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CARRERA: INGENIERIA EN ELECTRONICA Y TELECOMUNICACIONES
TEMA: CORRECCION PRUEBA 2
PARALELO:GR4

CORRECCION DE SEGUNDA PRUEBA

1.- Analizar si satisface las Ecuaciones de Cauchy-Riemann en el origen.

$$f(z) = \begin{cases} \dfrac{\bar{z}^5}{|z|^4} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$$

$$z = x+iy \Rightarrow \bar{z} = x - iy \Rightarrow \bar{z}^5 = x^5 - i5x^4y - 10x^3y^2 + i10x^2y^3 + 5xy^4 - iy^5$$

$$\Rightarrow |z| = \sqrt{x^2+y^2} \Rightarrow |z|^4 = (x^2+y^2)^2 = x^4 + 2x^2y^2 + y^4$$

$$f(z) = \frac{x^5 - i5x^4y - 10x^3y^2 + i10x^2y^3 + 5xy^4 - iy^5}{x^4 + 2x^2y^2 + y^4}$$

$$f(z) = \underbrace{\frac{x^5 - 10x^3y^2 + 5xy^4}{x^4+2x^2y^2+y^4}}_{u(x,y)} + i\underbrace{\frac{-5x^4y + 10x^2y^3 - y^5}{x^4+2x^2y^2+y^4}}_{v(x,y)}$$

Analizando las ECR

i) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$, ii) $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

i) $\dfrac{\partial u}{\partial x} = u_x = \lim\limits_{x\to 0} \dfrac{u(x,0)-u(0,0)}{x-0} = \lim\limits_{x\to 0}\left(\dfrac{\frac{x^5 - 10x^3(0)^2 + 5(x)(0)^4}{x^4+2x^2(0)^2+(0)^4} - 0}{(x-0)}\right)$

$$\frac{\partial u}{\partial x} = u_x = \lim_{x\to 0} \frac{\frac{x^5}{x^4}}{x} = \lim_{x\to 0} 1 = 1$$

$$\frac{\partial v}{\partial y} = v_y = \lim_{y\to 0} \frac{v(0,y)-v(0,0)}{y-0} = \lim_{y\to 0}\left(\frac{\frac{-5(0)^4y + 10(0)^2y^3 - y^5}{(0)^4+2(0)^2y^2+y^4} - 0}{y-0}\right)$$

$$= \lim_{y\to 0} \frac{-\frac{y^5}{y^4}}{y} = \lim_{y\to 0}(-1) = -1$$

$\dfrac{\partial u}{\partial x} \neq \dfrac{\partial v}{\partial y}$ ∴ Satisface

ii) $\dfrac{\partial u}{\partial y} = u_y = \lim\limits_{y\to 0} \dfrac{u(0,y)-u(0,0)}{y-0} = \lim\limits_{y\to 0} \dfrac{\frac{(0)^5 - 10(0)^3y^2 + 5(0)y^4}{(0)^4+2(0)^2y^2+y^4} - 0}{y-0}$

$$\frac{\partial u}{\partial y} = u_y = \lim_{y\to 0} \frac{\frac{0}{y^4}}{y} = \lim_{y\to 0}(0) = 0$$

$$\frac{\partial u}{\partial x} = U_x = \lim_{x\to 0} \frac{v(x,0) - v(0,0)}{x-0} = \lim_{x\to 0} \frac{\frac{-5x^4(0)+10x^2(0)^3-(0)^5}{x^4+2x^2(0)^2+(0)^4} - 0}{x-0}$$

$$\frac{\partial u}{\partial x} = V_x = \lim_{x\to 0} \frac{v(x,0) \cdot v(0,0)}{x-0} = \lim_{x\to 0} \frac{\frac{-5x^4(0)+10x^2(0)^3-(0)^5}{x^4+2x^2(0)^2+(0)^4} - 0}{x-0}$$

$$\frac{\partial u}{\partial x} = V_x = \lim_{x\to 0} \frac{\frac{0}{x^4}}{x} = \lim_{x\to 0} (0) = 0$$

$$\rightarrow \frac{\partial u}{\partial y} = -\frac{\partial u}{\partial x}$$

∴ Por i $f(z)$ no satisface las ecuaciones de C R en el origen

2. Halla $f(z)$ si $Re\{f'(z)\} = 3x^2 - 4y - 3y^2$ y $f(1+i) = 0$

Si $f(z) = u + iv$ → analitica

$$Re[f'(z)] = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3x^2 - 4y - 3y^2$$

