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Collatz Theory

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Abstract

Much of Collatz theory is based around a conjecture referred to as the Collatz conjecture, which is an unproven conjecture that states that for a value of *n* if *n* is an odd integer, then multiply by three and add one while if the value is an even number divide by two. The conjecture states that if these steps are followed for a continuous amount of time, the value will eventually reach one, which is known as the oneness property of numbers. In this paper, the Collatz Theory will be described with similar properties as the Collatz conjecture and an equation known as the Collatz Matrix equation.

Introduction

Collatz Theory revolves around the simple idea of integer properties and the properties of different numbers, both on real and complex planes. These properties, in relation with divisors and results of numeral properties, show a deep relation when presented on a multi-dimensional equation set. A multi-dimensional equation represents an evolving equation that is continuous based on the size of the matrix solution. Many properties of numbers appear when applied to a Collatz Matrix equation and are viewed in the perspective of Collatz Theory.

A Collatz Matrix equation is an equation that results in a variant amount of solutions. The following is the notation to be used for a Collatz Matrix equation.

$$\frac{\alpha}{\omega} = C(x)_{k \times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t), x = \frac{3a-1}{2\omega}$$

In these equations, α is the result of the Collatz Matrix equation, which its denominator is represented as ω. $C(x)_{k \times d}$ is the initiation of the equation, where the variable x is to be modified through the rules or parameters within the brackets. $s(k_p, d_p)$ refers to the initiating point within the matrix solution that the variable x lies. $S(k_t, d_t)$ retrieves the value at a specific coordinate within the matrix solution, which in turn becomes the value for α.

This is only the standard Collatz Matrix equation. There is also a complex Collatz Matrix equation, which is the following.

$$\frac{\alpha}{\omega} = \acute{C}(x)_{k \times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t), x = \frac{3a - \omega}{2\omega}$$

These equations derive the same notations and parameters except the notation of $\acute{C}(x)_{k \times d}$. This notation implies that both negative and decimal-form values can exist in a complex Collatz Matrix. In a Collatz Matrix equation and for the Collatz solution, for the value of x if a value is to be placed adjacent, above the initial value of x in the Matrix solution, the value would be divided by two. If a value were to be placed adjacent to and below the initial value of x, the value would be equal to the initial value of x multiplied by two. If a value were to be placed adjacent to and left of the initial value of x, then the the value would be equal to the variable x minus one and divided by three. If a value were to be placed adjacent to and right of the initial value of x, then the value would be equal to three times the value of x plus one.

Here is an example below to show the process.

$$-\frac{1}{1} = \acute{C}(x)_{k \times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t), x = -2$$

$$\acute{C}(-2)_{3\times 3} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & -2 & -5 \\ 0 & -4 & 0 \end{bmatrix}$$

Solution for x

It can be seen that within the Collatz Matrix equation there is a way to solve for the x value for the quadratic equation that is formed by finding the determinant of the rules or formulas within the brackets. In order to solve for *x*, α and ω must be known. If they are known, to solve for x, the two variables must be inputted into the equations for *x*.

$$x = \frac{3\alpha + \omega}{2\omega}$$

The variable *x* displayed here is different than the input. This *x* variable represents the solutions to the determinant of the formula rules within the Collatz Matrix equation.

$$y = \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \end{Bmatrix}$$

$$y = (x^2 \times \frac{x}{2}) - (\frac{x-1}{3} \times (3x+1))$$

$$y = \frac{2x+1}{3}$$

Though this is the quadratic equation that is a result of the determinant of the parameters within the Collatz matrix equation, there is no variable *y* that is the answer to the equation. Instead of y, the following would be the result.

$$D(k_t, d_t) = \frac{2x+1}{3}$$

Just as the x variable works, $D(k_t d_t)$ does not just contain one solution. It carries multiple solutions for various matrix solutions that can be produced by a Collatz Matrix equation. With multiple solutions,$D(k_t d_t)$ also equals $\frac{\alpha}{\omega}$, where ω is the denominator of α. This also would mean that ω is the denominator of $D(k_t d_t)$.

2x2 Collatz Matrix solution conjecture

A Collatz Matrix equation that is to have matrix solutions that have the dimensions of a width and height of two, the following Collatz Matrix equation would express these two solutions.

$$C(x)_{2\times2}\begin{Bmatrix}\frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x\end{Bmatrix}, s(1,1)$$

Where the matrix solutions for this Collatz Matrix equation would be the following.

$$C(x)_{2\times2} = \begin{bmatrix}x & a \\ 0 & b\end{bmatrix}, \begin{bmatrix}x & 0 \\ a & b\end{bmatrix}$$

Where the solutions of each matrix solution a and b are results of the variable x.
This conjecture only applies to 2×2 matrix solution, as stated in the conjecture name. It also only applies to equations where $s(1, 1)$.

This conjecture also ma

kes an analysis on the values of variables a and b. For the matrix solutions of x and $x + 1$, the value of a in the matrix solution located in the first matrix solution of $x + 1$ will always be three greater than the first matrix solution's a variable. However, the value of a will increase for every addition of n in $x + n$.

