

Seminario di Algebra & Geometria
Dipartimento di Matematica “Guido Castelnuovo”
SAPIENZA Università di Roma
A.A. 2024/2025

Mercoledì 25 Settembre 2024 - ore 14:00

Oksana Yakimova (Universität Jena)

A bi-Hamiltonian nature of the Gaudin algebras

Let h be a direct sum of n copies of a simple Lie algebra g . In 1994, Feigin, Frenkel, and Reshetikhin constructed a large commutative subalgebra of the enveloping algebra $U(h)$. This subalgebra, which is an image of the Feigin—Frenkel centre, contains quadratic Gaudin Hamiltonians and therefore is known as a Gaudin subalgebra. By now it has been studied from various points of view and numerous generalisations have been obtained. We look at the ‘classical’ version of a Gaudin algebra, i.e., at its image in the symmetric algebra $S(h)$. This image, say C , is Poisson-commutative and can be obtained from a suitable pair of compatible Poisson brackets on $S(h)$ via the Lenard—Magri scheme. An advantage of the Lenard—Magri approach is a well-developed geometric machinery. For example, it allows us to show that C is algebraically closed in $S(h)$. We will discuss also a generalisation to a non-reductive setting.

Mercoledì 2 Ottobre 2024 - ore 14:00

Dmitri Panyushev (MPI Bonn)

Projections of nilpotent orbits in simple Lie algebras

Let G be a simple algebraic group and $O \subset \mathfrak{g} = \text{Lie}(G)$ a nilpotent orbit. If H is a reductive subgroup of G , then $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}$, where $\mathfrak{m} = \mathfrak{h}^\perp$. We consider the natural projections $\varphi : \bar{O} \rightarrow \mathfrak{h}$ and $\psi : \bar{O} \rightarrow \mathfrak{m}$ and two related properties of (H, O) :

(P1): $\bar{O} \cap \mathfrak{m} = \{0\}$;

(P2): H has a dense orbit in O .

We prove that $(P1) \Rightarrow (P2)$ for all O and the converse holds for $O_{\min}$, the minimal nilpotent orbit. If $(P1)$ holds, then φ is finite and $[\varphi(e), \psi(e)] = 0$ for all $e \in O$. Here the closure of $\varphi(O)$ is the closure of a nilpotent H -orbit O' and the orbits O and O' are “shared” in the sense of Brylinski—Kostant (1994). We provide a classification of all

pairs (H, O) with property $(P1)$ and discuss relations between O and O' . In particular, we point out an omission in the list of “shared” orbits.

Mercoledì 9 Ottobre 2024 - ore 14:00

Anne Moreau (Université Paris-Saclay)

Isomorphisms between W-algebras

To any vertex algebra one can attach invariants of different nature: its automorphism group, its character (a formal series), its associated variety (a Poisson variety), etc. In this talk, I will explain how to exploit the connection between these invariants to obtain nontrivial isomorphisms between W-algebras at admissible levels. To study a more general setting, one can use totally different technics developed more recently.

Mercoledì 30 Ottobre 2024 - ore 14:00

Riccardo Piovani (Università di Torino)

L^2 Aeppli and Bott-Chern cohomology

Aeppli and Bott-Chern cohomologies are useful invariants on compact complex manifolds, especially if they do not admit Kahler metrics. In this seminar we will introduce generalisations of these objects on possibly non compact Hermitian manifolds, and analyse their main properties when the manifold is complete Kahler or a Galois covering of a compact complex manifold.

Mercoledì 6 Novembre 2024 - ore 14:00

Jarod Alper (University of Washington)

Tannaka Duality and GAGA Theorems in Algebraic Geometry

Tannaka Duality refers to the reconstruction of a compact group from its representations, while Serre's GAGA theorem relates coherent sheaves on an algebraic variety to its analytification. This talk will explore various incarnations of these two classical theorems in algebraic geometry with applications to moduli theory.

Mercoledì 27 Novembre 2024 - ore 14:00

Bernhard Hanke (Universität Augsburg)

Torsion obstructions to positive scalar curvature

In 1998, Thomas Schick discovered a purely homological obstruction to the existence of positive scalar curvature metrics on oriented closed smooth manifolds in terms of torality properties of their fundamental classes. We will combine this obstruction with the enlargeability obstruction to positive scalar curvature metrics introduced by Gromov and Lawson in the 1980s. As an application we construct new examples of manifolds which do not admit positive scalar curvature metrics, but whose Cartesian products admit such metrics. Several fundamental questions remain open. The most important is whether toral manifolds of dimension at least 4 with finite fundamental groups of odd order admit positive scalar curvature metrics.

