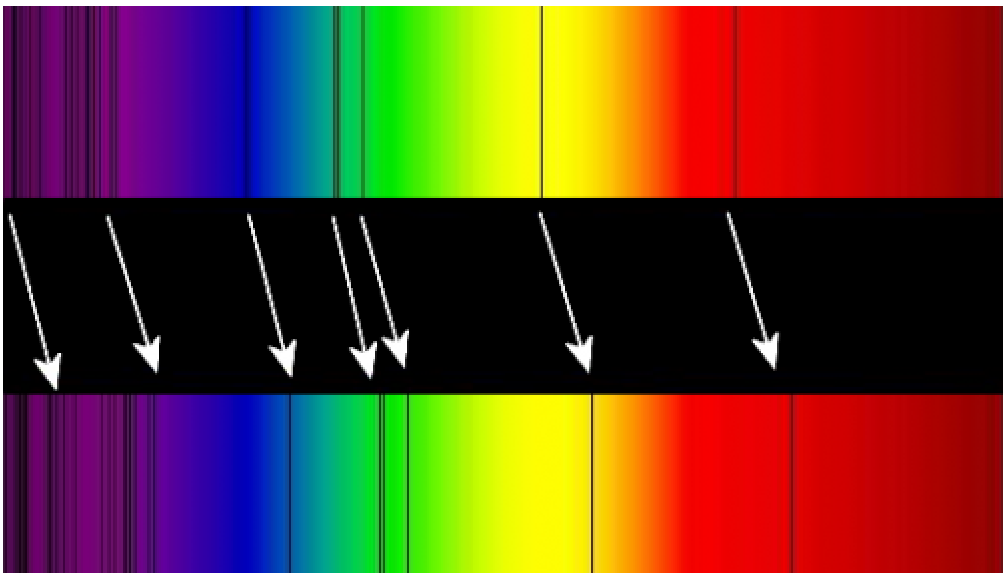
Date: _______________ **Name:**___________________

Discovering Exoplanetary Characteristics with RV (and Transit) Data

Recall that changes in the radial velocity of a star as it's "wobbled" by a planet will Doppler shift the star's spectra. Astronomers then measure this shift in order to derive the radial velocity of the star. The shift in a star's spectrum can be observed by measuring the wavelength of known absorption lines (such as the Frauenhofer lines, Balmer series, etc.) and comparing them to the known wavelength of those lines in the laboratory.



