CALCULUS

Grade 12

Prerequisite: Pre-Calculus Honors; Teacher's Recommendation

5 credits

ABSTRACT

The purpose of this five-credit course is to present the concepts of a full year of college-level *Calculus*. Students begin on an intuitive level and move to a rigorous examination of these concepts. Topics explored include functions, limits, derivatives and techniques of integration. Applications to real world situations in science, engineering and business are stressed. This approach is intended to generate self-confidence in using mathematics in applied programs as well as developing an enthusiasm for further rigorous study of mathematics in college.

This course contains college-level material. Extensive use is made of advanced graphing calculators and computer laboratory lessons are included.

Born on: NJSLS September 2017
Revised and BOE Approved: August 24, 2021
Aligned to NJSLS-Mathematics 2016 & NJSLS-CLKS 2020

UNIT #: *Unit Title* Number of Days	**Unit 1:** *Limits and Derivatives* 50 Days	**Unit 2:** *Integration* 65 Days	**Unit 3:** *Problem-Solving* 50 Days
STAGE 1: DESIRED RESULTS			
ESTABLISHED GOALS: *(NJSLS-Mathematics)*	**Algebra** A-SSE.A.1.a, b A-SSE.A.2 A-SSE.B.3.a-c A-SSE.B.4 A-APR.A.1 A-APR.B.2 A-APR.B.3 A-APR.C.4 A-APR.C.5 A-APR.D.6 A-APR.D.7 **Functions** F-IF.A.1 F-IF.A.2 F-IF.A.3 F-IF.B.4 F-IF.B.5 F-IF.B.6 F-IF.C.7.a-e F-IF.C.8.a, b F-IF.C.9 F-TF.A.1 F-TF.A.2 F-TF.A.3 F-TF.A.4	**Geometry** G-SRT.A.2 G-SRT.C.6 G-SRT.C.7 G-SRT.C.8 G-GMD.A.1 G-GMD.A.2 G-GMD.A.3 G-GMD.B.4 G-MG.A.1 G-MG.A.2 G-MG.A.3 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1	**Algebra** A-CED.A.1 A-CED.A.2 A-CED.A.3 A-CED.A.4 **Functions** F-BF.A.1.a-c F-BF.A.2 F-BF.B.3 F-BF.B.4.a-d F-BF.B.5 F-LE.A.1.a-c F-LE.A.2 F-LE.A.3 F-LE.A.4 F-LE.B.5 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8

	F-TF.B.5 F-TF.B.6 F-TF.B.7 F-TF.C.8 F-TF.C.9 **Number and Quantity** N-RN.A.1 N-RN.A.2 N-RN.B.3 N-Q.A.1 N-Q.A.2 N-Q.A.3 **Geometry** G-GPE.B.4 **Mathematical Practices** MP.1 MP.2 MP.3 MP.4 MP.5 MP.6 MP.7 MP.8 **Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6	8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3	**Technology** 8.1.12.A.3 8.1.12.C.1 8.1.12.D.1 8.1.12.D.2 8.1.12.F.1 8.2.12.E.1 **Career Readiness, Life Literacies, and Key Skills** 9.1.12.PB.1 9.2.12.CAP.6 9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3

	9.4.12.CT.1 9.4.12.CT.2 9.4.12.CT.3		
ENDURING UNDERSTANDINGS: *(Students will understand that . . .)*	• Errors generated to acceptable limits and analytical integration are comparable. • Limits from data and analytical tables are just estimates. • Limits are used for the process to develop the integral. • Properties of first and second derivatives of functions and their graphs are not coincidental • Correlate analytical results to those obtained on graphing calculators.	• Definite integrals are used to determine area and calculate geometric probabilities. • There are different methods of numerical integration. • Compare areas obtained geometrically to those computed by definite integrals. • Use Riemann sums to find the area under a curve. • Use definite integrals to determine area and volume. • Use Sigma notation to represent sequences. • Riemann sums can be used to demonstrate the value of area under a curve when examining total emissions, population and per capita emissions.	• How to apply the techniques of calculus to economic analysis such as profit/loss investigations. • Use calculus to model applications in the physical sciences (e.g. rectilinear motion). • Calculus is a tool for applications in Engineering, Science, Economics and other disciplines. • Optimization problems are solved using absolute and relative extrema. • Solve volume problems. • Utilize sample AP tests/solutions to prepare for the AP exam. • Use the Internet to obtain information and solve problems.
ESSENTIAL QUESTIONS: *(What provocative questions will foster inquiry, understanding, and transfer of learning?)*	• What does the limit represent? • What is the difference quotient? • How do you construct the difference quotient for a given function and simplify the resulting expression? • What does the first and second derivative test tell us about the graph of a function?	• What is the difference between a definite and indefinite integral? • How do you construct an integral from the limit of the addition of a sequence? • How do you approximate the area under a curve geometrically by constructing a finite number of rectangles and calculating the total area in those rectangles?, • How do you compare two different approximations of area under a curve by using a different number of rectangles?	• Is there a way to visualize what a derivative is? • What does the graph of a function tell about the equation? • How can calculus be used to solve problems in business and economics? • What methods involving integrals can be used to find the volume of a solid? • How can the concept of limits be applied in mathematics?