Mercoledì 4 Dicembre 2024 - ore 14:00

Gianluca Pacienza (Université de Lorraine)

Sulla congettura del cono relativa per famiglie di varietà olomorfe simplettiche.

Nel seminario presenterò un lavoro in collaborazione con A. Horing e Z. Xie nel quale studiamo la congettura del cono relativa per famiglie di varietà K -triviali con irregolarità nulla. Dopo aver introdotto e motivato la congettura e la sua versione relativa spiegherò le grandi linee della principale applicazione dei nostri risultati: la dimostrazione della congettura quando le fibre appartengono ad una delle famiglie note di varietà olomorfe simplettiche.

Mercoledì 11 Dicembre 2024 - ore 14:00

Olivier Benoist (ENS Paris)

On the rationality of real conic bundles

Deciding whether a given algebraic variety is rational (birational to projective space) is an important problem in algebraic geometry. Over the field of real numbers, this problem is particularly interesting for varieties that are known to be rational over the complex numbers, as it then has an arithmetic flavour. In this talk, I will review the techniques that are available, and focus on concrete examples of real conic bundles for which I will provide new positive and negative results. This is joint work with Alena Pirutka.

Mercoledì 8 Gennaio 2025 - ore 14:00

Guido Pezzini (Sapienza Università di Roma)

Verso la classificazione delle varietà log-omogenee

Le varietà log-omogenee sono state introdotte da Brion nel 2007, come generalizzazione delle varietà algebriche complesse omogenee complete. Queste ultime sono descritte dal Teorema di Borel-Remmert come prodotti di varietà abeliane e varietà delle bandiere, d'altra parte le varietà delle bandiere sono ben note e classificate in modo combinatorio grazie agli strumenti della teoria dei gruppi algebrici riduttivi. Brion ha dimostrato un analogo teorema di struttura per le varietà log-omogenee, dove però le varietà delle bandiere sono sostituite da varietà più generali e non ancora ben comprese. Nel seminario discuteremo un progetto di ricerca finalizzato alla classificazione combinatoria di queste varietà.

Mercoledì 15 Gennaio 2025 - ore 14:00

Fabrizio Barroero (Università Roma Tre)

On the Zilber-Pink conjecture for complex abelian varieties and distinguished categories

The Zilber-Pink conjecture is a very general statement that implies many well-known results in diophantine geometry, e.g., Manin-Mumford, Mordell-Lang, André-Oort and Falting's Theorem. It explains the behaviour of intersections between an algebraic variety and a countable family of "special varieties" in an ambient space that usually is an abelian variety, a Shimura variety or even a family of abelian varieties. After a general introduction to the problem, I will report on joint work with Gabriel Dill in which we proved that the Zilber-Pink conjecture for a complex abelian variety A can be deduced from the same statement for its trace, i.e., the largest abelian subvariety of A that can be defined over the algebraic numbers. This gives some unconditional results, e.g., the conjecture for curves in complex abelian varieties (over the algebraic numbers this is due to Habegger and Pila) and the conjecture for arbitrary subvarieties of powers of elliptic curves that have transcendental j -invariant. While working on this project we realised that many definitions, statements and proofs were formal in nature and we came up with a categorical setting that contains most known examples and in which (weakly) special subvarieties can be defined and Zilber-Pink and Mordell-Lang statements can be formulated. We obtained some conditional as well as some unconditional results.

Mercoledì 12 Febbraio 2025 - ore 14:00

Luca Schaffler (Università di Roma Tre)

Fineness and smoothness of a KSBA moduli of marked cubic surfaces

The moduli space of cubic surfaces marked by their 27 lines admits multiple compactifications arising from different perspectives. By work of Gallardo-Kerr-Schaffler, it is known that Naruki's cross-ratio compactification is isomorphic to the normalization of the Kollár, Shepherd-Barron, Alexeev (KSBA) compactification parametrizing pairs $(S, (1/9+\epsilon)D)$, where D is the sum of the 27 marked lines on S , along with their stable degenerations. In this talk, we show that the normalization assumption is unnecessary by proving that this KSBA compactification is smooth. Additionally, we show it is a fine moduli space. This is achieved by studying the automorphisms and the Q-Gorenstein obstructions of the stable pairs it parametrizes. This is joint work with Hanlong Fang and Xian Wu.