$$u'x + u'y = 3x^2 - 4y - 3y^2$$

$$u_x = \int (3x^2 - 4y - 3y^2)\,dx = x^3 - 4xy - 3xy^2 + g(y)$$

$$u_y = \int (3x^2 - 4y - 3y^2)\,dy = 3x^2y - 2y^2 - y^3 + h(x)$$

$$u = u_x + u_y = x^3 - 4xy - 3xy^2 + g(y) + 3x^2y - 2y^2 - y^3 + h(x)$$

$$u = x^3 - 4xy - 2y^2 - y^3 + g(y) + h(x)$$

Por ECR.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Rightarrow u'x = 3x^2 - 4y + h'(x)$$

$$\Rightarrow V_y = \int (3x^2 - 4y + h'(x))\,dy = 3x^2y - 2y^2 + yh'(x)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \Rightarrow u'y = -4x - 4y - 3y^2 + g'(y)$$

$$\Rightarrow V_x = \int (4x + 4y + 3y^2 - g'(y))\,\partial y = 2x^2 + 4xy + 3xy^2 - xg'(y)$$

$$\Rightarrow V = V_x + V_y = 2x^2 + 4xy + 3xy^2 - xg'(y) + 3x^2y - 2y^2 + yh'(x)$$

$\Rightarrow V = 2x^2 + 3x^2 y + 4xy + 3xy^2 - 2y^2 - xg'(y) + yh'(x)$

$$\Rightarrow f(z) = x^3 - 4xy - 2y^2 - y^3 + g(y) + h(x) + i(2x^2 + 3x^2 y + 4xy + 3xy^2 - 2y^2 - xg'(y) + yh'(x))$$

Si $f(1, i) = 0$

$$0 + 0i = 1^3 - 4(1)(1) - 2(1)^2 - 1^3 + g(y) + h(x) + i(2(1)^2 + 3(1)^2 + 4(1)(1) + 3(1)(1)^2 - 2(1)^2 - (1)g'(y) + (1)h'(x))$$

$0 = -6 + g(1) + h(1) \quad \wedge \quad i(0) = i(10 - g'(1) + h'(1))$

$\Rightarrow g(1) + h(1) = 6 \qquad g'(1) - h'(1) = 10$

$$f(z) = x^3 - 4xy - 2x^2 - y^3 + 6 + i(2x^2 + 3x^2 y + 4xy + 3xy^2 - 2y^2 - 10)$$

3.- Diga si $f(z) = \tan z$ es analítica

$f(z) = \dfrac{\operatorname{sen}(z)}{\cos(z)}$ Dominio de la función

$D_f = \mathbb{C} - \{ z / \cos(z) = 0 \}$

$\cos z \neq 0$

$z \neq \frac{\pi}{2}$

$z \neq \frac{3\pi}{2}$

$D_f = \mathbb{C} - \{ \frac{\pi}{2}, \frac{3\pi}{2} \}$

$$\operatorname{sen} z = \frac{e^{z} - e^{-z}}{2i} = \frac{e^{x}(\cos y + i\operatorname{sen} y) - e^{-x}(\cos y - i\operatorname{sen} y)}{2i} = \frac{e^{x}\cos y - e^{-x}\cos y + i(e^{x}\operatorname{sen} y + e^{-x}\operatorname{sen} y)}{2i}$$

$$\operatorname{sen} z = (e^{x}\operatorname{sen} y + e^{-x}\operatorname{sen} y) + i(e^{-x}\cos y - e^{x}\cos y) = \frac{1}{2}(\operatorname{sen} y(e^{x} + e^{-x}) + i\cos y(e^{-x} - e^{x}))$$

$$\cos z = \frac{e^{z} + e^{-z}}{2} = e^{x}(\cos y + i\operatorname{sen} y) + e^{-x}(\cos y - i\operatorname{sen} y) = \frac{e^{x}\cos y + e^{-y}\cos y + i(e^{x}\operatorname{sen} y - e^{-x}\operatorname{sen} y)}{2}$$

