IAL A level
Statistics 2
Hypothesis Test QP

1.Jan 2025-5

5. A meteor shower occurs for several weeks every summer. One Monday night during this meteor shower, Chris goes out to look for meteors and sees an average of 15 meteors per hour.

(a) Find the probability that in a random 20-minute period of time on this Monday night Chris sees

(i) at least 6 meteors

(ii) no more than 3 meteors.

(4)

Chris is told by the local astronomy club that there will be more meteors to be seen per hour on the following Friday night than on Monday night.

Chris decides to use data from the Friday night to test, at a 5% significance level, if there is evidence to support the claim of the local astronomy club.

Chris plans to spend 30 minutes looking for meteors on the Friday night.

(b) Write down suitable null and alternative hypotheses that Chris can use.

(1)

(c) Find the critical region for the test and state its associated probability.

(3)

On the Friday night, Chris sees 12 meteors in the 30 minutes.

(d) State the conclusion to the test that can be made using this observation.
Give a reason for your answer.

(2)

2.Jan 2024-3

3. Rowan believes that 35% of type *A* vacuum tubes shatter when exposed to alternating high and low temperatures.

Rowan takes a random sample of 15 of these type *A* vacuum tubes and uses a two-tailed test, at the 5% level of significance, to test his belief.

(a) Give **two** assumptions, in context, that Rowan needs to make for a binomial distribution to be a suitable model for the number of these type *A* vacuum tubes that shatter when exposed to alternating high and low temperatures.

(2)

(b) Using a binomial distribution, find the critical region for the test.
You should state the probability of rejection in each tail, which should be as close as possible to 0.025

(3)

(c) Find the actual level of significance of the test based on your critical region from part (b)

(1)

Rowan records that in the latest batch of 15 type *A* vacuum tubes exposed to alternating high and low temperatures, 4 of them shattered.

(d) With reference to part (b), comment on Rowan's belief. Give a reason for your answer.

(1)

Rowan changes to type *B* vacuum tubes. He takes a random sample of 40 type *B* vacuum tubes and finds that 8 of them shatter when exposed to alternating high and low temperatures.

(e) Test, at the 5% level of significance, whether or not there is evidence that the proportion of type *B* vacuum tubes that shatter when exposed to alternating high and low temperatures is lower than 35%
You should state your hypotheses clearly.

(5)

3.June 2024-3

3 Jian owns a large group of shops. She decides to visit a random sample of the shops to check if the stocktaking system is being used incorrectly.

(a) Suggest a suitable sampling frame for Jian to use.

(1)

(b) Identify the sampling units.

(1)

(c) Give one advantage and one disadvantage of taking a sample rather than a census.

(2)

Jian believes that the stocktaking system is being used incorrectly in 40% of the shops.

To investigate her belief, a random sample of 30 of the shops is taken.

(d) Using a 5% level of significance, find the critical region for a two-tailed test of Jian's belief.
You should state the probability in each tail, which should each be as close as possible to 2.5%

(3)

The total number of shops, in the sample of 30, where the stocktaking system is being used incorrectly is 20

(e) Using the critical region from part (d), state what this suggests about Jian's belief. Give a reason for your answer.

(1)

Jian introduces a new, simpler, stocktaking system to all the shops.

She takes a random sample of 150 shops and finds that in 47 of these shops the new stocktaking system is being used incorrectly.

(f) Using a suitable approximation, test, at the 5% level of significance, whether or not there is evidence that the proportion of shops where the stocktaking system is being used incorrectly is now **less than** 0.4
You should state your hypotheses and show your working clearly.

(7)

4.Jan 2023-3

3. *Superbounce* is a manufacturer of tennis balls.

It knows from past records that 10% of its tennis balls fail a bounce test.

(a) Find the probability that from a random sample of 10 of these tennis balls

(i) at least 4 fail the bounce test

(ii) more than 1 but fewer than 5 fail the bounce test.

(4)

The managing director makes changes to the production process and claims that these changes will reduce the probability of its tennis balls failing the bounce test.

After the changes were made a random sample of 50 of the tennis balls were tested and it was found that 2 failed the bounce test.

(b) Test, at the 5% significance level, whether or not this result supports the managing director's claim.

(4)

In a second random sample of n tennis balls it was found that none failed the bounce test. As a result of this sample, the managing director's claim is supported at the 1% significance level.

(c) Find the smallest possible value of n

(3)

5.Oct 2023-5

5. A supermarket receives complaints at a mean rate of 6 per week.

(a) State one assumption necessary, in order for a Poisson distribution to be used to model the number of complaints received by the supermarket.

(1)

(b) Find the probability that, in a given week, there are

(i) fewer than 3 complaints received by the supermarket,

(ii) at least 6 complaints received by the supermarket.

(3)

In a randomly selected week, the supermarket received 12 complaints.

(c) Test, at the 5% level of significance, whether or not there is evidence that the mean number of complaints is greater than 6 per week.
State your hypotheses clearly.

(5)

Following changes made by the supermarket, it received 26 complaints over a 6-week period.

(d) Use a suitable approximation to test whether or not there is evidence that, following the changes, the mean number of complaints received is less than 6 per week.
You should state your hypotheses clearly and use a 5% significance level.

(7)

6.Jan 2022-3

3 A photocopier in a school is known to break down at random at a mean rate of 8 times per week.

(a) Give a reason why a Poisson distribution could be used to model the number of breakdowns.

