# AP Precalculus 1.8 | Worksheet B | Teacher Explanation & Key

**Teaching sequence: (1) factor; (2) identify original denominator exclusions; (3) solve numerator zeros; (4) determine end behavior from degrees; (5) build a sign chart.**

A canceled factor creates a hole, not a zero. For ≥ or ≤, include defined zeros only; never include denominator zeros.

1. Find zeros of $f(x) = \frac{x-5}{x+2}$

Step 1 — Factor: Numerator: x − 5; denominator: x + 2.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: −2. Answer: x = 5.

End behavior: lim as x → ∞ = 1; lim as x → −∞ = 1. Compare leading terms to explain the limit.

Check: substitute any sample x-value from a selected interval to confirm its sign.

2. Find zeros of $f(x) = \frac{x^2}{x^2+2x-24}$

Step 1 — Factor: Numerator: $x^2$; denominator: $x^2 + 2x - 24$.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: −6, 4. Answer: x = 0.

End behavior: lim as x → ∞ = 1; lim as x → −∞ = 1. Compare leading terms to explain the limit.

Check: substitute any sample x-value from a selected interval to confirm its sign.

3. Find zeros of $f(x) = \frac{x^2+x-6}{x^2+3x-10}$

Step 1 — Factor: Numerator: $x^2 + x - 6$; denominator: $x^2 + 3x - 10$.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: −5, 2. Answer: x = −3.

Check: substitute any sample x-value from a selected interval to confirm its sign.

4. Find zeros of $f(x) = \frac{x^2 - 25}{x^2 + 2x - 15}$

Step 1 — Factor: Numerator: $x^2 - 25$; denominator: $x^2 + 2x - 15$.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: -5, 3. Answer: $x = 5$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

5. Find zeros of $f(x) = \frac{x^3 - 16x}{3x^2 + 8x - 3}$

Step 1 — Factor: Numerator: $x^3 - 16x$; denominator: $3x^2 + 8x - 3$.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: -3, 1/3. Answer: $x = -4$, $x = 0$, $x = 4$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

6. Find zeros of $f(x) = \frac{x^2 + x - 12}{x + 4}$

Step 1 — Factor: Numerator: $x^2 + x - 12$; denominator: $x + 4$.

Step 2 — Set the numerator equal to zero, then reject values where the ORIGINAL denominator is zero.

Excluded values: -4. Answer: $x = 3$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

**7. Analyze $f(x) = \frac{x^2 + 3x - 18}{x^2 + 2x}$**

Step 1 — Factor: Numerator: $x^2 + 3x - 18$; denominator: $x^2 + 2x$.

Domain: all real x except -2, 0. Zeros: -6, 3.

Horizontal asymptote: $y = 1$. y-intercept: none.

Sign chart: $(-\infty, -6)$: test $x = -7 \rightarrow$ positive; $(-6, -2)$: test $x = -4 \rightarrow$ negative; $(-2, 0)$: test $x = -1 \rightarrow$ positive; $(0, 3)$: test $x = 3/2 \rightarrow$ negative; $(3, \infty)$: test $x = 4 \rightarrow$ positive.

Answer for $f(x) > 0$: $(-\infty, -6) \cup (-2, 0) \cup (3, \infty)$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

**8. Analyze $f(x) = \frac{x^2 - 9}{x - 3}$**

Step 1 — Factor: Numerator: $x^2 - 9$; denominator: $x - 3$.

Domain: all real x except 3. Zeros: -3.

Horizontal asymptote: none (end behavior is not constant). y-intercept: (0, 3).

Sign chart: $(-\infty, -3)$: test $x = -4 \rightarrow$ negative; $(-3, 3)$: test $x = 0 \rightarrow$ positive; $(3, \infty)$: test $x = 4 \rightarrow$ positive.

Answer for $f(x) < 0$: $(-\infty, -3)$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

**9. Analyze $f(x) = \frac{2x + 8}{x^2 + 3x - 10}$**

Step 1 — Factor: Numerator: $2x + 8$; denominator: $x^2 + 3x - 10$.

Domain: all real x except -5, 2. Zeros: -4.

Horizontal asymptote: $y = 0$. y-intercept: $(0, -4/5)$.

Sign chart: $(-\infty, -5)$: test $x = -6 \rightarrow$ negative; $(-5, -4)$: test $x = -9/2 \rightarrow$ positive; $(-4, 2)$: test $x = -1 \rightarrow$ negative; $(2, \infty)$: test $x = 3 \rightarrow$ positive.

Answer for $f(x) < 0$: $(-\infty, -5) \cup (-4, 2)$.

Check: substitute any sample x-value from a selected interval to confirm its sign.

10. Graph inequality for f(x) = $\frac{x-4}{x+1}$ <0 0

Step 1 — Factor: Numerator: x − 4; denominator: x + 1.

Step 2 — Mark critical numbers: -1, 4. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -1): test x = -2 → positive; (-1, 4): test x = 3/2 → negative; (4, ∞): test x = 5 → positive. Equality includes numerator zeros when defined.