$$\cos z = \frac{1}{2}(\cos y(e^{x} + e^{-x}) + i\operatorname{sen} y(e^{x} - e^{-x}))$$

$$\tan z = \frac{\frac{1}{2}(\operatorname{sen} y(e^{x} + e^{-x}) + i\cos y(e^{-x} - e^{x}))}{\frac{1}{2}(\cos y(e^{x} + e^{-x}) + i\operatorname{sen} y(e^{x} - e^{-x}))} \cdot \frac{\cos y(e^{x} + e^{-x}) - i\operatorname{sen} y(e^{x} - e^{-x})}{\cos y(e^{x} + e^{-x}) - i\operatorname{sen} y(e^{x} - e^{-x})}$$

$$\tan z = \frac{\operatorname{sen} y \cos y\,(e^{x}+e^{-x})^2 + \operatorname{sen} y \cos y\,(e^{-x}-e^{x})(e^{-x}-e^{x}) + i\,(\cos^2 y\,(e^{-x}-e^{x})(e^{x}+e^{-x}) - \operatorname{sen}^2 y\,(e^{x}+e^{-x})(e^{x}-e^{-x}))}{\cos^2 y\,(e^{x}+e^{-x})^2 + \operatorname{sen}^2 y\,(e^{x}-e^{-x})^2}$$

$$\tan z = \frac{\operatorname{sen} y \cos y\,(e^{2x}+e^{-2x}+2+2-e^{2x}-e^{-2x}) + i(e^{-2x}+e^{2x})(\cos^2 y+\operatorname{sen}^2 y)}{\cos^2 y\, e^{2x} + \cos^2 y\, e^{-2x} + 2\cos^2 y + \operatorname{sen}^2 y\, e^{2x} + \operatorname{sen}^2 y\, e^{-2x} - 2\operatorname{sen}^2 y}$$

$$\tan z = \frac{4\operatorname{sen} y \cos y + i(e^{-2x}+e^{2x})}{(e^{2x}+e^{-2x})(\cos^2 y+\operatorname{sen}^2 y) + 2(\cos^2 y - \operatorname{sen}^2 y)} = \frac{4\operatorname{sen} y\cos y + i(e^{-2x}+e^{2x})}{e^{2x}+e^{-2x}+2\cos 2y}$$

$$\tan z = \frac{4\operatorname{sen} y \cos y}{e^{2x}+e^{-2x}+2\cos 2y} + \frac{i\,e^{-2x}+e^{2x}}{e^{2x}+e^{-2x}+2\cos 2y}$$

Analizando las ECR.

$$\text{i)}\ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \text{ii)}\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{-4\operatorname{sen} y\cos y\cdot(2e^{2x}-2e^{-2x})}{(e^{2x}+e^{-2x}+2\cos 2y)^2}$$

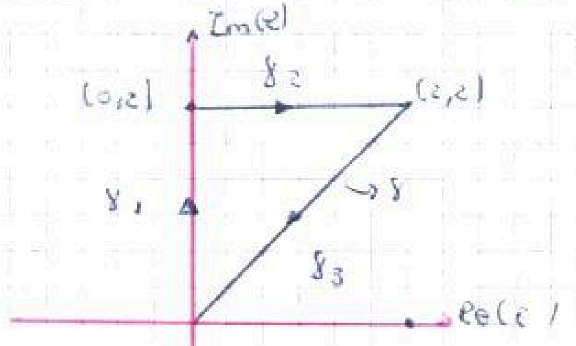
$$\frac{\partial v}{\partial y} = \frac{-(e^{-2x}+e^{2x})(2)(-\operatorname{sen} 2y)(2)}{(e^{2x}+e^{-2x}+2\cos 2y)} = \frac{4\operatorname{sen} 2y(e^{-2x}+e^{2x})}{(e^{2x}+e^{-2x}+2\cos 2y)^2}$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$ no satisface las ECR

$\therefore f(z) = \tan z$ no es analítica

4.- Evaluar la siguiente integral de línea

$\int_\gamma f(z)\,dz$, si $f(z) = \bar{z}^2$, y γ es la frontera del triángulo de vértices $(0,0)$, $(2,2)$ y $(0,2)$



Si $z = x+iy \Rightarrow \bar{z} = x - iy$

$\bar{z}^2 = x^2 - y^2 - i2xy$

Dom $f : \mathbb{C} \Rightarrow f$ es analítica.

