Advanced Math
Date:
Period:

Name:

3.0-3.5 Review

Show your steps. No calculators will be allowed on the quiz.

The Review only has Level I questions on it, which will account for about 80% of the quiz points.

The Review does NOT include any Level II or Level III questions, which will account for about 20% of the quiz points. You may study your homework questions for samples of Level II and Level III concept questions.

Matching with reason 1-4: The graphs of four polynomial functions are shown. Below are 4 polynomial functions. Match the function with the graph. **Explain** your reasoning with at least one complete sentence.

↖ Explanations may vary - A sample is given for #1 & #2

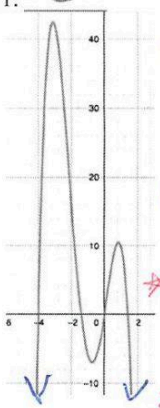
A. $f(x) = -2x^3 - 8x^2 + 4x + 16$

B. $f(x) = 2x^3 + 8x^2 - 4x - 8$

C. $f(x) = -2x^4 - 8x^3 + 4x^2 + 16x$

D. $f(x) = 2x^4 + 8x^3 - 4x^2 - 16x$

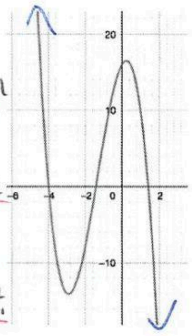
1. C



Answers may vary

The graph is most like a parabola that opens down (both ends point down) so f(x) is an even degree polynomial with a negative leading coefficient.

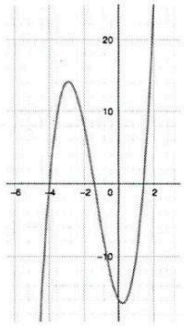
2. A



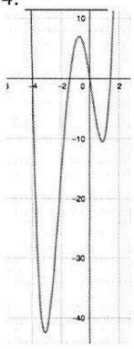
Reasons may vary

The graph is most like a line with negative slope (ends point opposite directions) so f(x) is an odd degree polynomial with a negative leading coefficient.

3. B



4. D



- Remember to explain 3 & 4
- Sample explanations were not given

5. Find the polynomial function with $g(-4) = g(3) = 0$ and $g(5) = 8$. You may leave your equation in "factored form".

$g(-4)=0$ and $g(3)=0$

so $x=-4$ and $x=3$ are zeros

$g(5)=8$ (input, output)

$g(x) = A(x+4)(x-3)$ create factors based on zeros

$8 = A(5+4)(5-3)$

$8=A(9)(2)$

$\frac{8}{18} = \frac{A(18)}{18}$

so $A = \frac{8}{18}$ or $\frac{4}{9}$ (use given point to find "A")

$g(x) = \frac{4}{9}(x+4)(x-3)$ write answer

6. Find the polynomial function with $g(-4) = g(1) = g(3) = 0$ and $g(4) = 5$. You may leave your equation in "factored form".

$g(x) = A(x+4)(x-1)(x-3)$ create factors based on given zeros

$5 = A(4+4)(4-1)(4-3)$

$5 = A(8)(3)(1)$

$\frac{5}{24} = \frac{A(24)}{24}$

$\frac{5}{24} = A$ (use given point $g(4)=5$ to find "A")

$g(x) = \frac{5}{24}(x+4)(x-1)(x-3)$ write answer

7. Given $f(x) = (x-3)(x+3)(x+5) = x^3 + 5x^2 - 9x - 45$, estimate each of the following. Answers such as about 3 or a really big positive number are fine. Explain your reasoning.

★ Remember f(x) equals the factored form & also simplified form so use form that is easier to compute for given input

a. $f(3.001) \approx$ 0

$f(3.001) \approx f(3) = (3-3)(3+3)(3+5) = 0$ (0)

3.001 is almost 3 and f(3) can be quickly computed in factored form

b. $f(-4.999) \approx$ 0

$f(-4.999) \approx f(-5) = (-5-3)(-5+3)(-5+5) = 0$ (0) (similar reasoning to a)

c. $f(0.001) \approx$ -45

$x=.001$ is close to $x=0$ so $f(.001)$ can be estimated by finding $f(0)$ in the simplified version