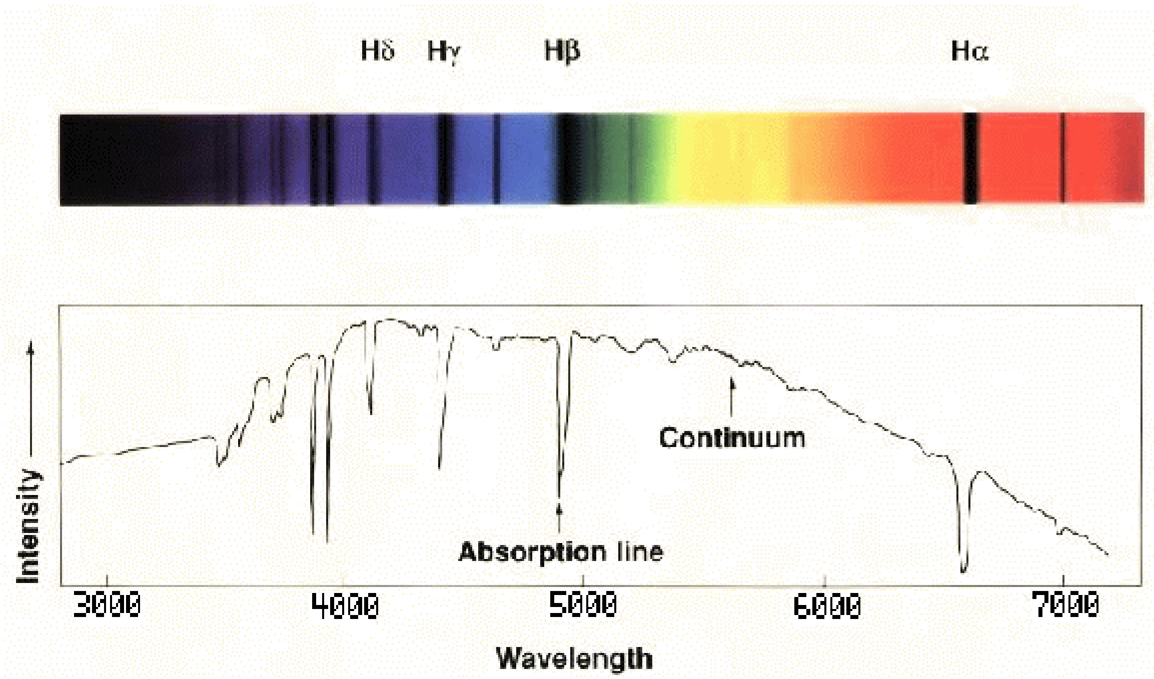
Credit: Wikimedia Commons

The radial velocity can then be calculated by the following equation:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v_r}{c}$$

where $\Delta\lambda$ is the difference in wavelength, λ_0 is the rest wavelength or the known wavelength of the line, v_r is the radial velocity, and c is the speed of light.

The illustration above is helpful for a visualization of how spectral lines shift as a result of the Doppler effect, but stellar spectra as measured by instruments and analyzed by astronomers look more like the following illustration:



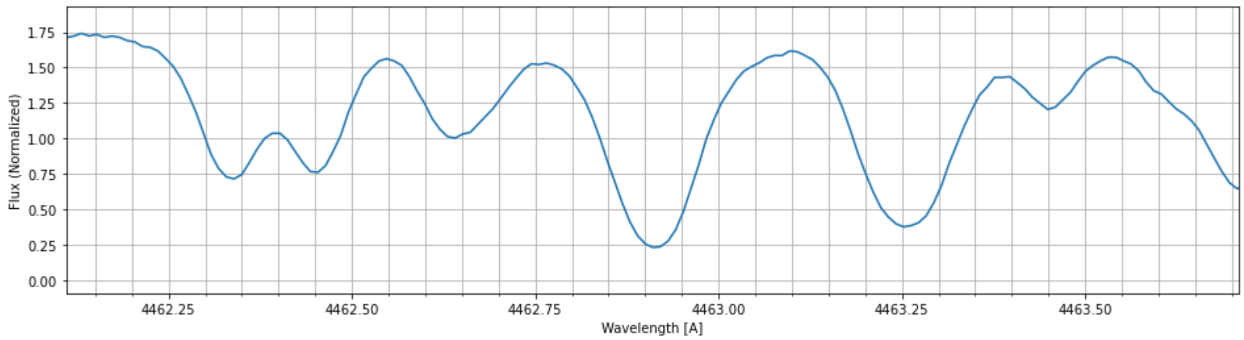
Credit: Gene Smith, UCSD

In this graph, the x-axis corresponds to wavelength, while the y-axis corresponds to the intensity or flux of the light measured from the star in that wavelength. Here, absorption lines appear as dips in the flux. Lines can vary greatly in width and depth – some are broader, or narrower – which are affected by a number of physical processes, and some lines very close together can appear blended, but the center of a line is considered the wavelength measured for that line.