Answer: (-1, 4).

Graph interpretation: choose the x-values where the curve is below or on the x-axis when equality is allowed; never include a vertical asymptote.

Check: substitute any sample x-value from a selected interval to confirm its sign.

11. Graph inequality for f(x) = $\frac{-x-3}{x-2}$ >=0 0

Step 1 — Factor: Numerator: -x − 3; denominator: x − 2.

Step 2 — Mark critical numbers: -3, 2. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -3): test x = -4 → negative; (-3, 2): test x = -1/2 → positive; (2, ∞): test x = 3 → negative. Equality includes numerator zeros when defined.

Answer: [-3, 2).

Graph interpretation: choose the x-values where the curve is above or on the x-axis when equality is allowed; never include a vertical asymptote.

Check: substitute any sample x-value from a selected interval to confirm its sign.

12. Solve inequality for f(x) = $\frac{x-4}{x+3}$ >0 0

Step 1 — Factor: Numerator: x − 4; denominator: x + 3.

Step 2 — Mark critical numbers: -3, 4. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -3): test x = -4 → positive; (-3, 4): test x = 1/2 → negative; (4, ∞): test x = 5 → positive. Equality includes numerator zeros when defined.

Answer: (−∞, -3) ∪ (4, ∞).

Check: substitute any sample x-value from a selected interval to confirm its sign.

13. Solve inequality for f(x) = $\frac{5}{x-2}$ <=0 0

Step 1 — Factor: Numerator: 5; denominator: x − 2.

Step 2 — Mark critical numbers: 2. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, 2): test x = 1 → negative; (2, ∞): test x = 3 → positive. Equality includes numerator zeros when defined.

Answer: (−∞, 2).

Check: substitute any sample x-value from a selected interval to confirm its sign.

14. Solve inequality for f(x) = $\frac{x^2+3x-4}{x+2}$ <0 0

Step 1 — Factor: Numerator: x² + 3x − 4; denominator: x + 2.

Step 2 — Mark critical numbers: -4, -2, 1. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -4): test x = -5 → negative; (-4, -2): test x = -3 → positive; (-2, 1): test x = -1/2 → negative; (1, ∞): test x = 2 → positive. Equality includes numerator zeros when defined.

Answer: (−∞, -4) ∪ (-2, 1).

Check: substitute any sample x-value from a selected interval to confirm its sign.

15. Solve inequality for f(x) = $\frac{x^2-10x+25}{x+3}$ >=0 0

Step 1 — Factor: Numerator: x² − 10x + 25; denominator: x + 3.

Step 2 — Mark critical numbers: -3, 5. Denominator zeros are always excluded.

Step 3 — Test signs: (−∞, -3): test x = -4 → negative; (-3, 5): test x = 1 → positive; (5, ∞): test x = 6 → positive. Equality includes numerator zeros when defined.

Answer: (-3, ∞).

Check: substitute any sample x-value from a selected interval to confirm its sign.

**16. Solve inequality for f(x) = $\frac{x^2-2x-8}{x^2+4x-5}$ <=0 0**

**Step 1 — Factor: Numerator: x² − 2x − 8; denominator: x² + 4x − 5.**

**Step 2 — Mark critical numbers: -5, -2, 1, 4. Denominator zeros are always excluded.**

**Step 3 — Test signs: (−∞, -5): test x = -6 → positive; (-5, -2): test x = -7/2 → negative; (-2, 1): test x = -1/2 → positive; (1, 4): test x = 5/2 → negative; (4, ∞): test x = 5 → positive. Equality includes numerator zeros when defined.**

**Answer: (-5, -2] ∪ (1, 4].**

**Check: substitute any sample x-value from a selected interval to confirm its sign.**

## Teacher video resources — links for each problem type

Zeros, intercepts and numerator factors (Questions 1–6): https://www.khanacademy.org/v/graphs-of-rational-functions-zeros

Asymptotes, canceled factors, graphing (Questions 7–11): https://www.khanacademy.org/v/finding-asymptotes-example

Solving rational inequalities by test intervals (Questions 7–16): https://www.khanacademy.org/math/precalculus-tx/xecf67014514b60db%3Arational-functions-equations-and-inequalities/xecf67014514b60db%3Asolving-rational-equations-and-inequalities/v/solving-factored-form-rational-inequalities

Inequalities read directly from a graph (Questions 10–11): https://www.khanacademy.org/math/precalculus-tx/xecf67014514b60db%3Arational-functions-equations-and-inequalities/xecf67014514b60db%3Asolving-rational-equations-and-inequalities/v/solving-rational-inequalities-from-a-graph

AP Precalculus Topic 1.8 lesson video: https://www.youtube.com/watch?v=K90mV6M7aNw

User-provided 1.8A worksheet worked-solutions video (original, not these new numbers): https://drive.google.com/file/d/1wFQk-ySME9DVIRhhrIRQJWGiEyA-HxF2/view

Video links explain the same METHODS, but may use different example values than these original questions.