1.6 Sequence/Series/Sigma Notation

Review: write the formulas for Arithmetic and Geometric Sequences and Series.

Ex 1 An arithmetic sequence has 5 and 13 as its first two terms respectively.

(a) Write down, in terms of *n*, an expression for the *n*th term, u_n.

(b) Find the number of terms of the sequence which are less than 400.

Ex 2 The ratio of the fifth term to the twelfth term of a sequence in an arithmetic progression is $\frac{6}{13}$. If each term of this sequence is positive, and the product of the first term and the third term is 32, find the sum of the first 100 terms of this sequence.

Sum to Infinity of a Geometric Series

Consider 0.3 + 0.03 + 0.003 + 0.0003 + ... , what is the sum?

For $S_n = \frac{u_1(1-r^n)}{1-r}$, $S_\infty = \frac{u_1}{1-r}$ for $|r| < 1$

Ex 3 Find the sum to infinity of the geometric series $-12 + 8 - \frac{16}{3} + ...$

Ex 4 The sum of an infinite geometric series is $13\frac{1}{2}$, and the sum of the first three terms is 13. Find the first term.

Sigma Notation

$$\sum_{i=0}^{n} u_i = u_0 + u_1 + u_2 + ... + u_n$$

Ex 5 Evaluate $\sum_{r=1}^{7} r + 1$

Properties of Sigma Notation

$\sum_{i=0}^{n} ca_i = c\sum_{i=0}^{n} a_i$ (you can factor out constants)

$\sum_{i=1}^{n} c = cn$

$\sum_{i=0}^{n} a_i \pm b_i = \sum_{i=0}^{n} a_i \pm \sum_{i=0}^{n} b_i$ (the sum of a sum is the same as the sum of a sum)

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Ex 6 Evaluate $\sum_{i=1}^{100} (3-2i)^2$

Homework