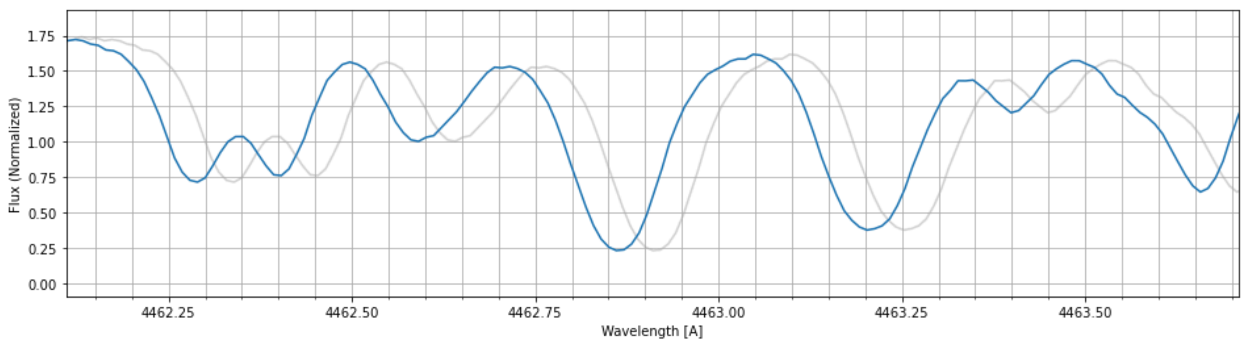
Measured stellar spectra can be very broad, containing thousands of lines, depending on the wavelength range of the instrument used to detect them. In the following exercises, we will zoom in and look at one small section of an example stellar spectrum to measure the radial velocity of a star with an exoplanet. Then we will use these measurements, in combination with other information we know about the star and planet from our data, to extract physical characteristics of our planet.

Exercise 1: Measuring RV shifts

Below is a section of a stellar spectrum at rest. Mark the wavelengths of each line you can identify; these are your rest/reference wavelengths for those lines.



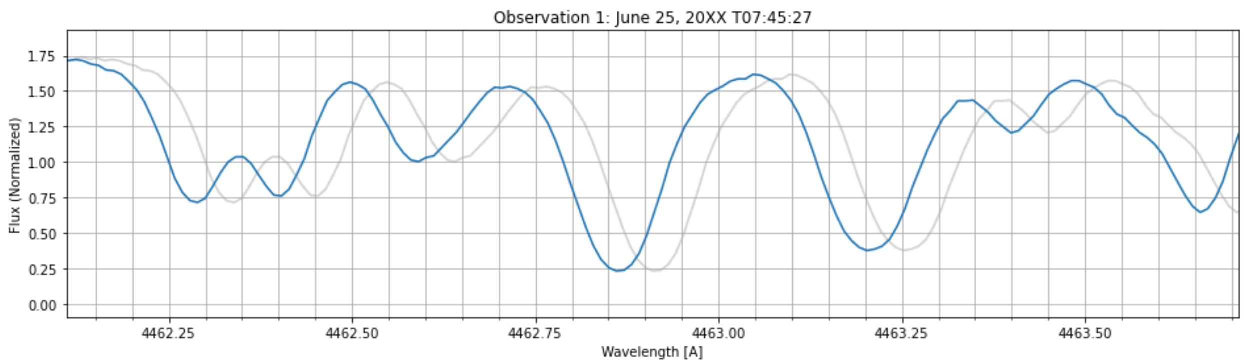
Now pretend you've received the following observation of this same star. For convenience, the reference spectrum is plotted underneath this new observation.



Using a ruler, measure the shift and find Δλ for each of the lines you identified earlier. Then, using the equation for radial velocity, calculate the radial velocity indicated by each of these shifts and average your results. What is the radial velocity of this star in this observation?

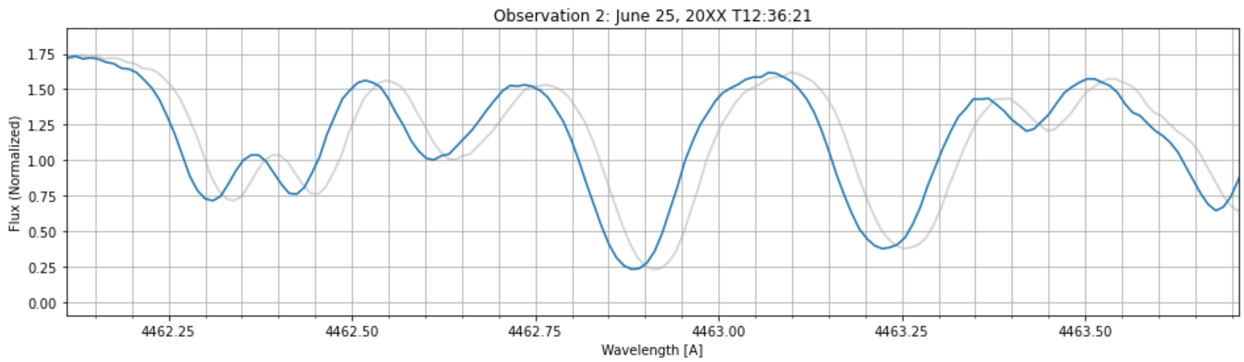
Exercise 2: Measuring RV over time

When measuring RVs of stars, we are usually interested in the RVs measured over time to search for periodicities, i.e. possible planets! Below are 10 different observations of the same star taken at different times. For the first observation, let's take the same observation that you already analyzed above, given a date, and record the RV you found:

