MTH603
Assignment#2

**Question No. 1:** Consider the following table of values

| x | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
|---|---|---|---|---|---|---|---|
| F(x) | 23 | 93 | 259 | 559 | 1031 | 1713 | 2643 |

Use Newton's backwards difference formula to compute *f* (11) with and *p* = 0.5

Solution

| x | Y=fx | $\nabla y_n$ | $\nabla^2 y_n$ | $\nabla^3 y_n$ | $\nabla^4 y_n$ | $\nabla^5 y_n$ | $\nabla^6 y_n$ |
|---|---|---|---|---|---|---|---|
| 2 | 23 | | | | | | |
| 4 | 93 | 70 | | | | | |
| 6 | 259 | 166 | 96 | | | | |
| 8 | 559 | 300 | 134 | 38 | | | |
| 10 | 1031 | 472 | 172 | 38 | 0 | | |
| 12 | 1713 | 682 | 210 | 38 | 0 | 0 | |
| 14 | 2643 | 930 | 248 | 38 | 0 | 0 | 0 |

Newton's backwards Formula

$$yx = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(P+3)}{4!}\nabla^4 y_n$$

$$+\frac{p(p+1)(p+2)(P+3)(P+4)}{5!}\nabla^5 y_n + \frac{p(p+1)(p+2)(P+3)(P+4)(P+5)}{6!}\nabla^6 y_n$$

$$yx = 2643 + 0.5(930) + \frac{0.5(0.5+1)}{2}(248) + \frac{0.5(0.5+1)(0.5+2)}{6}(38)$$

$$y_x = 2643 + 465 + 93 + 11.875$$

$$= 3212.875$$

**Question No. 2:** Find Newton's forward difference table from the following data.

| $x$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 0.9048 | 0.8187 | 0.6408 | 0.4703 |

Solution:

| x | Y=fx | $\nabla y_n$ | $\nabla^2 y_n$ | $\nabla^3 y_n$ | $\nabla^4 y_n$ |
|---|---|---|---|---|---|
| 0.0 | 1 | | | | |
| 0.1 | 0.9048 | -0.0952 | | | |
| 0,2 | 0.8187 | -0.0861 | -0.0091 | | |
| 0.3 | 0.6408 | -0.1779 | -0.0918 | -0.0827 | |
| 0.4 | 0.4703 | -0.1705 | -0.0074 | -0.0844 | -0.0017 |

$$yx = y_0 + p\nabla y_0 + \frac{p(p-1)}{2!}\nabla^2 y_0 + \frac{p(p-1)(p-2)}{3!}\nabla^3 y_0 + \frac{p(p-1)(p-2)(P-3)}{4!}\nabla^4 y_0$$

$$p = \frac{(x-x_0)}{h}$$

$$= \frac{(x-0.0)}{0.1} = (10x-0)$$

$$= 1 + 10x - (0.0952) + \frac{10x(10x-1)}{2!}(-0.0091) + \frac{10x(10x-1)(10x+2)}{3!}(-0.0827)$$

$$+ \frac{10x(10x-1)(10x+2)(10x+3)}{4!}(-0.0017)$$

$$= 1 + 0.952x + \frac{-0.182x^2 + 0.091x}{2} + \frac{-16.54x^3 + 24.81x^2 + 0.091x}{6} +$$

$$\frac{-0.068x^4 + 4.08x^3 + 1.19x^2 + 0.102x}{24}$$

$$= 1 + 0.952x + \frac{2.184x^2 + 1.092x - 66.16x^3 + 99.24x^2 - 6.616x - 0.068x^4 - 4.08x^3 + 1.19x^2 + 0.102x}{24}$$

$$= \frac{1}{24} - 0.068x - 70.24x + 100.43x + 8.762x$$

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