		• How can we use antiderivatives to model variables in climate change?	• How do the graphs of the first and second derivatives relate to the function graph? • How is the rate of change reflected in its table and graph?
STAGE 2: ASSESSMENT EVIDENCE			
PERFORMANCE TASKS: *(Through what authentic performance tasks will students demonstrate the desired understandings?) (By what criteria will performances of understanding be judged?)*	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning. Independent assignments Written responses Real-world problems Non-routine problems	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning. Independent assignments	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning. Independent assignments

	Collaborative problem- solving Write own problems and assessments The Match Game (A) Each student is given a note card with the graph of f, f', or f". Students work with others to find the other two students holding cards that are related to their own. Each member of the group explains their graph and the relationship to the other two group members.	Written responses Real-world problems Non-routine problems Collaborative problem- solving Write own problems and assessments Land Assessment. Students will estimate the area beneath a curve using the Riemann sum, the RAM program and the definite integral. They will have to be able to compare and discuss the different methods.	Written responses Real-world problems Non-routine problems Collaborative problem- solving Write own problems and assessments Students readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
OTHER EVIDENCE: *(Through what other evidence (e.g. quizzes, tests, academic prompts, observations, homework, journals) will students demonstrate achievement of the desired*	• Pre/Post-assessments • Quizzes • Unit tests • Multiple choice problem • Teacher observations • Math journals • Think-Alouds • Homework	• Pre/Post-assessments • Quizzes • Unit tests • Multiple choice problem • Teacher observations • Math journals • Think-Alouds • Homework	• Pre/Post-assessments • Quizzes • Unit tests • Multiple choice problems • Teacher observations • Math journals • Think-Alouds • Homework

results?) *(How will students self-assess their learning?)*	• Discussions • Open-ended problems • Peer/Self Evaluation Rubrics • At the end of each chapter in the unit, checkpoint assessments will be administered. • Benchmark assessments • LinkIt!	• Discussions • Open-ended problems • Peer/Self Evaluation Rubrics • At the end of each chapter in the unit, checkpoint assessments will be administered. • Benchmark assessments • LinkIt!	• Discussions • Open-ended problems • Peer/Self Evaluation Rubrics • At the end of each chapter in the unit, checkpoint assessments will be administered. • Benchmark assessments • LinkIt!
RESOURCES:	• Calculus Early transcendentals - Eighth Edition Single Variable • Chromebooks • www.derivative-calculator.net • www.desmos.com • www.savvasrealize.com • Tiered Worksheets • Research in the Media Center • Educational websites • Educational software • Overhead TI-89 calculator • Advanced Graphing Calculator (TI-89 or equivalent) • Computer Laboratory Time	• Calculus Early transcendentals - Eighth Edition Single Variable • Khanacademy.org to find instructional videos on processes of integration and finding area between curves • Desmos.com to find the intersection of two graphs • Youtube.com • Chromebooks • www.derivative-calculator.net • www.desmos.com • www.savvasrealize.com • Tiered Worksheets • Research in the Media Center • Educational websites • Educational software • Overhead TI-89 calculator • Advanced Graphing Calculator (TI-89 or equivalent) • Computer Laboratory Time	• Calculus Early transcendentals - Eighth Edition Single Variable • Chromebooks • www.derivative-calculator.net • www.desmos.com • www.savvasrealize.com • Youtube.com • Tiered Worksheets • Research in the Media Center • Educational websites • Educational software • Overhead TI-89 calculator • Advanced Graphing Calculator (TI-89 or equivalent) • Computer Laboratory Time
STAGE 3: LEARNING PLAN			