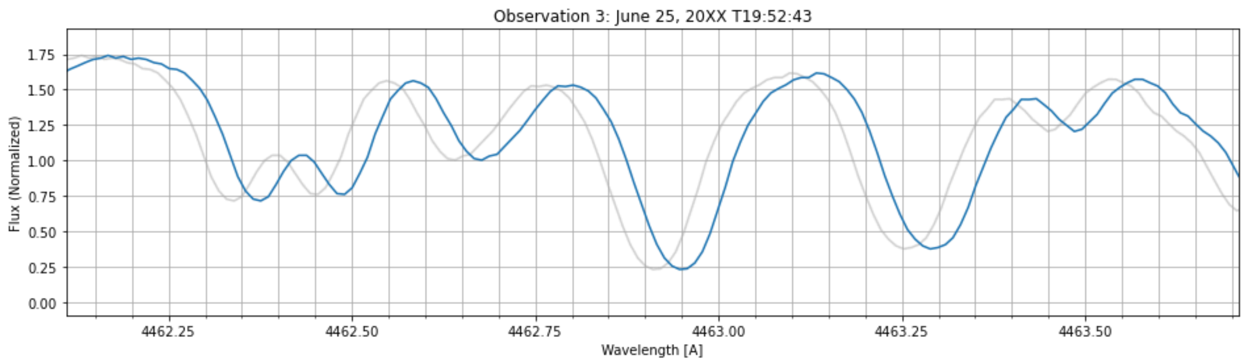


RV:_____________________________

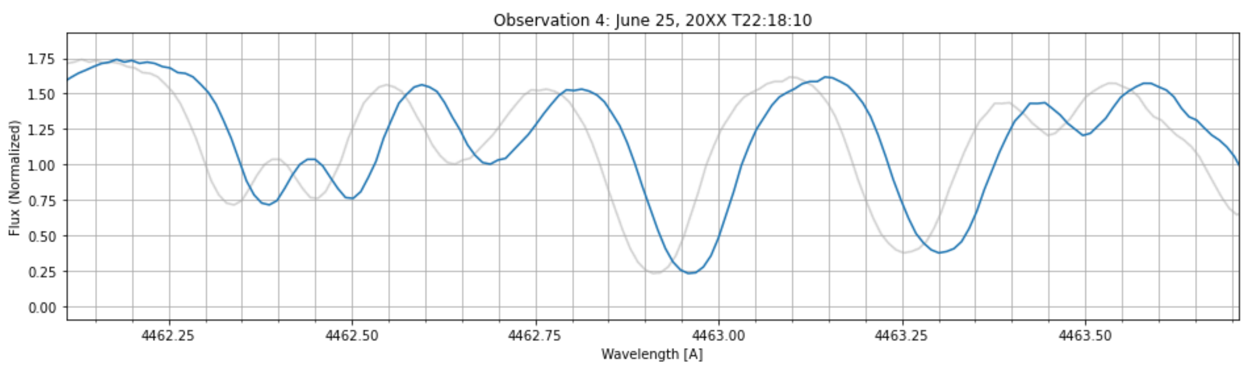
For the following 9 observations, use the same procedure you used in Exercise 1 to measure Δλ and calculate and record the radial velocity of the star for each observation.