Now, for every odd value of x beginning at $x = 1$, comparing the value of a of the first matrix solution to the first matrix solution of the value of a in the first matrix solution of $x + 2$, the value of a of the first matrix solution will always be a Collatz number.

Collatz Numbers

Collatz Numbers are a set of numbers that are found while carrying out Collatz operations with any number that is a whole number. Here is a list of some Collatz numbers:

$$4, 10, 16, 22, 28, 34...$$

Any Collatz number can be found taking any odd value x and carrying out the formula $3x + 1$. Since one Collatz number is six more than the previous Collatz number, this means that subtracting any Collatz number three continuously will have the Collatz number eventually equal one.

Collatz Number Conjecture

As established with the rules of Collatz numbers, subtracting any Collatz number three continuously will have the Collatz number eventually equal one. This means that all odd numbers that are turned into Collatz Numbers would be able to have this oneness property automatically. However, even numbers act differently. The Collatz transformation, or applying the $3x + 1$ rule, for even numbers would have to be applied after two times in order to become Collatz numbers. This means that all numbers will eventually equal one when applied to the Collatz algorithm. However, this assumes that any number will eventually equal to a Collatz number with the Collatz algorithm.

Fundamental theorem of arithmetic and Collatz Number Conjecture

The fundamental theorem of arithmetic states that all numbers can be broken down into an *n* amount of prime factors. For example, the value of 100 can be broken down into the following prime number factors.

$$2 \times 2 \times 5 \times 5 = 100$$

As seen with these prime factors, there are two even factors and two odd factors. As described with the Collatz Number conjecture, the even numbers have to be multiplied by three and added to one twice in order to become a Collatz number, while the odd factors only have to go through this process once. Since all prime numbers are odd, except for 2, this means that all numbers can become Collatz Numbers because they have the build up of prime numbers that can become Collatz numbers.

Odd and Even Collatz Numbers

Collatz Numbers, as defined within this paper, are numbers that subtracted by six until they equal four(though this definition is broad, it works as a beginning point). Now, as explored before, Collatz numbers can be found by taking any odd positive integer and multiplying it by three and adding one. However, for any even positive integer that has this process carried out upon it will not become a Collatz number, but when the process is carried out again, it does become a Collatz Number. This is why there is the existence of both odd and even Collatz Numbers. Simply put, in order to list the Collatz numbers the very first positive integer of the Collatz Number list for odd or even must be added to six. Here is the comparison between odd and even Collatz Numbers.

$$4, 10, 16, 22, 28, 34, 40...$$

$$7, 13, 19, 25, 31, 37, 43...$$

This only applies to the positive spectrum of a number line. The Collatz numbers are different for the negative spectrum of the number line.

$$-2, -8, -14, -20, -26, -32, -38...$$

$$-5, -11, -17, -23, -29, -35, -41...$$

Multiple Solution equations

With various matrix solutions, that means there are multiple solutions for *x* in the quadratic determinant, not just two solutions. For example, if $s(2,3) = 4$ for a 4 × 4 collatz matrix equation, the following would be possible matrix solutions for $D(2,1)$.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 3 & 10 \\ 0 & 0 & 6 & 20 \\ 0 & 4 & 13 & 40 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 3 & 0 \\ 0 & 0 & 6 & 19 \\ 0 & 4 & 0 & 38 \\ 0 & 8 & 25 & 76 \end{bmatrix}$$

Other solutions could be as follows.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 13 & 40 \\ 0 & 0 & 0 & 80 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 17 \\ 1 & 4 & 11 & 34 \\ 2 & 7 & 22 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 4 & 11 & 34 \\ 2 & 7 & 22 & 68 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 7 & 0 \\ 0 & 4 & 14 & 0 \\ 0 & 9 & 28 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 7 & 0 \\ 0 & 4 & 14 & 0 \\ 0 & 0 & 28 & 85 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 7 & 0 \\ 0 & 4 & 14 & 43 \\ 0 & 0 & 0 & 86 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 11 \\ 0 & 2 & 7 & 22 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 2 & 8 & 0 \\ 0 & 4 & 16 & 0 \\ 0 & 0 & 32 & 97 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 2 & 8 & 0 \\ 0 & 4 & 16 & 49 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 2 & 8 & 25 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 4 & 13 \\ 0 & 2 & 0 & 26 \\ 0 & 4 & 17 & 52 \\ 0 & 11 & 34 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 & 13 \\ 0 & 2 & 0 & 26 \\ 0 & 4 & 17 & 52 \\ 0 & 0 & 34 & 103 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 19 \\ 0 & 4 & 12 & 38 \\ 0 & 8 & 25 & 76 \end{bmatrix}$$

Since all other solutions resulted in $D(2,1)$ equaling zero, the following would be the solution for α.

$$\alpha = 0, 1$$

Much of the time, the initial position of the value within the matrix solution will determine how many solutions there will be available for the variable α. For example, if the initial value begins at $s(1,1)$ there would more likely be many more solutions.