(1)

The headteacher of the school replaces the photocopier with a refurbished one and wants to find out if the rate of breakdowns has increased or decreased.

(b) Write down suitable null and alternative hypotheses that the headteacher should use.

(1)

The refurbished photocopier was monitored for the first week after it was installed.

(c) Using a 5% level of significance, find the critical region to test whether the rate of breakdowns has now changed.

(3)

(d) Find the actual significance level of a test based on the critical region from part (c).

(2)

During the first week after it was installed there were 4 breakdowns.

(e) Comment on this finding in the light of the critical region found in part (c).

(2)

7.June 2022-4

4. Past evidence shows that 7% of pears grown by a farmer are unfit for sale.

This season it is believed that the proportion of pears that are unfit for sale has decreased. To test this belief a random sample of n pears is taken. The random variable Y represents the number of pears in the sample that are unfit for sale.

(a) Find the smallest value of n such that $Y = 0$ lies in the critical region for this test at a 5% level of significance.

(3)

In the past, 8% of the pears grown by the farmer weigh more than 180 g. This season the farmer believes the proportion of pears weighing more than 180 g has changed. She takes a random sample of 75 pears and finds that 11 of them weigh more than 180 g.

(b) Test, using a suitable approximation, whether there is evidence of a change in the proportion of pears weighing more than 180 g.
You should use a 5% level of significance and state your hypotheses clearly.

(6)

8.Oct 2022-3

3. A company produces packets of sunflower seeds. Each packet contains 40 seeds. The company claims that, on average, only 35% of its sunflower seeds do not germinate.

A packet is selected at random.

(a) Using a 5% level of significance, find an appropriate critical region for a two-tailed test that the proportion of sunflower seeds that do not germinate is 0.35
You should state your hypotheses clearly and state the probability, which should be as close as possible to 2.5%, for each tail of your critical region.

(4)

(b) Write down the actual significance level of this test.

(1)

Past records suggest that 2.8% of the company's sunflower seeds grow to a height of more than 3 metres.
A random sample of 250 of the company's sunflower seeds is taken and 11 of them grow to a height of more than 3 metres.

(c) Using a suitable approximation test, at the 5% level of significance, whether or not there is evidence that the proportion of sunflower seeds that grow to a height of more than 3 metres is now greater than 2.8%
State your hypotheses clearly.

(5)

9.June 2021-1

1. *Spany* sells seeds and claims that 5% of its pansy seeds do not germinate. A packet of pansy seeds contains 20 seeds. Each seed germinates independently of the other seeds.

(a) Find the probability that in a packet of *Spany*'s pansy seeds

(i) more than 2 but fewer than 5 seeds do not germinate,

(ii) more than 18 seeds germinate.

(5)

Jem buys 5 packets of *Spany*'s pansy seeds.

(b) Calculate the probability that all of these packets contain more than 18 seeds that germinate.

(2)

Jem believes that *Spany*'s claim is incorrect. She believes that the percentage of pansy seeds that do not germinate is greater than 5%

(c) Write down the hypotheses for a suitable test to examine Jem's belief.

(1)

Jem planted all of the 100 seeds she bought from *Spany* and found that 8 did not germinate.

(d) Using a suitable approximation, carry out the test using a 5% level of significance.

(6)

10.Oct 2020-4

4. In a peat bog, Common Spotted-orchids occur at a mean rate of 4.5 per m^2

(a) Give an assumption, not already stated, that is required for the number of Common Spotted-orchids per m^2 of the peat bog to follow a Poisson distribution.

(1)

Given that the number of Common Spotted-orchids in 1 m^2 of the peat bog can be modelled by a Poisson distribution,

(b) find the probability that in a randomly selected 1 m^2 of the peat bog

(i) there are exactly 6 Common Spotted-orchids,

(ii) there are fewer than 10 but more than 4 Common Spotted-orchids.

(4)

Juan believes that by introducing a new management scheme the number of Common Spotted-orchids in the peat bog will increase. After three years under the new management scheme, a randomly selected 2 m^2 of the peat bog contains 11 Common Spotted-orchids.

(c) Using a 5% significance level assess Juan's belief. State your hypotheses clearly.

(5)

Assuming that in the peat bog, Common Spotted-orchids still occur at a mean rate of 4.5 per m^2

(d) use a normal approximation to find the probability that in a randomly selected 20 m^2 of the peat bog there are fewer than 70 Common Spotted-orchids.

(3)

Following a period of dry weather, the probability that there are fewer than 70 Common Spotted-orchids in a randomly selected 20 m^2 of the peat bog is 0.012

A random sample of 200 non-overlapping 20 m^2 areas of the peat bog is taken.

(e) Using a suitable approximation, calculate the probability that at most 1 of these areas contains fewer than 70 Common Spotted-orchids.

(3)

11.Jan 2019-4

4. At a shop, past figures show that 35% of customers pay by credit card. Following the shop's decision to no longer charge a fee for using a credit card, a random sample of 20 customers is taken and 11 are found to have paid by credit card.

Hadi believes that the proportion of customers paying by credit card is now greater than 35%

(a) Test Hadi's belief at the 5% level of significance. State your hypotheses clearly.

(5)

For a random sample of 20 customers,

(b) show that 11 lies less than 2 standard deviations above the mean number of customers paying by credit card.
You may assume that 35% is the true proportion of customers who pay by credit card.

(4)