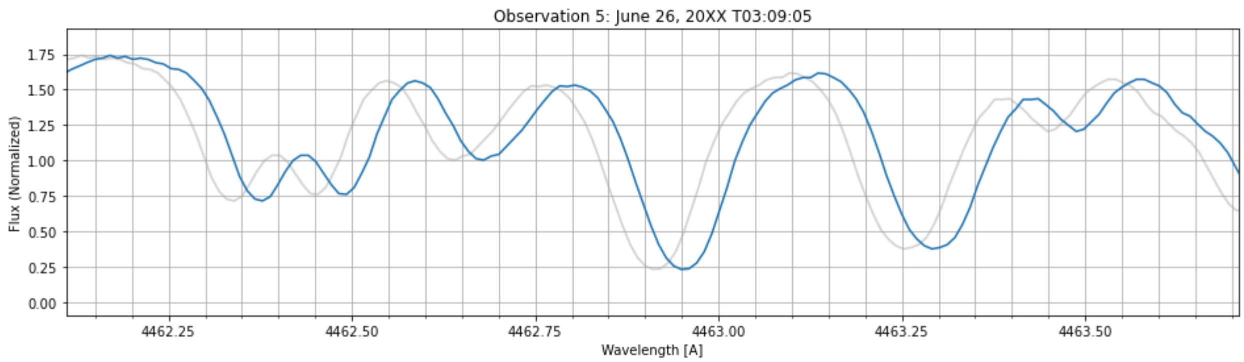
RV:______________________________


Observation 3: June 25, 20XX T19:52:43
Flux (Normalized)
Wavelength [A]

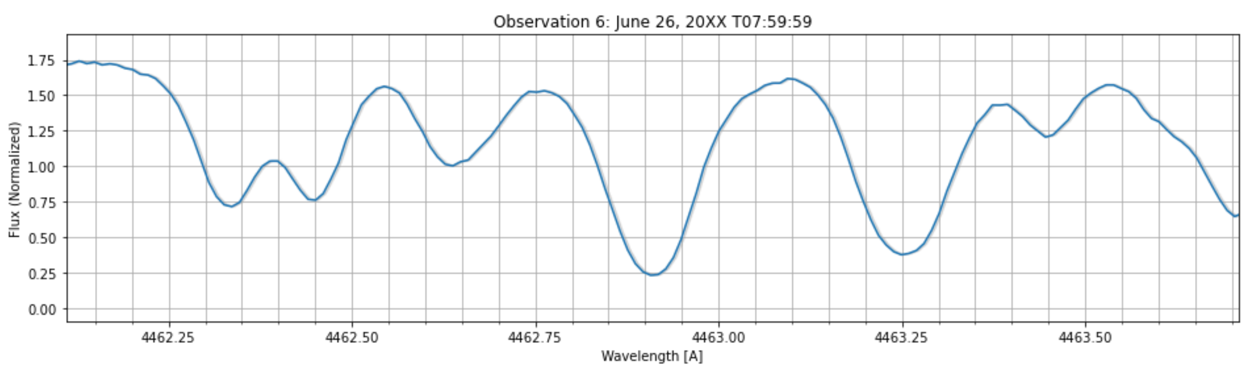
RV:_______________________________


Observation 4: June 25, 20XX T22:18:10
Flux (Normalized)
Wavelength [A]

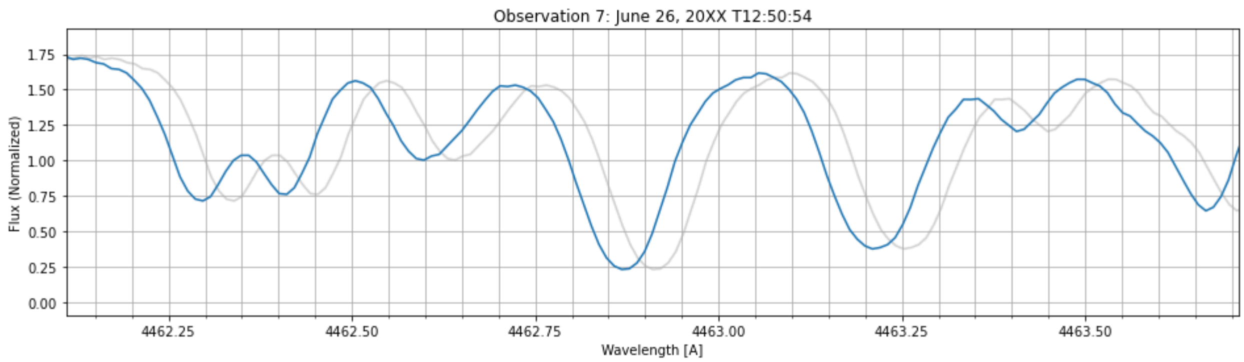
RV:_______________________________


Observation 5: June 26, 20XX T03:09:05
Flux (Normalized)
Wavelength [A]
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
4462.25
4462.50
4462.75
4463.00
4463.25
4463.50

