Background

Informally, the [Hopf link](#) is the subset of Euclidean space consisting of two rings that are "linked" as in two links in a chain.

In my third year of undergrad I learned from Luca Migliorini a beautiful computation of the fundamental group of the complement of the Hopf link inside Euclidean space, using the fact that the preimage of two points through the [Hopf fibration](#) is precisely a Hopf link. In particular, the two obvious generators (taking a loop through only one ring and the other loop through the other ring) somehow turn out to commute up to homotopy, which distinguishes this space to the somewhat similar "complement of a wedge of two circles", which has two similarly natural generators that in fact do not commute.

In the context of a minicourse on creating mathematical animations, I set out to animate an explicit "deformation" of these two paths inside Euclidean space that would allow me to see with my own two eyes that those two generators indeed commute. Here is this animation.

It is crude and probably badly implemented, but then again the two weeks during which I dedicated myself to this project amount to the totality of my exposure to javascript , so I hope you will cut me some slack :)

I would like to thank Gianmarco Todesco for providing his libraries and teaching the brief minicourse which introduced me to these tools.

How to play the animation:

Download the folder…
https://drive.google.com/drive/folders/1EQ4j-dfjyrIcIChNgpkbYHPYOEagKN1U?usp=sharing
…and double-click the html file !
You may drag around your cursor so as to rotate the point of view, which is necessary to appreciate some of the steps of the deformation.

What is happening:

The two generators of the fundamental group are color-coded. The base-point is represented by that black ball. Note that at some point the orange path almost fades out: it is to make the viewer focus on the green path, since from then until the end of the animation only the green is deformed.