SKILLS AND TOPICS: *(What specific activities will students do and what skills will students know as a result of the unit?)*	• Find limits graphically, numerically, and analytically: • Limits at a point • One-sided limits • Infinite limits • Apply the Limit Definition of the Derivative • Find the derivative of a function using the following methods: Power, Product, Quotient, Chain, and Implicit Differentiation • Apply the concept of derivative to solve related rates word problems. • Find extrema on open and closed intervals. • Find points of discontinuity. • Find intervals where a function is increasing, decreasing or constant. • Apply The First Derivative Test. • Apply The Second Derivative Test. • Apply Rolle's Theorem. • Apply the Mean Value Theorem • Apply various tests and knowledge of increasing, decreasing, extrema and concavity to sketch complex curves without using a calculator.	• Find the antiderivative of a function. • Find the area under a curve using definite integrals. • Approximate the area under a curve using Riemann Sums: inscribed rectangles, circumscribed rectangles, and midpoint • Approximate the area under a curve using the Trapezoidal Rule and determine the error. • Apply The Fundamental Theorem of Calculus. • Apply The Second Fundamental Theorem of Calculus. • Integrate using the u-substitution method. • Apply concepts of differentiation and integration to: Logarithmic Functions, Exponential Functions, and Inverse Functions • Solve differential equations. • Find the area of a region between two curves. • Find the volume of a three dimensional solid of revolution using: the disk, washer, and shell methods	• Apply concepts of extreme values to solve optimization word problems. • Solve problems involving growth and decay. • Find the area of a known cross-section. • Find the volume of a solid given the base and cross-section. • Apply L'Hopital's rule.
CROSS-CURRICULAR / DIFFERENTIATION: *(What cross-curricular (e.g. writing, literacy, math, science, history, Career Readiness, Life Literacies, and Key Skills, technology) learning activities are included*	**English Language Arts** **(RI.11-12.1, W.11-12.1.B)** • Solving word problems, translating, explanations during problem solving, understanding vocabulary regarding approaching infinity **Social Studies**	**English Language Arts** **(RI.11-12.1, W.11-12.1.B)** • Solving word problems, translating, explanations during problem solving **Science** **(HS-ESS3-6)** • Problem solving, scientific notation and applications	**English Language Arts** **(RI.11-12.1, W.11-12.1.B)** • Solving word problems, translating, explanations during problem solving **Science** **(HS-ESS3-6)** • Problem solving, scientific

in this unit that will help achieve the desired results?) (What type of differentiated instruction will be used for Sp.Ed./504, ELL, G&T, At-Risk students?)	**(6.3.12.A.2 , 6.3.12.D.2; 6.2.12.C.6.b)** ● Current event problems ● Reading and interpreting graphs, economic applications ● Tiered Lessons ● Project Based Learning ● Multimedia presentations to show ending behavior ● Open-ended responses ● Conclusions and analysis of exploratory activities <u>Special Education/504</u>: ● Allow students to use notes ● Provide students with whiteboard to graph functions ● Provide modifications and accommodations as listed in the student's IEP/504 plan ● Position student near helping peer or have quick access to teacher ● Modify or reduce assignments/texts by creating partial graphs when graphing limits ● Reduce length of assignment for different mode of delivery ● Increase one-to-one time ● Utilize working contract between you and student at risk ● Prioritize tasks ● Provide manipulatives, graphic organizers and math journals to keep track of formulas and	**Social Studies** **(6.3.12.A.2 , 6.3.12.D.2; 6.2.12.C.6.b)** ● Current event problems ● Reading and interpreting graphs, economic applications ● Tiered Lessons to adhere to different levels in the process of integration ● Project Based Learning ● Multimedia presentations ● Open-ended responses ● Conclusions and analysis of exploratory activities such as visualizing the area under a curve <u>Special Education/504</u>: ● Provide modifications and accommodations as listed in the student's IEP/504 plan ● Position student near helping peer or have quick access to teacher ● Modify or reduce assignments/texts ● Reduce length of assignment for different mode of delivery ● Break down tasks of integration to separate questions to allow students to complete one task before moving on to another ● Utilize working contract between you and student at risk ● Prioritize tasks ● Provide manipulatives ● Use graphic organizers to show various methods of integration that	notation and applications **Social Studies** **(6.3.12.A.2 , 6.3.12.D.2; 6.2.12.C.6.b)** ● Current event problems ● Reading and interpreting graphs, economic applications ● Tiered Lessons ● Project Based Learning ● Multimedia presentations ● Open-ended responses ● Conclusions and analysis of exploratory activities <u>Special Education/504</u>: ● Provide modifications and accommodations as listed in the student's IEP/504 plan ● Position student near helping peer or have quick access to teacher ● Modify or reduce assignments/texts ● Reduce length of assignment for different mode of delivery ● Increase one-to-one time ● Utilize working contract between you and student at risk ● Prioritize tasks ● Provide manipulatives ● Use graphic organizers ● Use interactive math journals ● Use online resources for skill building ● Provide teacher notes outlining