Since the complex Collatz Matrix equations accept decimal and negative solutions, there are multiple solutions for a complex Collatz Matrix equation. However, all complex Collatz Matrix equations with the same $n \times n$ size will have the same amount of solutions.

$$\frac{\alpha}{\omega} = \acute{C}(x)_{k \times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t), x = \frac{3a - \omega}{2\omega}$$

Here is an example when the variable $x = 5$ and $k = 3,\ d = 3$.

$$\begin{bmatrix} \frac{1}{2} & \frac{5}{2} & \frac{11}{2} \\ 1 & 5 & 11 \\ 2 & 7 & 22 \end{bmatrix}$$

Though there are more matrix solutions, this matrix solution is going to be what is focused on right now. If $\frac{2x+1}{3} = D(2,1)$, then $\frac{2x+1}{3} = \frac{5}{2}$. Now, the variable x needs to be solved for to find the solutions for the Collatz Matrix equation. In this case, the solution for x is $\frac{13}{4}$.

Collatz Series and the Infinite

A Collatz series is a Collatz Matrix equation that involves infinite sums, where the multiple solutions of a Collatz Matrix equation are added together to make one matrix solution. Here is what a Collatz series would look like.

$$P_{k\times d} = \sum_{n=1}^{\infty} C(n)_{k\times d} \begin{Bmatrix} \frac{n}{2} & \frac{n-1}{3} \\ 3n+1 & 2n \end{Bmatrix}, s(k_p, d_p)$$

With these type of equations, the initial value will not equal the original initial value set up in the equation. Therefore, there needs to be an equation to determine the amount of solutions within a given Collatz Matrix equation and then use that value to determine the starter value for the final solution.

$$s(k_p, d_p) = S_c n$$

However, this series notation denotes a system of matrix solutions that make the problem have more matrix solutions to define the equation. With integration, there is only one solution involved, which the following example displays.

$$k = \int_1^{\infty} \frac{3C(x)_{k\times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t)\ dx}{3x^4 - 2x^3 - x^2}$$

Collatz integral equations will not have the same graphical system as regular equations will. This is due to how Collatz-Matrix equations, in general, have multiple matrix solutions. The same integration can be applied to three and four dimensional Collatz Matrix equations.

$$l = \int_1^{\infty} \frac{6C(x)_{k\times d\times z} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \\ \frac{xi}{2} & \frac{xi-i}{3} \end{Bmatrix}, s(k_p, d_p, z_p), D(k_t, d_t, z_t)\ dx}{-3x^6 + 5x^5 - x^4 - x^3}$$

The default notation for a Collatz integral would be the following.

$$k = \int_{1}^{\infty} \frac{mC(x)_{k \times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t)\ dx}{n}$$

In this notation, *m* refers to the dimensional factor and *n* refers to the dimensional equation that develops the dimensional factor. The dimensional factor can be determined from the dimensional equation by using the following notation.

$$d_f = \frac{n}{m}$$

Dimensions have an important role of the integral Collatz Matrix equation. The integration makes each equation of each dimension unique to its dimension. This is called the dimensional identity. There are also multi-dimensional integral Collatz equations, which would look like the following.

$$l = \iint_{1}^{\infty} \frac{3C(x)_{k \times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3y+1 & 2y \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t)\ dx\ dy}{3x^2y^2 - 3xy^2 + x^2y - xy}$$

Since this type of integral Collatz equation is different than other Collatz-Matrix equations, this integral Collatz-Matrix equation will have two matrix solutions for each matrix solution; there will be the *x* and *y* matrix solution for each matrix solution. The *x* and *y* matrix solutions are dependent on each other because the position of each element is based on each other. For example, the following would be a matrix solution for the Collatz-Matrix equation shown above and if *x* equals three, the dimensions are 3×3, and if $s(0, 0)$.

$$x_{k\times d} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 13 & 0 & 0 \end{bmatrix} \; y_{k\times d} = \begin{bmatrix} 0 & 10 & 0 \\ 0 & 20 & 0 \\ 0 & 40 & 0 \end{bmatrix}$$

Of course, there are not only infinite integral Collatz equations, but also converging Collatz equations, which would look like the following. This means that the approaching value will be compared to the input of *x*.

$$l = \int_1^x \frac{3C(x)_{k\times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t)\ dx}{3x^4 - 2x^3 - x^2}$$

$$l = \int_1^x \frac{6C(x)_{k\times d\times z} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \\ \frac{xi}{2} & \frac{xi-i}{3} \end{Bmatrix}, s(k_p, d_p, z_p), D(k_t, d_t, z_t)\ dx}{-3x^6 + 5x^5 - x^4 - x^3}$$

$$l = \iint_1^x \frac{3C(x)_{k\times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3y+1 & 2y \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t)\ dx\ dy}{3x^2y^2 - 3xy^2 + x^2y - xy}$$

There can also be analysis on the i_x solutions for Collatz-Matrix equations. The equation for this would look like the following.

$$l = \iint_1^x \frac{3C(x)_{k\times d} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3y+1 & 2y \end{Bmatrix}, s(k_p, d_p), D(k_t, d_t), i_x\ dx\ dy}{3x^2y^2 - 3xy^2 + x^2y - xy}$$

