IBDP SL Mathematics

Applications & Interpretation

Year 1 – Semester 2 Final Examination - Paper 2

Question 1

[Maximum mark: 15]

ABC is a triangular field on horizontal ground. The lengths of AB and AC are 70 m and 50 m respectively. The size of angle BCA is 78°.

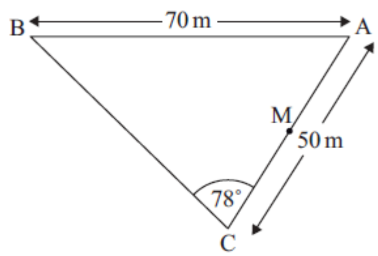


diagram not to scale

(a) Find the size of angle ABC. *[3]*

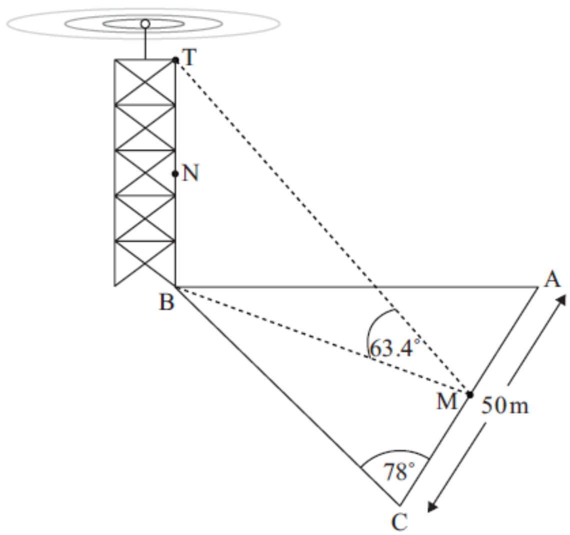
(b) Find the area of the triangular field. *[4]*

M is the midpoint of AC.

(c) Find the length of BM. *[3]*

A vertical mobile phone mast, TB, is built next to the field with its base at B. The angle of elevation of T from M is 63.4°. N is the midpoint of the mast.

diagram not to scale



(d) Calculate the angle of elevation of N from M. *[5]*

(a) $\dfrac{70}{\sin 78} = \dfrac{50}{\sin A\hat{B}C}$ *(M1)(A1)*

Note: Award *(M1)* for substituted sine rule, *(A1)* for correct substitution.

$A\hat{B}C = 44.3^\circ \quad (44.3209\ldots)$ *(A1)(G3)* ***[3 marks]***

Note: If radians are used the answer is 0.375918…, award at most *(M1)(A1)(A0)*.

(b) area $\Delta ABC = \dfrac{1}{2} \times 70 \times 50 \times \sin(57.6790\ldots)$ *(A1)(M1)(A1)*(ft)

Notes: Award *(A1)*(ft) for their 57.6790… seen, *(M1)* for substituted area formula, *(A1)*(ft) for correct substitution.
Follow through from part (a).

$= 1480\,\text{m}^2 \quad (1478.86\ldots)$ *(A1)*(ft)*(G3)* ***[4 marks]***

Notes: The answer is $1480\,\text{m}^2$, units are required. 1479.20… if 3 sf used.
If radians are used the answer is $1554.11\ldots\text{m}^2$, award *(A1)*(ft)*(M1)(A1)*(ft)*(A1)*(ft)*(G3)*.

(c) $BM^2 = 70^2 + 25^2 - 2 \times 70 \times 25 \times \cos(57.6790\ldots)$ *(M1)(A1)*(ft)

Notes: Award *(M1)* for substituted cosine rule, *(A1)*(ft) for correct substitution. Follow through from their angle in part (b).

$BM = 60.4\,(\text{m}) \quad (60.4457\ldots)$ *(A1)*(ft)*(G2)* ***[3 marks]***

Notes: If the 3 sf answer is used the answer is 60.5 (m).
If radians are used the answer is 62.5757…(m), award *(M1)(A1)*(ft)*(A1)*(ft)*(G2)*.

(d) $\tan 63.4^\circ = \dfrac{\text{TB}}{60.4457...}$ *(M1)*

Note: Award *(M1)* for their correctly substituted trig equation.

$\text{TB} = 120.707...$ *(A1)*(ft)

Notes: Follow through from part (c). If 3 sf answers are used throughout, $\text{TB} = 120.815...$
If $\text{TB} = 120.707...$ is seen without working, award *(A2)*.

$\tan \text{N}\hat{\text{M}}\text{B} = \dfrac{\left(\dfrac{120.707...}{2}\right)}{60.4457...}$ *(A1)*(ft)*(M1)*

Notes: Award *(A1)*(ft) for their TB divided by 2 seen, *(M1)* for their correctly substituted trig equation.
Follow through from part (c) and **within part (d)**.