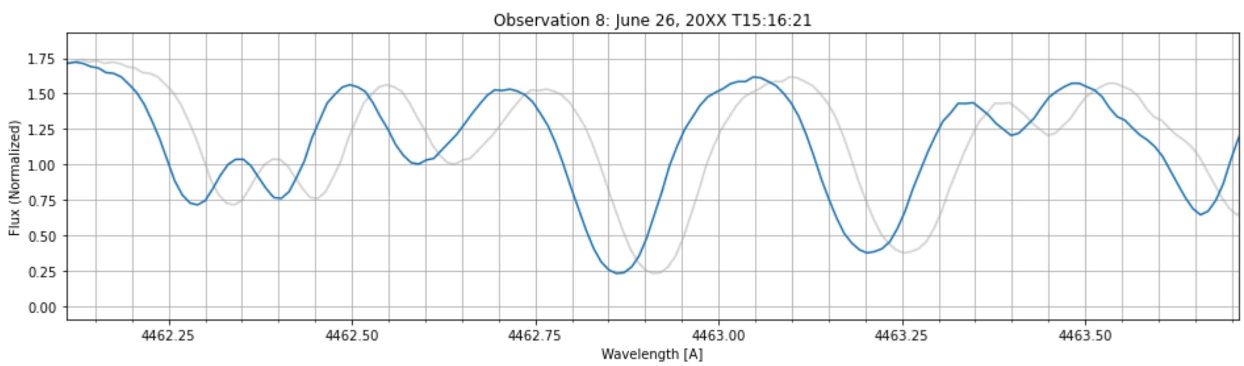
RV:______________________________


Observation 6: June 26, 20XX T07:59:59
Flux (Normalized)
Wavelength [A]
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
4462.25
4462.50
4462.75
4463.00
4463.25
4463.50

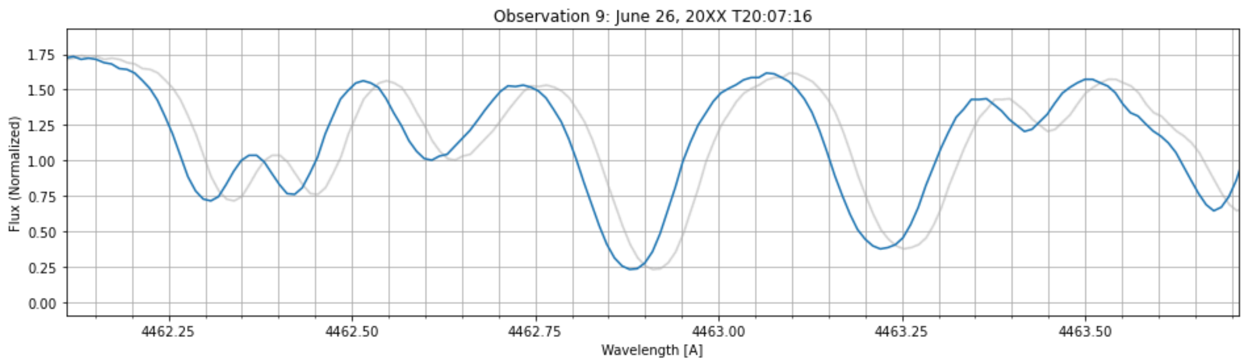
RV:______________________________


Observation 7: June 26, 20XX T12:50:54
Flux (Normalized)
Wavelength [A]
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
4462.25
4462.50
4462.75
4463.00
4463.25
4463.50