Numeral Evolution

Though such a simple concept, Collatz Theory also brings up the ideas of evolution and evolutionary models that exist in nature. With the previous example, there was three matrix solutions, each one being very unique, but similar to each other. Though not all the matrix solutions were presented, a basic understanding of the evolutionary model shows a deep mathematical understanding of naturalistic systems involving trait differentiations based on the environmental changes involved in the development or changes of a habitat or system. For example, in the matrix solutions the dimensions of the matrices involved was 4×4. Many factors are involved in the evolutionary growth of a system, including the initial variance and the derivative changes that occur constantly within a system, in this case, an environment. Of course, as the dimensions of a "virtual habitat" increase, the complexity of the possibilities increase as well. In the example above, the initial condition was of $x = 4$. Not only does the size of the habitat predict the inherent outcome, but the initial value has an effect on the outcome as well. The following evolutionary mathematical model represents the example talked about.

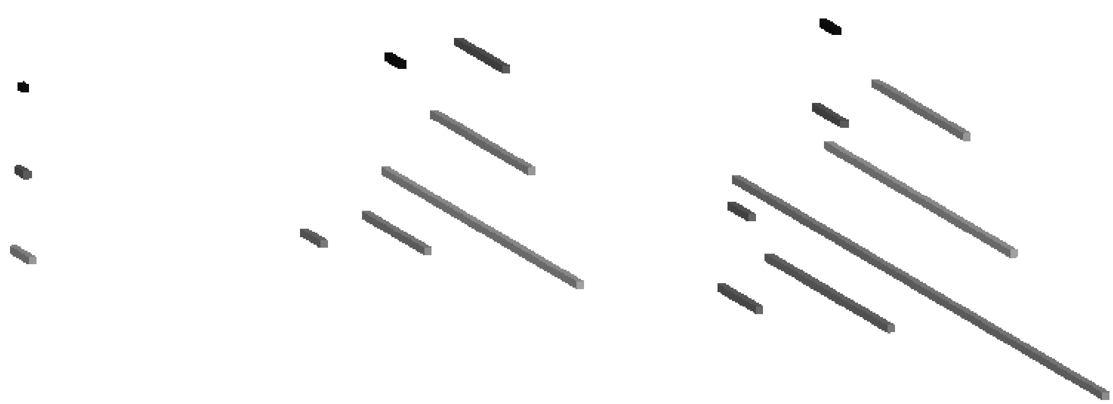


Figure 1 - Evolutionary model, each bar representing traits

If this model is to represent a species within an environment, each trait is represented by a block within a matrix. As seen within Figure 1, one trait on the bottom-right of the model continues to grow in the environment or system as it is the trait unique to it. However, because of a variance in the growth other traits spawn from based on the change in the system. This model represents the different traits that are developed over-time based on the possibilities available for the species. The species can be represented by the initial value of *x* within a Collatz Matrix equation. Figure 2 will give a better understanding the separation between the dormant/minor, unvaried, and identity traits.

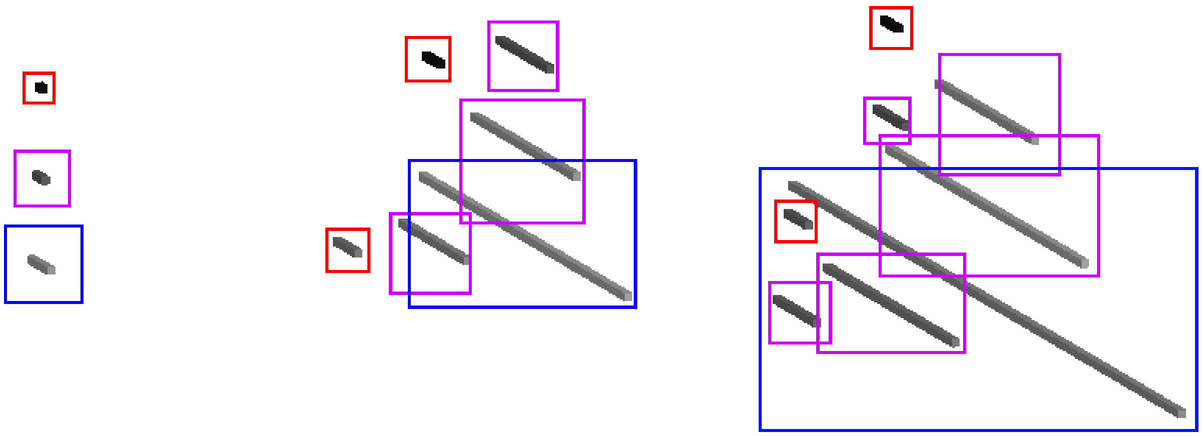


Figure 2 - This diagram identifies the different types of traits

As seen within Figure 2, there are common traits found within every evolved state of the Collatz model. The first Collatz model shows a primitive form of a species with only three common traits, which consist of the dormant/minor trait, unvaried trait, and the identity trait. The next model shows how other traits form from these three specific traits(of course this is variant on the area of the system). In the models, it can be seen that very few dormant/minor traits are found(only one is found in the first model, in the other two models only two are found), while there are is a major increase in unvaried traits. While this is the case, the identity trait is larger compared to the other traits, therefore there is no increase of identity traits. The identity trait only increases in size, unless a variance occurs within the system of the model.

