ea90;
Et90=1/(Sstar(2,2));
ut90=-(Sstar(2,3))*Et90;
Ga90=1/(Sstar(6,6));
Gt90=1/(Sstar(4,4));
```

```
Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_90=inv(Z)*[Gamma];



% Kf= fiber volume ratio= 99%
Kf=0.99;
S=1 ;                %composite model size
h1= sqrt(Kf.*S^2) ; % width and length of fiber
Vf = h1.^2 ;         %volume of fiber
Vc = S^2 ;           %volume of composite
Vm = Vc-Vf;          %volume of matrix
h2=S-h1;             % distance between each 2 fibers
h=h1+h2;
u12=h1*h2;
u11=h1*h1;
u22=h2*h2;
u21=h2*h1;
V=h^2;
% C for Fibers
C11f=Fp(1,1)+4*xf*Fp(1,2)^2;
C12f=2*xf*Fp(1,2);
C22f=xf+(0.5*Fp(1,3))/(1+Fp(1,4));
C23f=xf-(0.5*Fp(1,3))/(1+Fp(1,4));
C44f=Fp(1,5);
C66f=(C22f-C23f)/2;

% C For Matrix
C11m=Mp(1,1)+4*xm*Mp(1,2)^2;
C12m=2*xm*Mp(1,2);
C22m=xm+(0.5*Mp(1,3))/(1+Mp(1,4));
C23m=xm-(0.5*Mp(1,3))/(1+Mp(1,4));
C44m=Mp(1,5);
C66m=(C22m-C23m)/2;

% A
 A1= C22m*(1+h2/h1);
 A2=C23m*h1/h2;
 A3=C23m;
 A4=C22m*(h1/h2)+C22f;
 A5=C23f;
 A6=h2*C23m/h1;
 A7=C23f;
```

```
 A8=A6;
 A9=A4;
 A10=h1*C23m/h2;
 A11=A3;
 A12=A1;

 %D

D=A1*(A12*(A5*A7-A4*A9)+A6*A9*A10)+A2*(A4*A8*A12+A6*(A7*A11-A8*A10))+A3*(A4*A9*
A11+A5*(A8*A10-A7*A11));

 %T
 T1=-(A5*A8*A12+A6*A9*A11)/D;
 T2=(A2*A8*A12+A3*A9*A11-A1*A9*A12)/D;
 T3=(A1*A5*A12+A2*A6*A11-A3*A5*A11)/D;
 T4=(A1*A6*A9+A8*(A3*A5-A2*A6))/D ;
 T5=(A6*A9*A10+A12*(A5*A7-A4*A9))/D;
 T6=(-(A2*A7*A12+A3*A9*A10))/D;
 T7=(A3*A5*A10+A2*(A4*A12-A6*A10))/D;
 T8=(A2*A6*A7+A3*(A4*A9-A5*A7))/D;
 T9=(A4*A8*A12+A6*(A7*A11-A8*A10))/D;
 T10=(A1*A7*A12+A3*(A8*A10-A7*A11))/D;
 T11=(A3*A4*A11+A1*(A6*A10-A4*A12))/D;
 T12=-(A1*A6*A7+A3*A4*A8)/D;
 T13=(A4*A9*A11+A5*(A8*A10-A7*A11))/D;
 T14=(A1*A9*A10+A2*(A7*A11-A8*A10))/D;
 T15=(-(A1*A5*A10+A2*A4*A11))/D;
 T16=(A1*(A5*A7-A4*A9)+A2*A4*A8)/D;

%Q
Q1=u11*C12f*(T1+T9)-u12*C12m*(h2*T5/h1+h1*T9/h2)-u12*C12m*(h1*T1/h2+h2*T13/h1)+
u22*C12m*(T5+T13);
Q2=u11*C12f*(T2+T10)-u12*C12m*(h2*T6/h1+h1*T10/h2)-u12*C12m*(h1*T2/h2+h2*T14/h1
)+u22*C12m*(T6+T14);
Q3=u11*C12f*(T3+T11)-u12*C12m*(h2*T7/h1+h1*T11/h2)-u12*C12m*(h1*T3/h2+h2*T15/h1
)+u22*C12m*(T7+T15);
Q4=u11*C12f*(T4+T12)-u12*C12m*(h2*T8/h1+h1*T12/h2)-u12*C12m*(h1*T4/h2+h2*T16/h1
)+u22*C12m*(T8+T16);