$f(.001) \approx f(0) = 0^3 + 5(0)^2 - 9(0) - 45 = -45$ (0, 0, 0)

d. $f(1,000,000) \approx$ Big positive number

$f(x)$ is an odd degree polynomial with a positive leading coefficient so the $1x^3$ term has the biggest impact on output when x is big in magnitude. ($1 \cdot (\text{positive})^3 = 1 \cdot \text{positive} = \text{positive}$)

e. $f(-1,000,000) \approx$ Negative number big in magnitude

- similar to d except $1 \cdot (\text{negative})^3 = 1 \cdot \text{negative} = \text{negative}$

8. Given $f(x) = x^3 + 5x^2$, find the equation of the secant line with respect to $(1, f(1))$ and $(2, f(2))$.

Find 2 points:

$f(1) = 1^3 + 5(1)^2 = 1 + 5(1) = 6 \rightsquigarrow (1, 6)$

$f(2) = 2^3 + 5(2)^2 = 8 + 5(4) = 28 \rightsquigarrow (2, 28)$

Use the 2 points to find slope

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$

$\text{slope} = \frac{28 - 6}{2 - 1} = \frac{22}{1} = 22$

Write answer

- may use point slope form or slope intercept form (only one of the answers is needed)

$y - 6 = 22(x - 1)$ ← point-slope

OR

$y - 28 = 22(x - 2)$ ← point-slope

OR

$y = 22x - 16$ ← slope-intercept $y = mx + b$

9. Given $f(x) = x^3 + x^2$, find the equation of the secant line with respect to $(1, f(1))$ and $(4, f(4))$.

Find 2 points

$f(1) = 1^3 + 1^2 = 1 + 1 = 2 \rightsquigarrow (1, 2)$

$f(4) = 4^3 + 4^2 = 64 + 16 = 80 \rightsquigarrow (4, 80)$

Find slope $= \frac{\Delta y}{\Delta x}$

$\text{slope} = \frac{80 - 2}{4 - 1} = \frac{78}{3} = 26$

Write answer

$y - 2 = 26(x - 1)$ point-slope

OR

$y - 80 = 26(x - 4)$ point-slope

OR

$y = 26x - 24$ slope-intercept form

10. Given each $f(x)$, find $f'(x)$. In other words, find the expression for the slope of the tangent line.

A. $f(x) = x^2$

$f'(x) = 2x$

B. $f(x) = x^3$

$f'(x) = 3x^2$

C. $f(x) = -9x^2$

$f'(x) = -9(2x) = -18x$

D. $f(x) = -4x^3$

$f'(x) = -4(3x^2) = -12x^2$

E. $f(x) = 8x^3$

$f'(x) = 8(3x^2) = 24x^2$

C. $f(x) = x^2 + x^3$

$f'(x) = 2x + 3x^2$

#10 is based off of patterns learned in 3.5B Activity

11. Given $f(x) = x^3 + x^2$, find the equation of the tangent line at $(4, f(4))$.

Find a point

$f(4) = 4^3 + 4^2 = 64 + 16 = 80$

Find slope of tangent line

$f(x) = x^3 + x^2$

$f'(x) = 3x^2 + 2x$ $f'(x)$ is a formula to find slope of $f(x)$ at a point

$f'(4) = 3(4)^2 + 2(4)$ $f'(4)$ is the slope of tangent line at $x = 4$

$f'(4) = 3(16) + 8 = 48 + 8 = 56$

Write answer

$y - 80 = 56(x - 4)$ point-slope

OR

$y = 56x - 144$ slope-intercept form

point-slope → only one is needed → slope-intercept form

12. Given $f(x) = x^3 + 5x^2$, find the equation of the tangent line at $(2, f(2))$.

Find a point

$f(2) = 2^3 + 5(2)^2 = 8 + 5(4) = 8 + 20 = 28$

Find slope of tangent line at point

$f(x) = x^3 + 5x^2$

$f'(x) = 3x^2 + 5(2x) = 3x^2 + 10x$

$f'(2) = 3(2)^2 + 10(2) = 3(4) + 20 = 32$

Write answer

$y - 28 = 32(x - 2)$ point-slope

OR

$y = 32x - 36$ slope intercept

point-slope → only one is needed → slope intercept