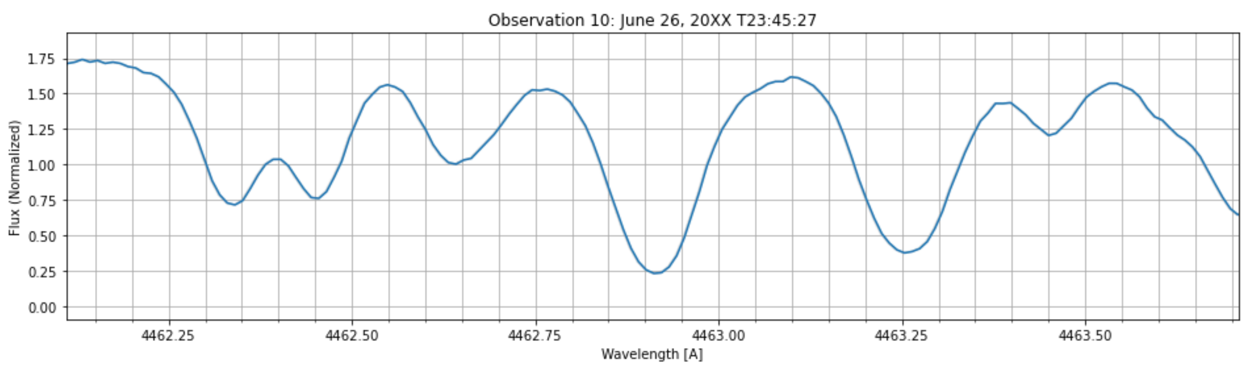
RV:______________________________


Observation 8: June 26, 20XX T15:16:21
Flux (Normalized)
Wavelength [A]
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
4462.25
4462.50
4462.75
4463.00
4463.25
4463.50

RV:______________________________


Observation 9: June 26, 20XX T20:07:16
Flux (Normalized)
Wavelength [A]

RV:______________________________


Observation 10: June 26, 20XX T23:45:27
Flux (Normalized)
Wavelength [A]

RV:______________________________

Now, plot your calculated RVs over time on the graph below:

Take a look at your plot. Do you see a periodicity in the RVs? By looking at a plot of RVs over time, we can estimate two important parameters of the orbit of a potential planet: the period, P, and the radial velocity semi-amplitude, K. Using this plot, estimate values for P and K:

Exercise 3: Calculating exoplanetary characteristics

We now have an estimate for the period and semi-amplitude of the orbit of our potential exoplanet. From here, we can now use these estimated parameters, combined with the other information we have at our disposal, to calculate estimates for other characteristics of our planet.

For these calculations, it is necessary to know some information about the planet's host star, namely its mass and radius. For convenience, we will assume that the star is a Sun-like star.

1. Orbital radius. We can calculate the orbital radius, a, of the planet using Kepler's Third Law:

$$P^2 = \left(\frac{4\pi^2}{G(M+m)}\right)a^3$$

We know P and M, and we can neglect m of the exoplanet for now since it is negligible compared to the mass of the star. Use the equation above to calculate a:

2. Mass of the planet. Recall the following equation, which we can use to calculate the mass of the planet:

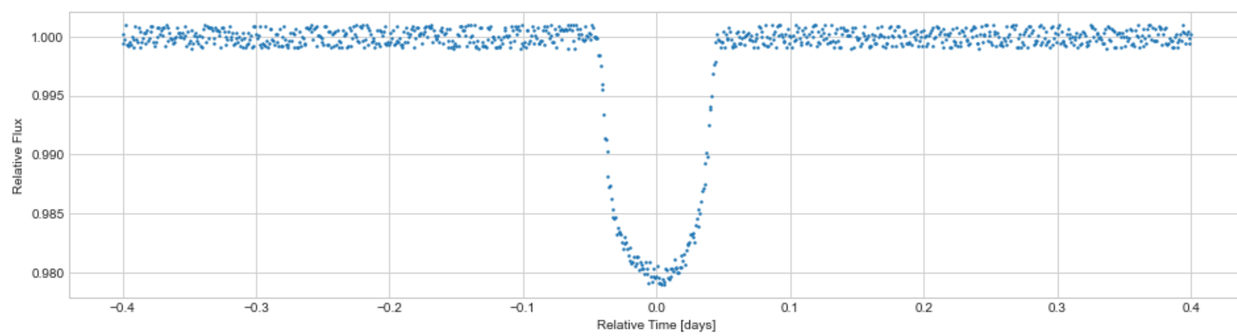
$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \cdot \frac{m \cdot sin(i)}{(M+m)^{2/3}}$$

