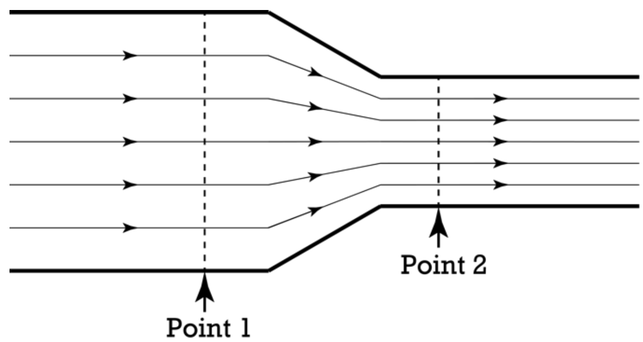
Flipping Physics Lecture Notes:
Continuity Equation for Ideal Fluid Flow - Derivation
http://www.flippingphysics.com/continuity-equation.html

Let's consider ideal fluid flow through a circular pipe which decreases in diameter. The streamlines of the fluid flow are shown in the figure. Remember in ideal fluid flow, the fluid is nonviscous and incompressible, and the flow is laminar and irrotational.

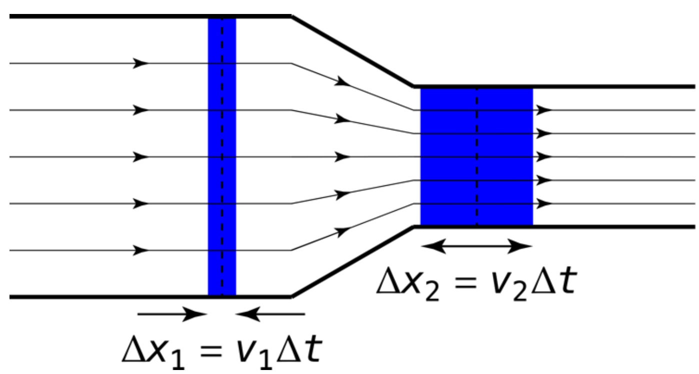


Assuming there are no losses of fluid in the circular pipe, the mass of the fluid flowing past point 1 must be the same as the mass of the fluid flowing past point 2. Because these masses are masses of fluids flowing past a point, let's identify these masses as Δm. Δm represents the amount of mass which passes each point during change in time, Δt.

We also know density equals mass over volume. In this case the volume is ΔV because it represents the volume of the fluid which passes by a point during change in time, Δt.

$$\rho = \frac{m}{V} = \frac{\Delta m}{\Delta V} \Rightarrow \Delta m_1 = \rho_1 \Delta V_1$$

During time Δt, the fluid will move a distance Δx. Using the equation for average velocity[1♥], we can determine that distance in terms of velocity, v[2♣], and Δt.



$$v_{avg} = \frac{\Delta x}{\Delta t}$$

$$\Rightarrow \Delta x_1 = v_1 \Delta t \ \& \ \Delta x_2 = v_2 \Delta t$$

The volume of the mass flowing through the pipe equals the cross-sectional area times the length of that volume of mass, Δx.

$$\Delta V_{cylinder} = (\text{Area})(\text{Length}) = A\Delta x \Rightarrow \Delta V_1 = A_1 v_1 \Delta t$$

Which we can substitute back into the mass equation.

[1♥] Given this is ideal fluid flow, the velocity of the fluid flow in the entire cylindrical pipe with constant diameter is considered to be constant. In the real world this is not the case. In the real world fluids have viscosity or internal friction. That makes it so fluid flowing near the sides of the pipe will be slowed down and will flow more slowly, and fluid in the middle of the pipe will flow more quickly.

[2♣] Yes, uppercase V for volume and lowercase v for velocity. Be very careful!

$$\Rightarrow \Delta m_1 = \rho_1 A_1 v_1 \Delta t$$

Because there are no losses of fluid within the pipe:

$$\Delta m_1 = \Delta m_2 \Rightarrow \rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t \Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Which brings us to the continuity equation of fluid flow:

$$\Rightarrow \rho A v = \text{constant}$$

The units for this are:

$$\rho A v \Rightarrow \left(\frac{kg}{m^3}\right)\left(m^2\right)\left(\frac{m}{s}\right) = \frac{kg}{s}$$

Which makes this a mass flow rate.

Ideal fluids are incompressible; therefore, the density of ideal fluids remains constant.

$$\text{If } \rho A v = \text{constant} \ \& \ \rho = \text{constant} \Rightarrow A v = \text{constant}$$

So, the continuity equation of ideal fluid flow is:

$$\Rightarrow A v = \text{constant}$$

The units for this are:

$$A v \Rightarrow \left(m^2\right)\left(\frac{m}{s}\right) = \frac{m^3}{s}$$

Which makes this a volumetric flow rate.

And we can show that, when the diameter of the pipe narrows, the speed of the fluid increases:

$$\Rightarrow A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \left(\frac{A_1}{A_2}\right) v_1$$

$$\text{if } A_1 > A_2 \Rightarrow \frac{A_1}{A_2} > 1 \Rightarrow v_2 > v_1$$

And you can see, when you watch the animation in the video, that is exactly what is happening.