% Q'=q
q1=u11*(C22f*T1+C23f*T9)-u12*(C22m*h2*T5/h1+C23m*h1*T9/h2)-u21*(C22m*h1*T1/h2+C
23m*h2*T13/h1)+u22*(C22m*T5+C23m*T13);
q2=u11*(C22f*T2+C23f*T10)-u12*(C22m*h2*T6/h1+C23m*h1*T10/h2)-u21*(C22m*h1*T2/h2
+C23m*h2*T14/h1)+u22*(C22m*T6+C23m*T14);
q3=u11*(C22f*T3+C23f*T11)-u12*(C22m*h2*T7/h1+C23m*h1*T11/h2)-u21*(C22m*h1*T3/h2
+C23m*h2*T15/h1)+u22*(C22m*T7+C23m*T15);
q4=u11*(C22f*T4+C23f*T12)-u12*(C22m*h2*T8/h1+C23m*h1*T12/h2)-u21*(C22m*h1*T4/h2
+C23m*h2*T16/h1)+u22*(C22m*T8+C23m*T16);

%d= delta
d=h1*C44m+h2*C44f;
d1=u11*C66m+(u12+u21+u22)*C66f;
%b
b11=(u11*C11f+C11m*(u12+u21+u22)+(C12m-C12f)*(Q2+Q3))/V;
b12=((h/h1)*(C12m*u12+Q1*C22m+Q3*C23m)+((h/h2)*(C12m*u21+Q2*C22m+Q4*C23m)))/V;
```

```
b13=b12;
b22=((h/h1)*(C22m*(u12+q1)+q3*C23m)+(h/h2)*(C22m*(u21+q2)+q4*C23m))/V;
b23=((h/h1)*(C23m*(u21+q2)+q4*C22m)+(h/h2)*(C23m*(u12+q1)+q3*C22m))/V;
b33=b22;
b44=C44m*(C44f*(h*(u11+u21)+h2*(u12+u22))+C44m*(u12+u22)*h1)/(V*d);
b55=b44;
b66=C66f*C66m*h^2/d1;

% e
e11=b11;
e12=(b12+b13)/2;
e13=e12;
e22=(+3*(b22+b33)/8)+(b23/4)+(b66/2);
e33=e22;
e23=((b22+b33)/8)+(3*b23/4)-(b66/2);
e44=(b44+b55)/2;
e55=e44;
e66=(e22-e23)/2;

B=[e11 e12 e12 0 0 0;e12 e22 e23 0 0 0;e12 e23 e22 0 0 0;0 0 0 e44 0 0;0 0 0 0
e44 0;0 0 0 0 0 e66];
Sstar=inv(B);

Ea99=1/(Sstar(1,1));
ua99=-(Sstar(1,2))*Ea99;
Et99=1/(Sstar(2,2));
ut99=-(Sstar(2,3))*Et99;
Ga99=1/(Sstar(6,6));
Gt99=1/(Sstar(4,4));

Gmf=[(C11f*Fp(1,6)+2*C12f*Fp(1,7));C12f*Fp(1,6)+(C22f+C23f)*Fp(1,7);C12f*Fp(1,6
)+(C22f+C23f)*Fp(1,7);0;0;0];
Gmm=[(C11m*Mp(1,6)+2*C12m*Mp(1,7));C12m*Mp(1,6)+(C22m+C23m)*Mp(1,7);C12m*Mp(1,6
)+(C22m+C23m)*Mp(1,7);0;0;0];

