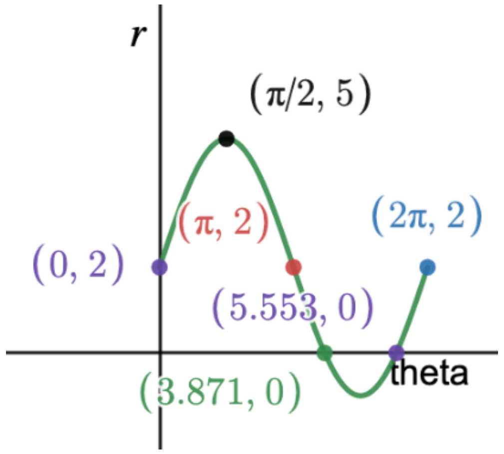
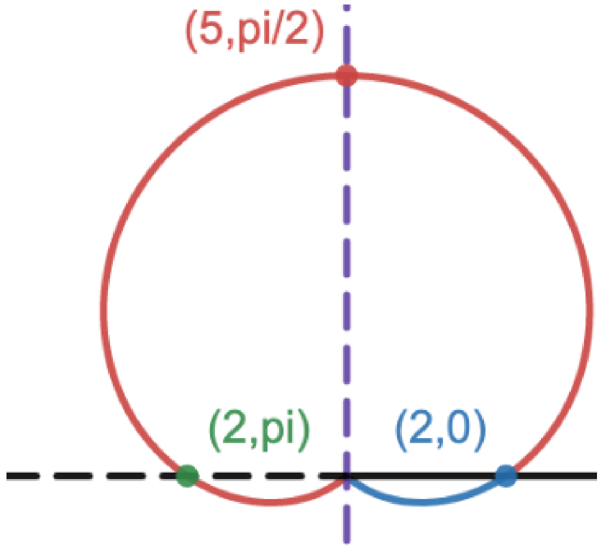
1.

$r = 2 + 3\sin\theta$

Sketch the curve and find the area enclosed by both vertical and horizontal tangents.





To find the tangent perpendicular to the initial line, solving $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 0$

$$\begin{aligned}
x &= r\cos\theta \\
x &= (2 + 3\sin\theta)\cos\theta \\
\frac{\mathrm{d}x}{\mathrm{d}\theta} &= (3\cos\theta)\cos\theta + (2 + 3\sin\theta)(-\sin\theta) \\
0 &= 3\left(1 - \sin^2\theta\right) - 2\sin\theta - 3\sin^2\theta \\
0 &= 6\sin^2\theta + 2\sin\theta - 3 \\
\sin\theta &\neq -\frac{1+\sqrt{19}}{6} \text{ as this gives negative } r \\
\sin\theta &= \frac{-1+\sqrt{19}}{6} \\
x &= (2 + 3\sin\theta)\cos\theta \\
&= \left[2 + 3\left(\frac{-1+\sqrt{19}}{6}\right)\right]\sqrt{1 - \left(\frac{-1+\sqrt{19}}{6}\right)^2} \\
&= 3.049
\end{aligned}$$

The coordinates of the vertical tangent.

$$\begin{aligned}
\theta &= \sin^{-1}\left(\frac{-1+\sqrt{19}}{6}\right) \approx 0.594 \\
r &= \left[2 + 3\left(\frac{-1+\sqrt{19}}{6}\right), \sin^{-1}\left(\frac{-1+\sqrt{19}}{6}\right)\right] \approx (3.679, 0.594) \\
r &= \left[2 + 3\left(\frac{-1+\sqrt{19}}{6}\right), \pi - \sin^{-1}\left(\frac{-1+\sqrt{19}}{6}\right)\right] \approx (3.679, 5.689)
\end{aligned}$$

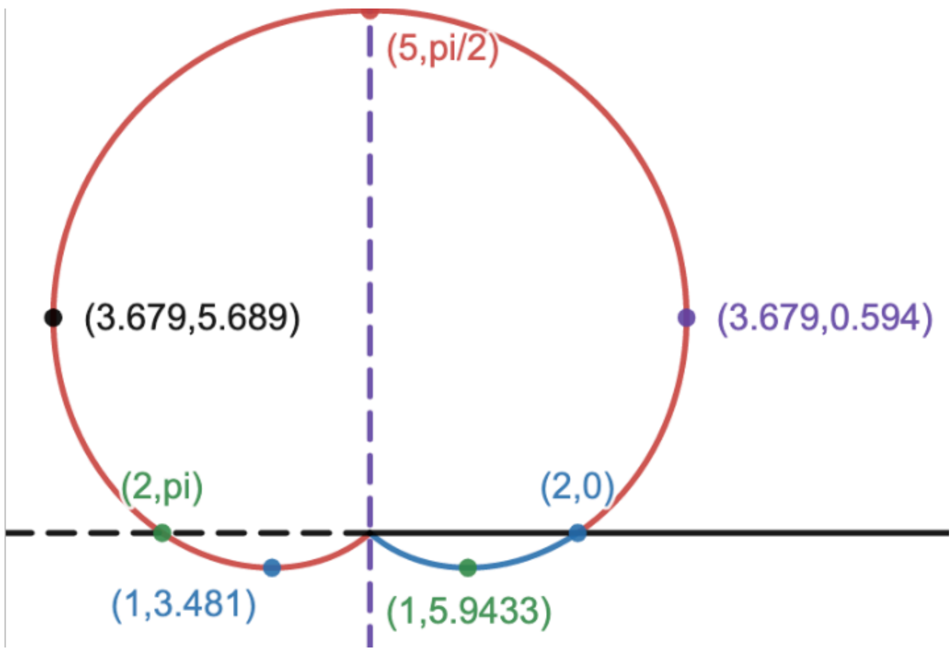
To find the tangent parallel to the initial line, solving $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$

$$
\begin{aligned}
y &= r\sin\theta \\
y &= (2+3\sin\theta)\sin\theta \\
\frac{\mathrm{d}y}{\mathrm{d}\theta} &= (3\cos\theta)\sin\theta + (2+3\sin\theta)(\cos\theta) \\
0 &= \cos\theta(6\sin\theta+2)
\end{aligned}
$$

$$
\begin{aligned}
\cos\theta &= 0 \\
\theta &= \frac{\pi}{2}, \frac{3\pi}{2} \\
r &= 2+3\sin\frac{\pi}{2} = 5 \\
r &= 2+3\sin\frac{3\pi}{2} = -1\,(\text{reject}) \\
\sin\theta &= -\frac{1}{3} \\
\theta &= 3.481,\ 5.943
\end{aligned}
$$

The coordinates of the vertical tangent.

$$
\begin{aligned}
r &= \left(5, \frac{\pi}{2}\right) \\
r &= \left[2+3\left(-\frac{1}{3}\right), 3.481\right] \approx (1, 3.481) \\
r &= \left[2+3\left(-\frac{1}{3}\right), 5.943\right] \approx (1, 5.943)
\end{aligned}
$$



$$\text{width} = 2\times 3.679\cos 0.594 = 6.098$$

$$\text{height} = 5-(\sin 3.481) = \frac{16}{3}$$

$$
\begin{aligned}
\text{area} &= (6.098)\left(\frac{16}{3}\right) \\
&= 32.5\,\text{units}^2
\end{aligned}
$$