	techniques of differentiation (i.e. product rule, quotient rule, behavior of limits) ● Use online resources for skill building and allow students to see notes of processes of differentiation and graphing) ● Use collaborative grouping strategies such as small groups to reinforce the process of differentiation ● NJDOE resources ELL: ● Place student next to same-language speaker, if possible ● Vocabulary aides: limits, derivatives ● Provide text to speech for math problems ● Provide ● Use of translation dictionary or software for differentiation and optimization ● Implement strategy groups ● Continually have progress update with students and teachers on a weekly basis ● Provide graphic organizers ● Allow students to use notes and examples ● Modification plan ● NJDOE resources ● Adapt a strategy-adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm ● Instruction will be based on language proficiency.	still arrive at the same core answer ● Use interactive math journals ● Use online resources for skill building ● Provide teacher notes on google classroom that students can print out and use in class and at home ● Use collaborative grouping strategies such as small groups ● Use online resources ● NJDOE resources ELL: ● Work with ELL teacher to ascertain difficulties students may have with integration at the advanced level especially with word problems ● Place student next to same-language speaker, if possible ● Vocabulary aides; integrals ● Provide text to speech for math problems ● Use of translation dictionary or software to help explain finding area between curves ● Implement strategy groups ● Confer frequently ● Provide graphic organizers ● Modification plan ● NJDOE resources ● Adapt a strategy-adjusting strategy for ELL: ● http://www.teachersfirst.com/content/esl/adaptstrat.cfm ● Instruction will be based on language proficiency. At Risk: ● Tiered interventions following RtI framework	how to set up and solve word problems ● Use collaborative grouping strategies to form a plan to relate to real world situations ● Use online resources to find similar problems that can be applied to problem solving with Calculus ● Break down word problems into smaller tasks that can be verified by the teacher before moving on ELL: ● Assign a partner to explain words that may not exist in a different language ● Place student next to same-language speaker, if possible ● Provide text to speech for long word problems and provide diagrams ● Use of translation dictionary or software ● Implement strategy groups ● Confer frequently ● Provide graphic organizers ● Modification plan ● NJDOE resources ● Adapt a strategy-adjusting strategy for ELL: http://www.teachersfirst.com/content/esl/adaptstrat.cfm ● Instruction will be based on language proficiency. At Risk: ● Tiered interventions following RtI

	At Risk: ● Tiered interventions following RtI framework ● Rtl Intervention Bank ● Use additional practice and textbook RTI resources ● NJDOE resources ● Create weekly check-ins outside class ● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org ● Provide students with opportunities to create projects such as ones that illustrate what the first and second derivative tell us about the graph of a function Gifted and Talented: ● Process should be modified: higher-order-thinking skills, open-ended thinking, discovery ● Utilize project-based learning for greater depth of knowledge and assign students to aid struggling students ● Utilize exploratory connections to higher grade concepts ● Contents should be modified: abstraction, complexity, variety, organization ● Learning environments should be modified: student-centered learning, independence, openness, complexity, groups varied ● Use of web-based resources such as http://www.tenmarks.com www.khanacademy.org	● Rtl Intervention Bank ● Use additional practice and textbook RTI resources ● NJDOE resources ● Create weekly check-ins outside class ● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org ● Keep in contact with parents of students and track their progress, noting each communication log. Involve guidance and other teachers as necessary Gifted and Talented: ● Process should be modified: higher-order-thinking skills, open-ended thinking, discovery ● Utilize project-based learning for greater depth of knowledge ● Utilize exploratory connections to higher grade concepts ● Contents should be modified: abstraction, complexity, variety, organization ● Learning environments should be modified: student-centered learning, independence, openness, complexity, groups varied ● Use of web based resources such as http://www.tenmarks.com www.khanacademy.org geogebra.org http://www.wolframalpha.com/calculators/integral-calculator/ ● NJDOE resources ● Assign projects that demonstrate real-world uses for integration to find area	framework ● Rtl Intervention Bank ● Use additional practice and textbook RTI resources ● NJDOE resources ● Create weekly check-ins outside class ● Utilize online resources such as http://www.tenmarks.com or www.khanacademy.org ● Have students keep a journal of their progress and which questions need to be addressed in and out of class Gifted and Talented: ● Process should be modified: higher-order-thinking skills, open-ended thinking, discovery ● Utilize project-based learning for greater depth of knowledge ● Utilize exploratory connections to higher grade concepts ● Contents should be modified: abstraction, complexity, variety, organization ● Products should be modified: real-world problems, audiences, deadlines, evaluation, transformations ● Learning environments should be modified: student-centered learning, independence, openness, complexity, groups varied ● Use of web-based resources such as http://www.tenmarks.com www.khanacademy.org geogebra.org desmos.com ● NJDOE resources

	geogebra.org ● NJDOE resources ● Allows students to explore topics on their own related to this unit and present to the rest of the class. Encourage links to other subjects		● Students can be assigned as tutors to help other students with difficult topic areas