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Fiddler Solution

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BASIC FIDDLER

Answer

$\frac{3}{4}$ or 75%

Solution

I went down a few roads trying to solve this problem. I was worried I would have to resort to computer programming again, which doesn't really satisfy me.

My initial approach was "build out a tree to account for all possibilities, and the probability of you winning is the probability path which is, hand wavyly, the "maximal-minimal path". Basically keeping your opponents actions the most rational, pick the path that maximizes your win.

However, that seemed too complicated. Then I had an insight that simplified the problem for me in a way that I could do it without programming, which is the kind of solution I like the best.

Before I go into the key insight, I want to state two key facts that help us solve the problem. They both have to do with the fact that the game is symmetric:

- If the score is tied (at any score), both players have a 50/50 chance of winning. There is no advantage in either direction as both players play rationally and both players have an equal chance of winning each hole.
- If player 1 has a probability p of winning when the score is X to Y , then player 2 has a probability of p of winning when the score is reversed: Y to X .

Now the key insight I had: if the score is 2-1 in either direction, the probability of either player winning is 50%. The player who is behind can force the probability to be 50/50.

Let's say player A is at 2 and player B is at 1. My reasoning is, at that point:

- There's no rational reason for B to not throw the hammer:
 - If A accepts the hammer, B either wins the match outright 2-3, or they lose 4-1. Since the probability of winning a particular hole is 50%, that means both scenarios are 50/50.
 - If A doesn't accept the hammer, the score changes to 2-2, which means the probability of winning is 50-50 anyway.
- If B doesn't throw the hammer, then at best B ends up 2-2 (a 50/50 scenario) and at worst, loses 3-2. This means throwing the hammer is strictly better.
- Now let's look at it from the perspective of A.
 - If they reject the hammer, they end up with a 2-2 score - so both players have a 50/50 chance.
 - If they accept the hammer, they end up with either 4-1 or 2-3 in a 50/50 chance.
- If B throws the hammer, whatever A does in response, the chance of both players winning is 50%

Now here's what's interesting. Because 2-1 is 50-50, it helps us simplify the lower score lines. Let's take 2-0 where A is ahead of B for example:

- It makes sense for B (on 0) to throw the hammer. For B, player A getting 1 pt or 2 pts doesn't matter - they win. So B has to maximize, which means going for 2 points. For A, if they lose with or without the hammer, 2-1 is 50/50 and 2-2 is also 50/50.
- This means that when the score is at 2-0, three things can happen:
 - B throws the hammer, A rejects, leaving the score at 2-1, which is 50-50
 - B throws the hammer, A accepts:
 - A wins the hole, so A wins 4-0
 - B wins the hole, so the score is 2-2, so 50/50
- There's no rational reason for A to reject the hammer and directly moving the odds to 50/50. So A will accept the hammer, and:
 - 50% of the time, A will win with $p = 1$
 - 50% of the time, A will win with $p = 0.5$
- This means if the score is at 2-0, A wins with 75% chance

We can resolve 1-0 similarly. Now In this scenario, I'm A, and I'm ahead 1-0.

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- It is rational for me (A) to throw the hammer. If I win the next hole I win 3-0 with certainty. If I lose the next hole, it's 1-2, which is 50-50
 - If I don't throw the hammer, I get 2-0 and 1-1 as outcomes, which are 75% and 50%, which are strictly worse.
 - If B rejects the hammer, I'm up 2-0 and a 75% favorite.
 - If B accepts the hammer, then:
 - 50% of the time I win with $p = 1$ (3-0)
 - 50% of the time I win with $p = 0.5$ (1-2)
 - So my probability of winning is $50\% * 1 + 50\% * 0.5 = 75\%$

EXTRA CREDIT

Answer

11/16 or 68.75%

Solution

This resolves similarly to the 3 point version. Without going over the rationalizations again, I'm only considering the scores in one direction (they are symmetric anyway)

4-3, 4-4 is 50-50

4-2, 4-1, 3-2, 3-1 is 75-25

3-0 is 87.5 - 12.5

2-2, 2-1 is 50-50

2-0 is 68.75-31.75

1-0 is 68.75 - 31.75 or **11/16**