Gamma1=[(Gmm(2,1)-Gmf(2,1))*(Q2+Q3)+u11*Gmf(1,1)+(u12+u21+u22)*Gmm(1,1)]/V;
Gamma2=[(Gmm(2,1)-Gmf(2,1))*(q2+q3)+u11*Gmf(2,1)+(u12+u21+u22)*Gmm(2,1)]/V;
Gamma3=Gamma2;
Gamma=[Gamma1;Gamma2;Gamma3;0;0;0];

Bstar=[b11 b12 b13;b12 b22 b23;b13 b23 b33];
Z=[b11 b12 b12 0 0 0;b12 b22 b23 0 0 0;b12 b23 b22 0 0 0;0 0 0 b44 0 0;0 0 0 0
b44 0;0 0 0 0 0 b66];
alphastar_99=inv(Z)*[Gamma];


k_values = [0 0.01 0.1 0.25 0.4 0.5 0.6 0.75 0.9 0.99];
Ea =[Ea0 Ea1 Ea10 Ea25 Ea40 Ea50 Ea60 Ea75 Ea90 Ea99];
ua=[ua0 ua1 ua10 ua25 ua40 ua50 ua60 ua75 ua90 ua99];
Et=[Et0 Et1 Et10 Et25 Et40 Et50 Et60 Et75 Et90 Et99];
ut=[ut0 ut1 ut10 ut25 ut40 ut50 ut60 ut75 ut90 ut99];
Ga=[Ga0 Ga1 Ga10 Ga25 Ga40 Ga50 Ga60 Ga75 Ga90 Ga99];
Gt=[Gt0 Gt1 Gt10 Gt25 Gt40 Gt50 Gt60 Gt75 Gt90 Gt99];
```

```
AlphaA=[alphastar_0(1,1) alphastar_1(1,1) alphastar_10(1,1) alphastar_25(1,1)
alphastar_40(1,1) alphastar_50(1,1) alphastar_60(1,1) alphastar_75(1,1)
alphastar_90(1,1) alphastar_99(1,1)];
AlphaT=[alphastar_0(2,1) alphastar_1(2,1) alphastar_10(2,1) alphastar_25(2,1)
alphastar_40(2,1) alphastar_50(2,1) alphastar_60(2,1) alphastar_75(2,1)
alphastar_90(2,1) alphastar_99(2,1)];
AlphaT1=[alphastar_0(3,1) alphastar_1(3,1) alphastar_10(3,1) alphastar_25(3,1)
alphastar_40(3,1) alphastar_50(3,1) alphastar_60(3,1) alphastar_75(3,1)
alphastar_90(3,1) alphastar_99(3,1)];

figure()
plot(k_values,AlphaA)
title('AlphaA Vs Vf')
xlabel('Vf')
ylabel('AlphaA')
xlswrite('alphaA.xlsx',[k_values(:),AlphaA(:)]);


figure()
plot(k_values,AlphaT)
title('AlphaT Vs Vf')
xlabel('Vf')
ylabel('AlphaT')
xlswrite('alphaT.xlsx',[k_values(:),AlphaT(:)]);

figure()
plot(k_values,Ea)
title('Ea Vs Vf')
xlabel('Vf')
ylabel('Ea')
xlswrite('Ea.xlsx',[k_values(:),Ea(:)]);


figure()
plot(k_values,Et)
title('Et Vs Vf')
xlabel('Vf')
ylabel('Et')
xlswrite('ET.xlsx',[k_values(:),Et(:)]);

figure()
plot(k_values,ua)
title('ua Vs Vf')
xlabel('Vf')
ylabel('ua')
xlswrite('ua.xlsx',[k_values(:),ua(:)]);

figure()
plot(k_values,ut)
title('ut Vs Vf')
xlabel('Vf')
ylabel('ut')
xlswrite('ut.xlsx',[k_values(:),ut(:)]);

figure()
```

```
plot(k_values,Ga)
title('Ga Vs Vf')
xlabel('Vf')
ylabel('Ga')
xlswrite('Ga.xlsx',[k_values(:),Ga(:)]);

figure()
plot(k_values,Gt)
title('Gt Vs Vf')
xlabel('Vf')
ylabel('Gt')
xlswrite('GT.xlsx',[k_values(:),Gt(:)]);
```