Within a Collatz model, there will only consist of one identity trait while the other traits will vary depending on the initial value. However, the Collatz model will most likely consist of mainly variant traits, which are traits that can be found in most species, such as a sense of smell or sense of sight. Dormant/minor traits will consist of hidden traits within species. Though a species will never completely lost a specific trait the trait will become either a minor trait or a dormant one. The initial trait, or initial point of evolution, can be a minor/dormant trait depending on its size ratio compared to the other traits found within the Collatz model.

These Collatz models do not only represent evolutionary systems. They also can represent uniqueness between two or more minds or forms of intelligence. For example, when there are twins born, these two people have very similar genetic traits. However, their mind is what makes them unique to each other. These unique traits are the Identity trait of each mind.

Evolutionary Collatz Sets/Subsets

With the Collatz models given above, it can be seen that there are traits that arise that are similar to other traits that existed before. These are known as set/subset traits. Within Collatz matrix solutions, each solution has a similar set of value if those paths were taken, however the values are off an *n* amount. However, in two of the solutions, the value 3 is located in both solutions. If each solution, in order of matrix solutions, were to be identified with the variables of A, B, and C the following would be the case.

$$B_{4\times4}(3,1) \in C_{4\times4}$$

$$C_{4\times4}(3,1) \in B_{4\times4}$$

Though the matrix solutions *B* and *C* are not completely the same, they have similar traits. This asserts a specific idea of how certain traits may appear in two different breeds within a species, showing the relationship between habitual environments, whether in similar or different environments or systems. These subsets can help organize certain matrix solutions, or "breeds" into families of traits. For example, for the example presented, there are 16 matrix solutions. However, many of the matrix solutions can be grouped up into families depending on their similarities. The following family sets, if each matrix solution is labeled matrix *A-P*.

$$A_f\ \{A_{4\times4}, G_{4\times4}, H_{4\times4}, I_{4\times4}, J_{4\times4}, K_{4\times4}, L_{4\times4}, M_{4\times4}, N_{4\times4}, O_{4\times4}\}$$

$$B_f\ \{B_{4\times4}, D_{4\times4}\}$$

$$C_f\ \{C_{4\times4}, P_{4\times4}\}$$

$$D_f\ \{E_{4\times4}, F_{4\times4}\}$$

$$\{A_f, B_f, C_f, D_f\} \subseteq S$$

With this Collatz Matrix equation, there are four "families" that exist for the amount of matrix solutions. For a two dimensional Collatz matrix equation, there can only be a maximum of four families, however there are not always four families for every Collatz matrix equation. Some Collatz Matrix equations may only consist of three or two families. For example, prime values, except two, will only have two families. The prime value two, however, has only three families.

Though there are only four families of this Collatz Matrix equation, this does not mean there are only four species. In fact, this Collatz Matrix equation has a total of 16 species. The A_f family consists of ten species while the other families only consist of two species. This means that the A_f family is the dominant family over all of the other families. This dominant family is known as the Identity-Base family, which means it is the set of species that establishes the unique trait among the species.

Similar observations can be made with the traits of specific species within families. Each species is defined by the unique traits that develop it. For example, though family A_f may consist of ten species, this does not mean that these ten species are the same. However, species in the same family consist of a similar origin. In family A_f, each species being the subset of this family all have an {4, 2} origin subset.

Within families, species can also be grouped into subfamilies, defining origin point from multiple factors involved. For example, in family A_f all the species and be classified into different categories within that family. Here would be how the species would be classified within A_f.

$$\alpha_s\left\{G_{4\times4}, H_{4\times4}, I_{4\times4}, J_{4\times4}\right\}$$

$$\beta_s\left\{A_{4\times4}, K_{4\times4}, L_{4\times4}, M_{4\times4}, N_{4\times4}, O_{4\times4}\right\}$$

$$\left\{\alpha_s, \beta_s\right\} \subseteq A_f$$

Many species can be broken down into smaller and smaller categories, however for the sake of presenting a system of classification there will only be subfamilies presented here.

Duality of Collatz Functions

Within functions, there is always the derivative of change, where there is constant motion between functions within a real time situation. This activity of change can bring up the idea of duality Collatz functions or the Duality of a Collatz function. This does not mean there are merely two Collatz functions competing or working together within a specific area of the environmental factors of the matrix solutions, but there are multiple working in partnered pairs in the evolutionary function or set of Collatz matrix equations. For example, again using the example from before, let there be two matrix solutions presented.

$$\begin{bmatrix} 0 & 0 & 0 & 11 \\ 0 & 2 & 7 & 22 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 3 & 10 \\ 0 & 0 & 6 & 20 \\ 0 & 4 & 13 & 40 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

...where $\Delta(3, 2) =- 1$

Within these two matrix equations at the coordinate *(3, 2)* the values are different or changed because, obviously, the two systems or structures are different. However, these two values are similar. If trying to find the average between the two values the average would be 6.5. This can relate the two matrix solution together with either one or two Collatz matrix equations. However, it is known that these two matrix equations came from the same Collatz matrix equation, therefore they are of the same origin. Since, however, multiple matrix solutions from multiple Collatz matrix equations can have similarities with their trait values, it is more difficult to trace the original Collatz matrix equation that the matrix equation was derived, making the Collatz matrix equation a one-way function.