$\gamma = \gamma_1 \cup \gamma_2 \cup \gamma_3$

Por propiedades

Curva Recta $\Rightarrow z(t) = z_0 + t(z_f - z_0)$

$\gamma_1 : z(t) = 0+0i + t(0+2i - 0 - 0i) = 2ti \Rightarrow z'(t) = 2i \quad , \; 0 \leq t \leq 1.$

continua en $\mathbb{C}$ $\quad z'(t) \neq 0 \rightarrow \gamma_1$ curva suave

$\gamma_2 \quad z(t) = 0+2i + t(2+2i - 0 - 2i) = 2t + i2 \Rightarrow z'(t) = 2 \quad , \; 0 \leq t \leq 1$

continua en $\mathbb{C}$ $\quad z'(t) \neq 0 \rightarrow \gamma_2$ curva suave

$\gamma_3 \quad z(t) = 2+2i + t(0+0i - 2 - 2i) = 2 - 2t + i(-2t+2) \; , \; 0 \leq t \leq 1$
continua en $\mathbb{C}$.

$z'(t) = -2 - 2i$

$z'(t) \neq 0 \rightarrow \gamma_3$ curva suave

$$\int_\gamma f(z)\, dz = \int_\gamma \bar{z}^2 dz = \int_{\gamma_1} \bar{z}^2 dz + \int_{\gamma_2} \bar{z}^2 dz + \int_{\gamma_3} \bar{z}^2 dz$$

$$\Rightarrow \int_\gamma \bar{z}^2 dz = \int_0^1 (\overline{2ti})^2 \cdot 2i\, dt + \int_0^1 (\overline{2t+2i})^2 \cdot 2\, dt + \int_0^1 (\overline{2-2t+i(-2t+2)})^2 (-2-2i)\, dt$$

$$\int_\gamma \bar{z}^2 dz = \int_0^1 (-4t^2)\, 2i\, dt + \int_0^1 (4t^2 - 4 - i8t)\, 2\, dt + \int_0^1 (8 - 8t + 8t^2 + i(-8 + 8t - 8t^2))\, dt$$

$$\int_\gamma \bar{z}^2 dz = -8i\int_0^1 t^2 dt + 8\int_0^1 t^2 dt - 8\int_0^1 t^2 dt - 8\int_0^1 dt - 16i\int_0^1 t\, dt + 8\int_0^1 dt - 8\int_0^1 t\, dt + 8\int_0^1 t^2 dt$$
$$- 8i\int_0^1 dt + 8i\int_0^1 t\, dt - 8i\int_0^1 t^2 dt$$

$$\int_\gamma \bar{z}^2 dz = -8i\left(\tfrac{1}{3} - 0\right) + 8\left(\tfrac{1}{3} - 0\right) - 8(1 - 0) - 16i\left(\tfrac{1}{2} - 0\right) + 8(1 - 0) - 8\left(\tfrac{1}{2} - 0\right) + 8\left(\tfrac{1}{3} - 0\right)$$

$$\int_\gamma \bar{z}^2 dz = \frac{4}{3} - i\,\frac{52}{3}$$

5_ Encuentre todos los números complejos z que cumplen la ecuación dada:

$\cosh(z) = 2$

5. $\cosh(z) = \dfrac{e^{z}+e^{-z}}{2} \Rightarrow \dfrac{e^{z}+e^{-z}}{2} = 2$

$$e^{z}+e^{-z} = 4$$

$$(e^{z})(e^{z}+e^{-z}) = 4e^{z}$$

$$e^{2z}+1 = 4e^{z}$$

$$e^{2z}-4e^{z}+1 = 0$$

$a=1$, $b=-4$, $c=1$, $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$e^{z} = \frac{4 \pm \sqrt{16-4}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

$$e^{z_1} = 2+\sqrt{3} \quad , \quad e^{z_2} = 2-\sqrt{3}$$

$$\ln(e^{z_1}) = \ln(2+\sqrt{3}) \Rightarrow z_1 = \ln(2+\sqrt{3}) = 1{,}317+0i$$

$$\ln(e^{z_2}) = \ln(2-\sqrt{3}) \Rightarrow z_2 = \ln(2-\sqrt{3}) = -1{,}317+0i$$