Mercoledì 19 Febbraio 2025 - ore 14:00

Valerio Proietti (University of Oslo)

Chern character and groupoid-equivariant bivariant K-theory

Building on previous work by Davis and Lück, and recent constructions of bivariant K-theory as a stable ∞ -category, I will sketch the construction of a Chern character running from groupoid-equivariant K-homology to (periodicized) homology groups of the given groupoid. Under the assumption that the isotropy groupoid is torsion free, this map is a rational isomorphism, thereby establishing a modified form of Matui's HK conjecture (after integral counterexamples have been found). This construction also computes the rational homotopy type of the algebraic K-theory spectrum of ample groupoids as defined in a recent work by X. Li. This is joint work-in-progress with M. Yamashita.

Mercoledì 26 Febbraio 2025 - ore 14:00

Nicola Cavallucci (EPFL Lausanne)

Rigidity theorems for CAT(0) spaces

I will present some rigidity theorems about cocompact CAT(0)-spaces. For instance, I will show that if a cocompact CAT(0) space is close enough to a Euclidean space, then it is isometric to it. In the first part of the talk I will define all the objects and I will provide examples. In the second part I will sketch some proof, using a careful analysis of converging

sequences of cocompact $CAT(0)$ spaces and of the groups acting on them. The talk is partially based on joint works with Andrea Sambusetti.

Mercoledì 5 Marzo 2025 - ore 14:00

Domenico Valloni (EPFL Lausanne)

Classi di Brauer supersingolari in caratteristica positiva

Lo scopo di questo talk è di definire e studiare certe classi di Brauer in caratteristica positiva che si possono costruire dalle forme differenziali. Dopo aver spiegato la costruzione e le prime proprietà di questi oggetti, dimostreremo che il gruppo di Brauer delle superfici K3 supersingolari può essere ottenuto in questo modo. Se il tempo permette, parleremo anche del ruolo che queste classi hanno nell'ostruzione di Brauer-Manin al principio locale-globale.

Mercoledì 12 Marzo 2025 - ore 14:00

Amnon Neeman (Milano Statale / Australian National University)

Vanishing negative K-theory and bounded t-structures

We will begin with a quick reminder of algebraic K-theory, and a few classical, vanishing results for negative K-theory. The talk will then focus on a striking 2019 article by Antieau, Gepner and Heller - it turns out that there are K-theoretic obstructions to the existence of bounded t-structures.

The result suggests many questions. A few have already been answered, but many remain open. We will concentrate on the many possible directions for future research.

Mercoledì 19 Marzo 2025 - ore 14:00

Tuan Ngo Dac (Université de Caen)

On moduli spaces of shtukas

In this talk we give an overview of the geometry of moduli spaces of shtukas for the general linear groups $GL(n)$. These moduli spaces have been instrumental in the

work of Drinfeld and Lafforgue on the Langlands correspondence for $GL(n)$ over function fields. We then explore the connection between the geometry of these moduli spaces and the wonderful compactification of De Concini-Procesi. This talk is based on joint work with Y. Varshavsky.

Mercoledì 02 Aprile 2025 - ore 14:00

Dieter Kotschick (LMU München)

Complex and almost complex structures on hyperbolic manifolds

According to the uniformisation theorem, in complex dimension one, there is an intimate connection between complex and hyperbolic structures. However, in higher dimensions, the two geometries diverge. We will discuss both the existence of almost complex structures on hyperbolic manifolds, and the potential obstructions to their integrability. In odd complex dimensions the discussion generalizes to certain stably parallelizable manifolds, of which the hyperbolic ones constitute a very special subclass.

Mercoledì 9 Aprile 2025 - ore 14:00

Aitor Iribar Lopez (ETH Zürich)

Noether-Lefschetz cycles on the moduli space of abelian varieties

We study A_g , the moduli space of principally polarized abelian varieties of dimension g . The tautological ring, generated by the Chern classes of the Hodge bundle, was fully determined by Gerard van der Geer in 1999, but the question of which geometrically defined cycles belong to this subring remains open. In 2024, Canning, Oprea and Pandharipande showed that $[A_1 \times A_5]$ is not tautological in A_6 , and later I showed that $[A_1 \times A_{g-1}]$ is not tautological for $g = 12$ or $g \geq 16$ even.