Measurement of Evolutionary Systems

The measurement of speed of system evolution is important to finding how long it takes for a Collatz Matrix equation to develop each matrix solution of each equation. This measurement can help get a better understanding of how specific values react in specific factors, such as area of matrix and the location of it within the matrix condition. The evolutionary speed of growth can be measured with the following equation.

$$E_g = \frac{F(s_1 + s2 + s3 + s4)}{4t}$$

In this equation, Eg is the measurement of evolutionary speed of growth and F is the amount of families within a Collatz Matrix equation multiplied by the amount of species within each family divided by the total time multiplied by four. For example, for the previous example, since there are four families, the first containing 10 species and the rest containing two species and the time that will be used in this example is 20 days. In this case, the evolutionary rate would be 4.4. In this case, this means that there were 4.4 species developed per day. Of course, this would most likely not occur in real time events, but the equation can, in fact, be used for real time situations.

The evolutionary rate equation can also find the system of equations for each family that exists within a Collatz-Matrix equation and could even find the variable set or equations for each species. The polynomial equations that can be derived from the parameter sets of Collatz-Matrix equations can help with this. For example, if there were to be a three-dimensional Collatz-Matrix equation the following would be the polynomial equation.

$$E_g = \frac{-2x^3 + x^2 + x}{18}$$

Using this set, there can be a evolutionary rate equation formed to define the characteristics of the families and their species.

$$E_g = \frac{x(-2x^2 + x + 1)}{4(4.5)}$$

$F = x$

$s_1 = -2x^2$

$s_2 = x$

$s_3 = 1$

$s_4 = 0$

$t = 4.5$

This equation can give a summary of a specific evolutionary process of a system of equations. In this case, for an x amount of families within a Collatz-Matrix equation within a 4.5 year span, the growth of the first family of species will be represented by $-2x^2$, the second family will be represented by x, the third family will be represented by a constant value of 1, and the fourth family will be represented as null, or non-existent. This set pattern can also be represented in the evolutionary formula notation, which would be the following.

$$I\left\{\frac{x}{2}, \frac{x-1}{3}, 3x+1, 2x, \frac{xi}{2}, \frac{xi-i}{3}\right\} \rightarrow S\left\{-2x^2, x, 1, 0\right\}$$

This formula would be a result of the following initial formula notation.

$$I\left\{p_1, p_2, p_3, \ldots, p_d\right\} \rightarrow S\left\{s_1, s_2, s_3, s_4, \ldots, s_f\right\}$$

This notation defines a system of equations which would describe the the growth of an enter set consisting of a certain amount of families. The same process can be done for regular Collatz-Matrix equations, which the following addresses.

$$E_g = \frac{2(x + \frac{1}{2} + 0 + 0)}{4(0.75)}$$

This shows the symmetry between the variables and variances of families and their species.

$F = 2$

$s_1 = x$

$s_2 = \frac{1}{2}$

$s_3 = 0$

$s_4 = 0$

$t = 0.75$

The following would be the notation to address the symmetry between the parameters and the results.

$$I\left\{\frac{x}{2}, \frac{x-1}{2}, 3x+1, 2x\right\} \rightarrow S\left\{x, \frac{1}{2}, 0, 0\right\}$$

Another example would be using a one-dimensional Collatz-Matrix equation which would look like the following.

$$\frac{\alpha}{\omega} = C(x)_k \left\{\begin{matrix} \frac{x-1}{3} & 3x+1 \end{matrix}\right\}, s(k_p, d_p), D(k_t, d_t)$$

$$\frac{\alpha}{\omega} = C(x)_k \left\{\begin{matrix} \frac{x}{2} & 2x \end{matrix}\right\}, s(k_p, d_p), D(k_t, d_t)$$

The reason why there are two equations for one-dimensional Collatz-Matrix equations is due to the variance of the one dimensional plane. For the first equation, the following would be the evolutionary rate equation.

$$E_g = \frac{3(x^2 + -\frac{2}{3}x + -\frac{1}{3} + 0)}{4(0.75)}$$

$F = 3$

$s_1 = x^2$

$s_2 = -\frac{2}{3}x$

$s_3 = -\frac{1}{3}$

$s_4 = 0$

$t = 0.75$

The following would be the evolution formula for this Collatz-Matrix equation.

$$I\left\{\frac{x-1}{3}, 3x+1\right\} \rightarrow S\left\{x^2, -\frac{2}{3}x, -\frac{1}{3}, 0\right\}$$

The next equation would be the representation of the alternate Collatz-Matrix equation.

$$E_g = \frac{x(x+0+0+0)}{4(0)}$$

$F = x$

$s_1 = x$

$s_2 = 0$

$s_3 = 0$

$s_4 = 0$

$t = 0$

From the data collected here, an evolution formula can be derived.

$$I\left\{\frac{x}{2}, 2x\right\} \rightarrow S\{x, 0, 0, 0\}$$

In this case, the rate of the first species is equal to the amount of a families there are. Therefore, x is equal to one. This means that x must always be equal to one because there is only one species within a set of families.

