

STAT 165/265: Forecasting

Discussion 10 Worksheet

4/11/2025

You will work through this sheet in class on Friday, 4/11 in groups of 2-3. The goal of today's discussion is for you to apply your knowledge of Martingales and Kelly betting.

Please submit the Discussion sheet for participation points on Gradescope under the "Discussion 11" assignment. Submit what you have at the end of the hour – it's ok if you didn't finish every question thoroughly!

For students who have contacted us about absences this Friday, please spend an hour on this sheet and submit to the same Gradescope assignment.

Part I. Beliefs as Martingales

Recall the setting from Lecture 17:

- Every week, you are trying to predict whether or not it will rain on Friday
- You make a first prediction on Monday
- After your Monday prediction, you watch a weather forecast on TV
- You make a second prediction on Tuesday, informed by the weather forecast

1. Suppose that every week, you make the following predictions:

- On Mondays, you predict a **50%** chance of rain
- On Tuesdays:
 - On a fraction f of weeks, the TV forecast says it will rain on Friday, so you predict a **60%** chance of rain
 - On the other weeks, the TV forecast says it will not rain on Friday, you predict a **30%** chance of rain

Type of week	Fraction of weeks	Monday prediction	Tuesday prediction
TV predicts rain	f (?)	50%	60%
TV predicts no rain	$1 - f$ (?)	50%	30%

For your predictions to be **well-calibrated**, on what fraction **f** of weeks must the TV forecast predict rain on Friday? (Hint: use the martingale property)

2. Suppose that every week, you make the following predictions:
- On Mondays, you predict a chance **p** of rain
 - On Tuesdays:
 - a. On **25%** of weeks, the TV forecast says it will rain on Friday, so you predict a **60%** chance of rain
 - b. On the other weeks, the TV forecast says it will not rain on Friday, you predict a **20%** chance of rain

Type of week	Fraction of weeks	Monday prediction	Tuesday prediction
TV predicts rain	25%	p (?)	60%
TV predicts no rain	75%	p (?)	20%

For your predictions to be **well-calibrated**, what prediction **p** should you make on Mondays? (Hint: use the martingale property)

Part II. Kelly Betting

For each one of the following bets:

- What fraction of your wealth should you bet to **maximize your expected wealth**?
- What fraction of your wealth does the **Kelly criterion** say you should bet?
- What fraction of your wealth would **you** want to bet?

1. You pay $\$n$, then:

- With probability 0.75, you win and are returned $\$2n$, so you made $\$n$ on net
- With probability 0.25, you receive nothing, so you lost $\$n$ on net

2. You pay $\$n$, then:

- With probability 0.5, you win and are returned $\$4n$, so you made $\$3n$ on net
- With probability 0.5, you receive nothing, so you lost $\$n$ on net

3. You pay $\$n$, then:

- With probability 0.4, you win and are returned $\$2n$, so you made $\$n$ on net
- With probability 0.6, you receive $\$0.5n$, so you lost $\$0.5n$ on net

4. You pay $\$n$, then:

- With probability 0.25, you win and are returned $\$3n$, so you made $\$2n$ on net
- With probability 0.5, you receive $\$0.5n$, so you lost $\$0.5n$ on net
- With probability 0.25, you receive nothing, so you lost $\$n$ on net