1.

One root of the equation $x^2 + px + q = 0$ is $2 - 3\mathrm{i}$. Find the values of *p* and *q*.

(3)

2.

Given $z^2 = 15 + 8\mathrm{i}$, $z = a + b\mathrm{i}$. Find the possible combinations of *a* and *b*, where *a* and *b* are integers.

(6)

3.

a. Find all possible values of the real numbers *a* and *b* which

$$2 + a\mathrm{i} = \frac{6 - 2\mathrm{i}}{b + \mathrm{i}}$$

(4)

b. Given that $w = -\frac{1}{2} + \frac{1}{2}\mathrm{i}$, find the modulus and the argument of $\frac{1}{1 + w}$, giving the argument in radians between $-\pi$ and π

(4)

4.

Given that $1 + 3\mathrm{i}$ is a root of the equation $z^3 + 6z + 20 = 0$

a. Find the other two roots of the equation,

(4)

b. Show, on a single Argand diagram, the three points representing the roots of the equation,

(1)

c. Prove that these three points are the vertices of a right-angled triangle.

(2)

5.

Show that

$$\sum_{r=1}^{n+2}(2r + 3) = (n + 2)(n + 6)$$

(3)

6.

Show that

$$\sum_{p=3}^{n}(4p + 5) = (2n + 11)(n - 2)$$

(3)

7.

a. Show that $\sum_{r=1}^{k}(4r-5)=2k^2-3k$

(2)

b. Find the smallest value of *k* for which $\sum_{r=1}^{k}(4r-5)>4850$

(2)

8.

a. Show that $\sum_{r=1}^{n}r(r+2)=\frac{n}{6}(n+1)(2n+7)$

(3)

b. Use the result, or otherwise, find in terms of *n*, the sum of
$3\log 2+4\log 2^2+5\log 2^3+...+(n+2)\log 2^n$

(2)

9.

If the roots of the equation $4x^3+7x^2-5x-1=0$ are $\alpha,\ \beta$ and γ, find the equation whose roots are $2\alpha+1,\ 2\beta+1$ and $2\gamma+1$

(5)

10.

The root of the equation $x^3+px^2+qx+30=0$ are in the ratios 2:3:5. Find the values of *p* and *q*.

(5)

11.

a. Sketch the locus $|z-3+\mathrm{i}|=4$

(2)

b. Find the maximum value of $|z|$ correct to 3 decimal places

(5)

12.

Find the cartesian equation for

a. $|z-4|<|z-2\mathrm{i}|$

(2)

b. $\arg(z-2+\mathrm{i})=\dfrac{\pi}{3}$

(3)

Shaded the region on separated Argand diagrams

a. $|z-4|<|z-2\mathrm{i}|$

(4)

b. $\dfrac{\pi}{6}<\arg(z-2+\mathrm{i})\leqslant\dfrac{\pi}{3}$

(4)