

## (Exponential) Population Growth

Sometimes things grow. When how fast a thing grows depends on how big the thing is, those things can grow very fast. A good example of this is the story of the inventor of chess asking for 2 grains of rice for the first square of the chess board, and then 4 for the next square, then 8 for the next square... that is, doubling each time:

1	2	4	8	16	32	64	128
256	512	1024	2048	4096	8192	16384	32768
65536	131K	262K	524K	1M	2M	4M	8M
16M	33M	67M	134M	268M	536M	1G	2G
4G	8G	17G	34G	68G	137G	274G	549G
1T	2T	4T	8T	17T	35T	70T	140T
281T	562T	1P	2P	4P	9P	18P	36P
72P	144P	288P	576P	1E	2E	4E	9E

The amount of rice grows unexpectedly fast. In total, there are  $2^{64}$  – 1 grains of rice on the board, which would be a pile of rice *larger than Mt. Everest*.

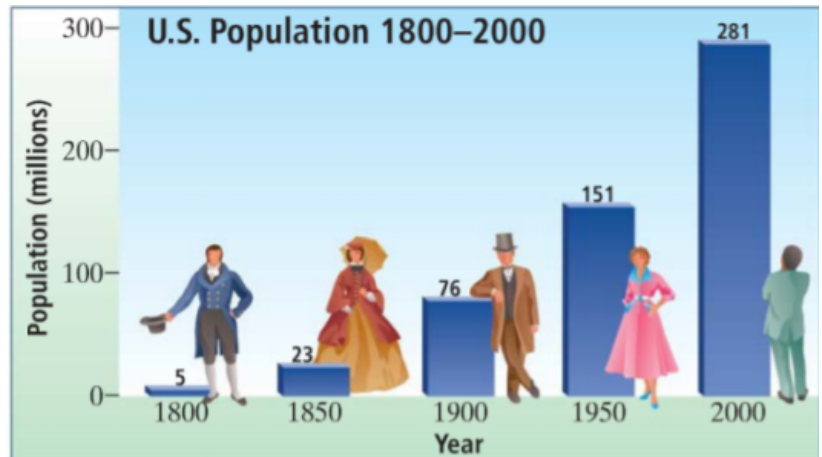
This example has limited practical use - things don't usually grow relative to squares on a chalkboard. They usually grow relative to time. For example, the population of the United States got bigger as the years went by. No surprise there. The population actually grew *faster* the more people there were. (That also shouldn't surprise you: the more people there are, the more babies that can be made, hence the faster the population grows). Here is a graph:

In the year 2000, the population was growing at a rate of 1.24%. That is, if the population had been 10,000, then the population would have grown by 1.24% of 10,000 people:

$$10,000 * 1.24\% = 124$$

so the next year there would have been

$$10,000 + 124 = 10,124 \text{ people.}$$



However, in the year 2000, there were (according to the graph) 281 million people. So in the year 2001, there were  $281,000,000 * 1.24\% + 281,000,000 = 284,484,400$  people.

**Here is how you make a function for population growth:** **first** you need to know (1) the starting population and (2) the rate that population is growing at (call this “a”). In this example, the starting population is 281 million and it is growing at a rate of 1.24%. **Second**, convert the rate to a decimal and add one, call this number “b”. Then plug in to:

$$p(t) = a * b^t, \quad \text{where } t \text{ is the number of years since you start counting.}$$

So in our case,  $b = 1 + 1.24\% = 1 + .0124 = 1.0124$ , and  $a = 281,000,000$ , giving

$$p(t) = 281,000,000 * 1.0124^t, \quad \text{where } t \text{ is “years after 2000.”}$$

name: \_\_\_\_\_

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### Questions:

(1) Using the model from the previous page, if we wanted to know the population in the year 2013, what value for  $t$  should we use?

(2) Plug in that value of  $t$  to the model to find the population in the year 2013 according to the model.

(3) Go to <http://www.census.gov/popclock/> and look at the actual population of the US. What is the actual population of the US in 2013? Did the model over- or under-estimate the population in 2013?

(4) Go to [www.wolframalpha.com](http://www.wolframalpha.com) and type in *population of California*. What was the population in 2012 (the most recent data) and the rate at which the population was growing then?

(5) Follow the instruction on the previous page to make a model for the population of California as a function of years after 2012. That is, plug in the correct  $a$  and  $b$  into:

$$p(t) = a * b^t$$

(6) You will be finishing college around 2022. What does your model predict the population of California will be then?