Collatz Symmetry

With evolution formulas, there is the symmetry behind the equations. For example, when there are two Collatz-Matrix equations involved there must be a symmetry between the two equations(which would depend on whether the two equations were a part of a duality). The following notation would be used to define a duality between two Collatz-Matrix equations.

$$S_1\left\{s_1, s_2, s_3, \dots, s_f\right\} \to S_2\left\{s_1, s_2, s_3, \dots, s_f\right\}$$

The symmetry equation or evolution formula implies that there will be a constant symmetry between these two species until unconnected in a habitat or environment. More than one species can also be added to each side, changing the spectrum of change for each species.

$$S_1\left\{s_1, s_2, s_3, \dots, s_f\right\}, S_2\left\{s_1, s_2, s_3, \dots, s_f\right\} \to S_3\left\{s_1, s_2, s_3, \dots, s_f\right\}$$

If this case happens, this changes how each side becomes affected by an addition to the whole equation. For example, if there is to be a whole change to the equation of multiplying by *n*, the amount of times the first side is multiplied by *x* is the amount of times the other side must be multiplied by *x*.

$$S_1\left\{xs_1, xs_2, xs_3, \dots, xs_f\right\}, S_2\left\{xs_1, xs_2, xs_3, \dots, xs_f\right\} \to S_3\left\{2xs_1, 2xs_2, 2xs_3, \dots, 2xs_f\right\}$$

This is to be represented with an analogy of two predators and one prey. If there are two species that work together within a system or habitat, there will soon be a resulting species that forms that consists of both acknowledged species. The symmetry implies that if there are two species on one side of the equation and only one on the other the one on the other end will be doubly affected because the two species on the one side were both affected by one specific instance.

With Collatz symmetry, there is a new way of carrying out operations within the evolution formula notation. Here is the list of what the operations would look like.

$$S_1\left\{s_1, s_2, s_3, ..., s_f\right\} + S_2\left\{d_1, d_2, d_3, ..., d_f\right\} \rightarrow\ S\{s_1d_1, s_2d_2, s_3d_3, ..., s_fd_f\}$$

$$S_1\left\{s_1, s_2, s_3, ..., s_f\right\} - S_2\left\{d_1, d_2, d_3, ..., d_f\right\} \rightarrow\ S\{\frac{s_1}{d_1}, \frac{s_2}{d_2}, \frac{s_3}{d_3}, ..., \frac{s_f}{d_f}\right\}$$

$$S_1\left\{s_1, s_2, s_3, ..., s_f\right\} \times S_2\left\{d_1, d_2, d_3, ..., d_f\right\} \rightarrow S\begin{Bmatrix} s_1 & s_2 & s_3 & \dots & s_f \\ d_1 & d_2 & d_3 & \dots & d_f \end{Bmatrix}$$

$$S_1\begin{Bmatrix} s_1 & s_2 & s_3 & \dots & s_f \\ d_1 & d_2 & d_3 & \dots & d_f \end{Bmatrix} \div S_2\left\{d_1, d_2, d_3, ..., d_f\right\} \rightarrow S\begin{Bmatrix} s_1 & s_2 & s_3 & \dots & s_f \end{Bmatrix}$$

Most of the operations done are simple and intuitive, however division is the most complex because of the specifications of Collatz symmetry to the evolution formula dealing with division.

Multi-Dimensional Collatz Matrix Equations

With regular Collatz Matrix equations, the evolutionary systems work on a two-dimensional world. This makes the matrix solutions more simplistic and more observational. However, once more dimensions are added to the Collatz Matrix equations and to their matrix solutions the complexity of the equations increase. This also means that in the equations the imaginary line of the plane of the number line must be added to the parameter formulas and equations within the Collatz Matrix equations. Here is an example of a three-dimensional Collatz Matrix equation.

$$\frac{\alpha}{\omega} = C(x)_{k\times d\times z}\begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \\ \frac{xi}{2} & \frac{xi-i}{3} \end{Bmatrix}, s(k_p, d_p, z_p), D(k_t, d_t, z_t)$$

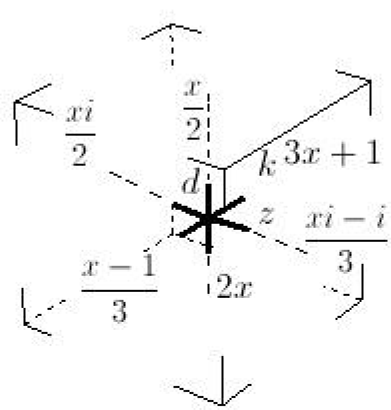
Then, from the parameters involved with these type of Collatz Matrix equations, a polynomial equation can be formed to determine the values of solutions.

$$(\frac{xi}{2})(\frac{xi-i}{3}) = \frac{-x^2+x}{6}$$

$$(\frac{2x+1}{3})(\frac{-x^2+x}{6})$$

$$\frac{-2x^3+x^2+x}{18}$$

Since a three-dimensional Collatz Matrix equation works on more complex planes of the number line, here is what a three-dimensional Collatz matrix solution would look like.