$\text{N}\hat{\text{M}}\text{B} = 45.0^\circ \quad (44.9563...)$ *(A1)*(ft)*(G3)*

Notes: If 3 sf are used throughout, answer is 45°.
If radians are used the answer is 0.308958…, and if full working is shown, award at most *(M1)(A1)*(ft)*(A1)*(ft)*(M1)(A0)*.
If no working is shown for radians answer, award *(G2)*.

OR

$\tan \text{N}\hat{\text{M}}\text{B} = \dfrac{\text{NB}}{\text{BM}}$ *(M1)*

$\tan 63.4^\circ = \dfrac{2 \times \text{NB}}{\text{BM}}$ *(A1)(M1)*

Note: Award *(A1)* for $2 \times \text{NB}$ seen.

$\tan \text{N}\hat{\text{M}}\text{B} = \dfrac{1}{2}\tan 63.4^\circ$ *(M1)*

$\text{N}\hat{\text{M}}\text{B} = 45.0^\circ \quad (44.9563...)$ *(A1)(G3)* *[5 marks]*

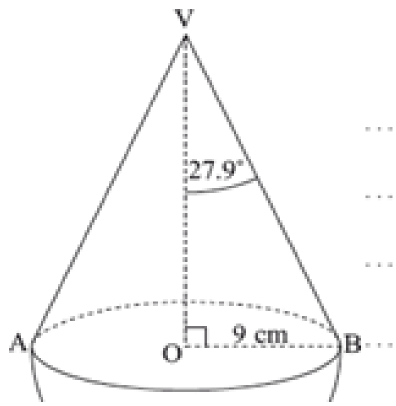
Notes: If radians are used the answer is 0.308958…, and if full working is shown, award at most *(M1)(A1)(M1)(M1)(A0)*. If no working is shown for radians answer, award *(G2)*.

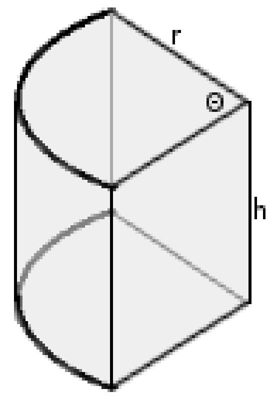
Total [15 marks]

Question 2 (11 marks)

The contents of a conical bin will be emptied into a cylindrical sector.

Calculate the volumes of each solid in order to answer: will the cylindrical sector overflow?





sector perimeter = 40 cm

radius = 14 cm

height = 15 cm

$\tan(27.9°) = 9/VO$

$VO = 9/\tan(27.9) = 16.99$

≈ 17 cm.

V cone

$\frac{1}{3}(\pi r^2) \cdot H$

$\frac{1}{3}\pi(9)^2 \cdot 17$

V = 1442 cm^3

Arc length = 40 – (2)(14) = 12

12 =

V = base area x height

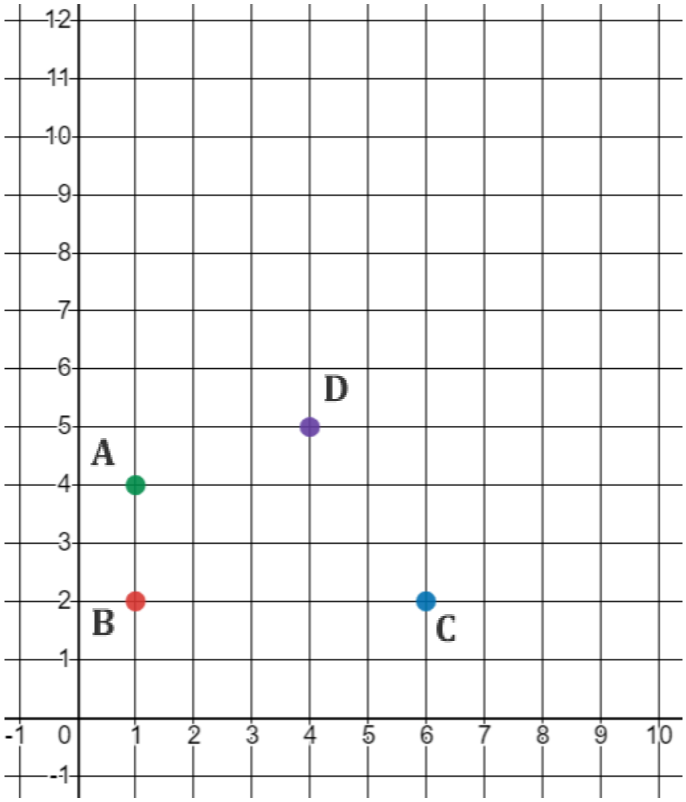
V = (1259.7 cm^3

The cylindrical sector will overflow as it's volume is less than that of the cone.