The cycle $[A_1 \times A_{g-1}]$ is one of the Noether-Lefschetz cycles on the moduli spaces. With Greer and Lian, we conjecture that these cycles are related to modular forms of weight $2g$.

A new technique, which was not available in 1999 is the existence of a projection operator by Canning, Oprea, Molcho and Pandharipande onto the tautological ring. This leads to interesting conjectures about Gromov-Witten invariants on a moving elliptic curve, which now have been proven in collaboration with Pandharipande and Tseng, and are also connected to the failure of the Gorenstein conjecture on the moduli space of curves of compact type

The talk will be an overview of the intersection theory of A_g and the moduli space of curves, and I will explain briefly the ideas behind some proofs.

Mercoledì 16 Aprile 2025 - ore 14:00

Antonella Perucca (Université du Luxembourg)

Almost 100 years of Artin's Conjecture

In 1927, Artin formulated his famous conjecture on primitive roots. The most basic question, which is still open, is as follows. For an odd prime number p , we say that 2 is a primitive root modulo p if every integer that is not a multiple of p is congruent to a power of 2 modulo p . Are there infinitely many primes p such that 2 is a primitive root modulo p ?

Artin's conjecture (that has been proven by Hooley in 1967 under GRH) says in particular that there is a positive density of primes p such that 2 is a primitive root modulo p . This density is conjectured to be Artin's constant, which is roughly 37%. Beyond the classical results, I will present very general results on the index map (joint work with Järvinen and Sgobba), an unexpected lower bound on the density (joint work with Shparlinski) and a computational project (joint work with Tholl).

Mercoledì 30 Aprile 2025 - ore 14:00

Antonio Trusiani (Chalmers University of Technology)

Extremal Kähler metrics on modifications

I will present the invariance of extremal Kähler manifolds under a suitable class of bimeromorphic morphisms. Focusing mostly on the cscK (constant scalar curvature Kähler) case, I will show how the main result is obtained as a consequence of a general uniform coercivity estimate for the Mabuchi energies of perturbed Kähler classes. I will then explain how such variational approach applies more generally to the class of weighted extremal metrics, modulo a log-concavity assumption on the first weight, and to any equivariant resolution of singularities of Fano type of a compact Kähler klt space whose weighted Mabuchi energy is assumed to be coercive. This is a joint work with S. Boucksom and M. Jonsson.

Mercoledì 7 Maggio 2025 - ore 14:00

Younghan Bae (University of Michigan)

Fourier transform, Abel-Jacobi theory and intersection theory on compactified Jacobians

Fourier analysis is a powerful tool in analysis. In the setting of abelian schemes, Fourier-Mukai transformation and the weight decomposition play a similar role. For degenerate abelian fibrations, the relative group structure disappears and understanding the intersection theory leads to many interesting questions, such as the $P=W$ conjecture, χ -independence phenomenon, and multiplicative splitting of the perverse filtration for the Beauville-Mukai system.

In this talk, I will connect Fourier transform between compactified Jacobians over the moduli space of stable curves and logarithmic Abel-Jacobi theory. As an application, I will compute the pushforward of monomials of divisor classes on compactified Jacobians via the twisted double ramification formula. Along the way, we will encounter instances of χ -independence and the multiplicativity of perverse filtration for compactified Jacobians. This is a joint work with Samouil Molcho and Aaron Pixton.

Mercoledì 14 Maggio 2025 - ore 14:00

Matthias Ludewig (Universität Greifswald)

Coarse cohomology of partitions

One of the challenges in the study of topological insulators is the generalization of invariants such as Chern numbers to the case of disordered (i.e., non translation-invariant) systems. In two dimensions, a proposal was made by Kitaev in his 2005 „Anyons“ paper, which uses a partition of space into three subsets and then measures the difference of clockwise and counter-clockwise hopping between these subsets. In recent work with Guo Chuan Thiang, we were able to interpret the Kitaev formula as a pairing between coarse cohomology and Roe algebra K -theory. As we explain in this talk, this provides a generalization to arbitrary dimensions, as well as a proof that the values of this Kitaev pairing are “quantized”, i.e., only take a discrete set of values.