Assume that the planet's orbit is on an inclination of $i = 90°$ and calculate m:

3. Radius of the planet. Calculating the radius of the planet requires knowing some information about the planet's transit. For convenience, we assume that this planet also transits its star and that we have transit data available. Then, to calculate the radius of the planet, we can use the equation:

$$drop = \frac{r^2}{R^2}$$

where *drop* is the measured fractional drop in the star's brightness/flux caused by the planet's transit. In the transit light curve below, the flux is normalized to 1, so the measured flux drop is also the measured fractional drop.



Using this data and the equation above, calculate *r* of the planet:

4. Density of the planet. Given density = mass/volume, calculate the density of the planet:

Look up the densities of the materials and Solar System planets listed below and fill in the table. How does your calculated density of your possible exoplanet compare to these? What guesses might you make about the composition of your planet?

Material/Planet	**Density**
Water	
Ice	
Iron	
Terrestrial rock(s)*	
Mercury	
Earth	
Jupiter	
Saturn	

** "Rock" is a general term that refers to many different types of minerals and mineral matter. Try looking for a range of and/or average densities of common rocks found on Earth.*

5. Equilibrium temperature of the planet. Finally, we can estimate the equilibrium temperature of the planet by using the following equation:

$$T_p = T_\odot (1 - A)^{1/4} \sqrt{\frac{R_\odot}{2a}}$$

Note the term A, which stands for the albedo of the planet. Recall that albedo is a measure of how much energy/light is absorbed or reflected by an astronomical body, and takes a value between 0 and 1 (where 0 = total absorption and 1 = total reflection). We don't know the albedo of our planet, but we can try to make a reasonable guess. Try looking up the albedos of the materials and Solar System planets whose densities you just found in the previous exercise, and use those combined with what you *do* know about our planet so far to make a reasonable guess for its albedo. Then use the equation above to calculate T_p :

Try looking up the equilibrium temperatures of Earth and other Solar System planets. How does your calculated T_p compare to these? What, if anything, might you be able to guess about the habitability of our planet based on this measurement?

Challenge Exercise (Optional): Changing stellar/planetary parameters

Think about what you've learned about how different planetary characteristics can be determined from RV and/or transit data and discuss the following questions with your neighbors. One additional parameter that we skipped over in this activity was the *duration* of the planet's transit across the star. Refer back to page 11 and note the duration of the planet's transit. Consider what additional characteristic(s) we might be able to determine from the transit duration (hint: think about the geometry of the orbit!).

For these exercises, we assumed that the star in the system we are looking at was a Sun-like star. Imagine instead that the star in this system you just looked at was an M dwarf/red dwarf, which are significantly smaller than the Sun. Assuming you received the exact same RV and transit data, how would this affect the planetary parameters you obtain from the equations above? What if the star is significantly larger?

Now imagine that we keep the star Sun-like, but the planet in the system is significantly smaller; say, an Earth-like planet. How might this change the RV and transit data we receive from the star? If you have time, try calculating the approximate radial velocity semi-amplitude, K, and flux drop we would measure as a result of an Earth-like planet's "wobble" of and transit across a Sun-like star. How do these values compare to the RV and transit data we observed for the system in the previous exercises? Estimate the uncertainties in the measurements/data you worked with in this activity and consider the limitations that these uncertainties place on the types of planets you can find with this data. What is the smallest planet you think you could detect with RV and/or transit data?

Useful constants/values and equations:

$G = 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$

$c = 3 \cdot 10^8 m\, s^{-1}$

$M_\odot = 1.989 \cdot 10^{30} kg$

$R_\odot = 6.957 \cdot 10^8 m$

$T_\odot = 5772K$

$m_E = 5.97 \cdot 10^{24} kg$

$r_E = 6.37 \cdot 10^6 m$

$V_{sphere} = \frac{4}{3} \pi r^3$