Now, since within the three-dimensional matrix solution there are also imaginary numbers not all the values will be whole numbers. This means that there is a limit to direction in accordance with the direction of structure. For example, if attempting to move forward within a matrix solution, which would result in an irrational complex value, and if entering another direction the direction must carry the value to a more rational state(or must take on the form of $ai + b$).

$$s(k_p, d_p) = 4$$

$$3(\frac{4i-i}{3}) + 1$$

$$4i - i + 1$$

$$3i + 1$$

With the complex plane of a Collatz matrix solution, once a value becomes complex it will remain a complex value and origin of another complex number.

Not only are there three-dimensional Collatz Matrix equations. Here is an example of a four-dimensional Collatz Matrix equation.

$$\frac{a}{\omega} = C(x)_{k\times d\times z\times u}\begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \\ \frac{xi}{2} & \frac{xi-i}{3} \\ 3xi+i & 2xi \end{Bmatrix}, s(k_p, d_p, z_p, u_p), D(k_t, d_t, z_t, u_t)$$

With a fourth-dimensional Collatz Matrix equation, the matrix solutions would be formed with tesseract matrices. When there is an increase in dimensional factors, there are more planes of numeral complexity that arise. In fact, after the fourth dimension there is no use of both complex or real numbers. These infinite planes of numbers are known as meta-numeral planes. For example, for a five-dimensional Collatz Matrix equation, there would be penta-numeral numbers. Penta-numeral numbers would be addressed with the variable ι_5. All the types of numbers would be addressed with ι having a subscript of the amount of dimensions involved.

The Meta-Numeral System

The meta-numeral system does not just refer to one plane of specific types of numbers. It represents the infinite range of types of numbers that exist based on the amount of dimensions worked with in a Collatz Matrix equation. Since there are an infinite set of meta-numeral type values, this means there are an infinite set of meta-numeral geometric shapes. A number that is to be located in a meta-numeral system would be represented by ι_n, where *n* represents the dimensions at which the meta-numeral value lies. Here is how a meta-numeral value would be represented.

$$a + b\iota_n$$

One thing to keep in mind, however, is the known meta-numeral lines. For example, there are quaternions, octonions, and sedenions. These three numeral systems are extensions of the complex number system and describe the next set of dimensions of matrices. The five-dimensions number line would replace the imaginary number *i* with *j* while the next one would consist of *k*. The complexity of Collatz Matrix equations increases when octonions and sedenions are used.

$$\frac{\alpha}{\omega} = C(x)_{k\times d\times z\times u\times j\times r\times t\times w\times\kappa\times\eta} \begin{Bmatrix} \frac{x}{2} & \frac{x-1}{3} \\ 3x+1 & 2x \\ \frac{xi}{2} & \frac{xi-i}{3} \\ 3xi+i & 2xi \\ \frac{xj}{2} & \frac{xj-j}{3} \\ 3j+j & 2xj \\ \frac{xk}{2} & \frac{xk-k}{3} \\ 3k+k & 2xk \\ \frac{e_x}{2} & \frac{e_x-e_1}{3} \\ 3e_x+e_1 & 2e_x \end{Bmatrix}, s(k_p, d_p, z_p, u_p, j_p, r_p, t_p, w_p, \kappa_p, \eta_p), D(k_t, d_t, z_t, u_t, j_t, r_t, t_t, w_t, \kappa_t, \eta_t)$$

For the polynomial equation for this Collatz Matrix equation, the following would be the operation to use.

$$(\frac{2x+1}{3})(\frac{-2x-1}{3})^3$$

$$\eta(x) = (\frac{2x+1}{3})(\frac{-2x-1}{3})^3$$

As seen within this function denoting the existence of a ten dimensional equation, there is a combination of meta-numerals being used in the system of equations. In order to treat meta-numerals as a set, meta-numerals would be mathematically represented by $\mathbb{U}$. Since all values are a subset of the real numbers system, meta-numerals are a subset of the real numbers system, or $\mathbb{U} \subseteq \mathrm{R}$.

Conclusion

The Collatz Theory has an array of statements dealing with both the properties of matrices, the parameters involving the Collatz conjecture, and the properties of numbers. It is able to analyze properties of numbers, which would allow numbers that seemingly are unrelated to be connected through the solutions given by a complete solution or the other solutions that come forth from the steps of evaluating a Collatz Matrix equation. This analysis of numbers could, in fact, help solve the Collatz conjecture, proving the conjecture true of false. This theory also can provide insight into other problems dealing with numeral properties, such as prime integers. Collatz Theory also can help the study of imaginary numbers, dealing with their properties and connections with other numbers.