Question 3

[Maximum mark: 10]

Four locations, marked as points A, B, C, and D are given on the co-ordinate plane below.



(a) Sketch the perpendicular bisectors of:

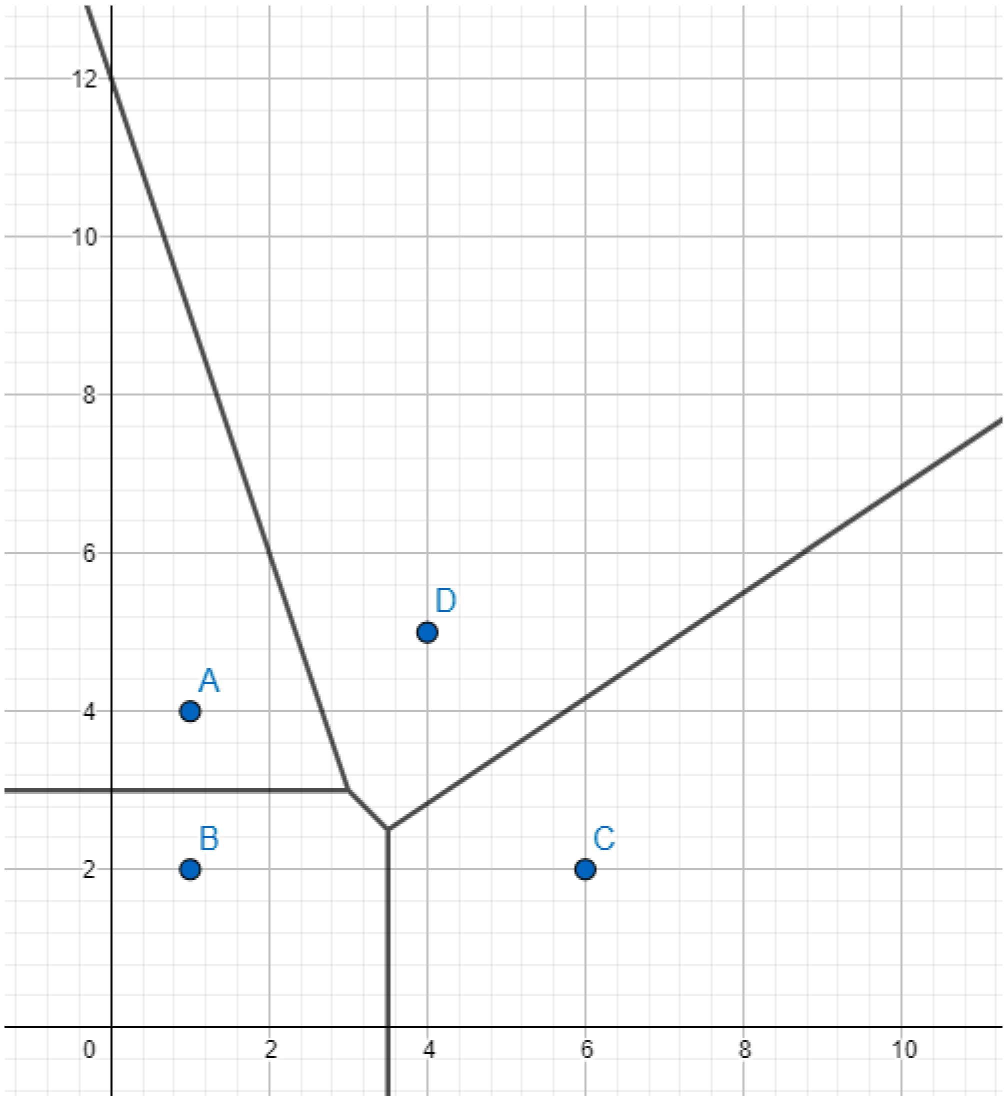
i) the segment connecting points A and B, and; *[1]*

ii) the segment connecting points B and C *[1]*

(b) The equation of the perpendicular bisector of the segment connecting points A and D is $y + 3x = 12$.
Using this equation, add this perpendicular bisector to the diagram. *[2]*

(c) Algebraically, find the equation of the perpendicular bisector of the segment connecting points D and C. *[4]*

(d) Complete the Voronoi diagram for points A, B, C, D *[2]*



(c) m.p. CD = ((6+4)/2, (2+5)/2) = (5,3.5)

slope CD = (5-2)/(4-6) = -3/2

therefore, perpendicular slope = 2/3

equation: y = mx + b

3.5 = 2/3(5) + b

3.5 = 10/3 + b

b = -1/6

y = 2/3 x - 1/6