Mercoledì 21 Maggio 2025 - ore 14:00

Leonid Ryvkin (Université Lyon 1)

Universal central extensions of the Lie algebra of volume-preserving vector fields

The goal of the talk is proving a conjecture of Claude Roger about the universal central extension of the Lie algebra of volume-preserving vector fields. In the beginning we will briefly review the notion of central extensions of Lie algebras and their link to Chevalley-Eilenberg-cohomology. We will then proceed to Roger's conjecture, which lies in the (continuous) infinite-dimensional setting. To solve it we will need a combination of analytical and geometrical methods, and maybe even a bit of representation theory.

Based on joint work with Bas Janssens and Cornelia Vizman.

Mercoledì 28 Maggio 2025 - ore 14:00

Andriy Regeta (Università di Padova)

A characterization of rationality and solvable subgroups in the Cremona group

In this talk I will present the following two results about the group of birational transformations (which we denote by $\text{Bir}(X)$) of an irreducible variety X :

The first one: If $\text{Bir}(X)$ is isomorphic to $\text{Bir}(\mathbb{P}^n)$, then X is rational.

The second one: a closed connected solvable subgroup of $\text{Bir}(X)$ has derived length less than or equal to $2 \dim X$ and the equality holds iff X is rational. Moreover, all Borel subgroups (maximal connected solvable subgroups) of $\text{Bir}(\mathbb{P}^n)$ of derived length $2n$ are conjugate. Furthermore, Borel subgroups of $\text{Bir}(\mathbb{P}^n)$ of derived length strictly smaller than $2n$ exist.

This talk is based on the joint work with C. Urech and I. Van Santen.

Mercoledì 4 Giugno 2025 - ore 14:00

Andrea Appel (Università di Parma)

The R-matrix for affine Yangians

The theory of Yangians was introduced by Drinfeld in the 1980s as a systematic approach to solving the Yang-Baxter equation: every irreducible finite-dimensional representation is equipped with a rational R-matrix obtained by normalising the

action of a universal R -matrix. In this talk, I will provide a generalisation of this result for Yangians of affine type and their representations in category O .

Mercoledì 11 Giugno 2025 - ore 14:00

Hélène Eynard-Bontemps (Université Grenoble Alpes)

Deforming foliations and commuting diffeomorphisms

A codimension one foliation of a 3-manifold is a partition of the manifold by immersed surfaces which locally pile up nicely like parallel affine planes but which can have a very complicated global behavior. I will start by presenting a beautiful proof by Thurston that any plane field on a 3-manifold can be homotoped to one that is tangent to a foliation (such a plane field is called integrable). In order to understand the space of all foliations existing on a given manifold, the next natural question is:

If two foliations have homotopic tangent plane fields, can they be connected by a path of integrable plane fields?

I will explain how this question can be reduced to the following problem of one-dimensional dynamics:

Can any two pairs of commuting diffeomorphisms of the interval be connected by a path of such pairs?

If time permits, I will unveil some of the subtleties hidden behind this still open question.

Mercoledì 18 Giugno 2025 - ore 14:00

Luca Francone (Università di Roma Tor Vergata)

Schemes of bands and their cluster structures

This talk is about a family of infinite dimensional affine schemes recently introduced by myself and Bernard Leclerc called schemes of (G, c) -bands. Here G denotes a simple simply laced and simply connected algebraic group, and c is a Coxeter element of its Weyl group. These schemes offer a geometric framework for understanding certain cluster algebras which play a central role in the representation theory of quantum affine algebras, their Borel subalgebras, and shifted quantum affine algebras. The goal of the talk is to introduce these schemes and explore their connections with cluster algebras and representation theory. Time permitting, I will mention some possible developments.

Mercoledì 25 giugno 2025 - ore 14:00

Margherita Pagano (Imperial College London)

Il ruolo dei primi di buona riduzione nell'ostruzione di Brauer–Manin

In questo seminario ci concentreremo sullo studio di punti razionali su varietà algebriche, un tema centrale nello studio delle soluzioni di equazioni polinomiali. Dopo una breve introduzione all'insieme di Brauer–Manin e al suo utilizzo nello studio di punti razionali, discuteremo come le proprietà della riduzione modulo p della varietà possano essere utilizzate per lo studio dell'ostruzione di Brauer–Manin. In particolare, esploreremo il ruolo dei primi di buona riduzione in questo contesto.