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B.Sc. (Hons.) Mathematics (Semester: 3rd)

ANALYSIS –I

Subject Code: BMATHS1322

Paper ID: 22131211

Time: 03 Hours **Maximum Marks: 60**

Instructions for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consists of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consists of 3 questions of 10 marks each. The student has to attempt any 2 questions out of it.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Show that if $f: A \to B$ and G, H are subsets of B, then $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$

b. Show that the set of irrational numbers is uncountable.

c. Give an example of an unbounded sequence that has a convergent subsequence.

d. Establish the convergence or divergence of the sequence $\{X_n\}$, where $X_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n}$ $for\ n \in N$

e. Test the convergence of the series $\sum \left(\frac{1}{n^{1+\frac{1}{n}}} \right)$

f. Show that the series $\sum_{n=2}^{\infty} \left(\frac{1}{n(\log n)^p} \right)$, p > 0 converges for p > 1, diverges for p ≤ 1

g. Give examples of functions f and g such that f and g do not have limits at point c, but both f+g and fg have limits at c.

h. If $f : [0,1] \to R$ is continuous and has only rational values, must f be constant. Prove your assertion.

i. Prove that every sequence of real numbers contains a monotone subsequence.

j. Show that if a, b are real numbers, then prove that max {a, b} = $\frac{1}{2}\left(a + b + |a - b|\right)$

Section – B **(5 Marks each)**

Q2. Show that there exists a positive real number x such that x2 = 2

Q3. If {Sn} be a sequence of positive real numbers such that Sn = $\frac{1}{2}(S_{n-1}+S_{n-2})\ \forall n>2$, then show that {Sn} converges. Also find Lim Sn

Q4. Test the convergence of the series:

$$1+\frac{\alpha.\beta}{1.\gamma}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}x^2+\frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{1.2.3\gamma(\gamma+1)(\gamma+2)}x^3+.......$$

for all positive values of x, α, β and γ

Q5. If $f:A\rightarrow R$ is uniformly continuous on subset A of R and if {Xn} is a Cauchy Sequence in A, then prove that {f(Xn)} is Cauchy sequence in R.

Q6. Let A and B be bounded subsets of R such that x ϵ A, y ϵ B $\Rightarrow$ x $\leq$ y. Prove that sup A $\leq$ inf B

Section – C **(10 Marks each)**

Q7. If $f:A\rightarrow R$ and let c be a cluster point of A. Then prove that the following are equivalent.

(i) $\lim_{x\rightarrow c} f = L$

(ii) For every sequence {X_n} in A that converges to c such that $X_n \neq$ c for all n in N, the sequence {f (X_n)} converges to L.

Q8. State and Prove Abel's Test

Q9. If S is a subset of R that contains at least two points and has the property:

If x, y $\in S\ and\ x<y, then\ \ [x,y]\subseteq S$.

Then S is an interval