CHAPTER 5

DAY 13 – DIFFERENTIABILITY – VARIOUS FORMULAS AND RESULTS

- Suppose f is a real valued function and c be a point in the domain of f. The derivative of f at c is defined by $\frac{f(c+h)-f(c)}{h}$, provided the limit exists.

- Let $y = f(x)$ be a function. Then the function defined by

$$f'(x) = \frac{f(x+h)-f(x)}{h}$$

Whenever the limit exists is defined to be the derivative of f.

- The derivative of $y = f(x)$ is denoted by $f'(x)$ or $\frac{d}{dx}(f(x))$ or $\frac{dy}{dx}$ or y'.

Derivatives of various functions

Sl.no	Function	Derivative
1	1, 2, 3, …constants	0
2	x	1
3	x^2	$2x$
4	x^3	$3x^2$
5	x^n	nx^{n-1}
6	$\sin sin\, x$	$\cos cos\, x$
7	$\cos cos\, x$	x
8	$\tan tan\, x$	$sec^2\, x$
9	$\cot cot\, x$	$-\ cosec^2\, x$
10	$cosec\, x$	$-\ \cot cot\, x\ cosec\, x$
11	$\sec sec\, x$	$\tan tan\, x\ sec\, x$
12	e^x	e^x
13	a^x	$a^x \log log\, a$
14	$\log log\, x$	$\frac{1}{x}$

15	x	$\frac{1}{\sqrt{1-x^2}}$
16	x	$-\frac{1}{\sqrt{1-x^2}}$
17	x	$\frac{1}{1+x^2}$
18	x	$-\frac{1}{1+x^2}$
19	x	$\frac{1}{x\sqrt{x^2-1}}$
20	x	$-\frac{1}{x\sqrt{x^2-1}}$
21	$\frac{1}{x}$	$-\frac{1}{x^2}$
22	$\sqrt{x} = x^{\frac{1}{2}}$	$\frac{1}{2\sqrt{x}}$

Algebra of derivatives

Let u and v be two differentiable functions. Then

1. $(u + v)' = u' + v'$
2. $(u - v)' = u' - v'$
3. $(uv)' = u'v + v'u$
4. $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$, whenever $v \neq 0$.

Note: Every differentiable function is continuous.

Questions

1. Differentiate the following functions with respect to x.
   a. $\sin sin\ 2x$
   b. $\tan tan\ 5x$
   c. $cosec\left(\sqrt{x}\right)$
   d. $\cos cos\ (2x + 4)$
   e. $\sin sin\ (\cos cos\ x)$
   f. $\sec sec\left(\tan tan\left(\sqrt{x}\right)\right)$

g. $\tan tan\ (\sec sec\ (x))$

h. $\sec sec\ x\ +\ \cot cot\ x$

i. $\sec sec \sqrt{x}\ -\ \tan tan\ x^2$

j. $x^2 . \sin sin\ x$

k. $\tan tan\ x\ . \sin sin\ x$

l. $\frac{\sin sin\ x}{\cos cos\ x}$

m. $\frac{\sin sin\ 4x}{\cos cos\ 2x}$