

Def Integral without fund thm:

http://matheducators.stackexchange.com/questions/11779/evaluating-integrals-geometrically-without-using-the-fundamental-theorem-of-cal/11784?noredirect=1#comment27606_11784

history: <http://mathpages.com/rr/s8-01/8-01.htm>

riemann shadows: mrhonner.com/archives/14528

Calc II

graphing slopes:

mathhombre.tumblr.com/post/115134292804/derivative-intuition-this-nice-idea-for

polar: <http://infinitesums.com/commentary/2014/3/11/sidewalk-chalk-three-techniques/>

<http://mathmisery.com/wp/2014/04/20/some-cute-integrals-u-substitutions/>

good definite integral questions:

<http://www.simplylearnt.com/AIEEE-exam/test-question/Definite-Integration>

Calculus I: Organized to keep students thinking

(Derivatives First, Derivative Rules anchored with good introductory problems)

Conventional textbooks start with a review chapter (boring), and a chapter on limits (alienating). Historically, calculus was invented to solve real problems in science. It's exciting, and we can bring it to students that way. Chapter One: Understanding the Derivative. (And a chapter zero on the history.)

Conventional textbooks devote just one section to understanding the derivative (maybe two, if you're lucky), and then there's an onslaught of 'derivative rules'. How about a whole chapter on

understanding the derivative, and then have the following chapters develop few properties of derivatives each, along with some work that uses them? Chapter Two: Graphing, and Derivatives of Polynomials, Chapter Three: Guitar Strings, Roller Coasters, Optimizing, and Derivatives of Trig Functions (along with product 'rule' and quotient 'rule'), Chapter Four: Exponential Growth and Decay, and Derivatives of everything else (includes implicit differentiation, related rates, ...)

I want to write this textbook that engages students in the exciting exploration of calculus, and keeps them thinking. One question I have is whether to introduce integration early - historically it came together with derivatives, because there were problems that needed it. But it may be simpler to bring it in after we've gotten solid in our handling of derivatives. Chapter Six: Estimating Areas, and getting serious about those Limits (includes Fundamental Theorem of Calculus, etc, etc)

The rest of this document is mainly links to cool lessons and ideas for calculus.

New Links (not yet organized)

<http://jliszka.github.io/2013/10/24/exact-numeric-nth-derivatives.html>

<http://freepdfdb.org/pdf/projects-that-integrate-engineering-physics-calculus-and-31365599.html>
(3 building projects)

<http://alwaysformative.blogspot.com/2013/04/shuffle-quiz.html> (velocity and distance)

<http://samjshah.com/2013/05/10/some-random-things-i-have-liked/>

<https://bowmanimal180.wordpress.com/2013/02/05/day-76-the-second-fundamental-theorem/comment-page-1/#comment-68>

<http://shawncornally.com/wordpress/?p=3340> (sound waves, chain rule)

<http://kalamitykat.com/2013/02/19/intro-to-projectile-motion/>

<http://bowmanimal180.wordpress.com/2013/03/07/day-96-how-do-we-find-the-area-of-something-curved/> (duxford museum)

<http://bowmanimal180.wordpress.com/2013/03/03/day-92-intro-to-related-rates/comment-page-1/#comment-76>

<http://thephysicsvirtuosi.com/posts/trigonometric-derivatives.html>

<http://www.epsilon-delta.org/2013/03/introduction-to-tangent-lines.html>

<http://threesixty360.wordpress.com/2013/03/29/third-derivatives-in-the-news-again/>

<http://bowmanimal180.wordpress.com/2013/03/20/day-105-the-results-of-our-area-contest/>

<http://bowmanimal180.wordpress.com/2013/03/17/day-102-integration-folding-story/>

<http://bowmanimal180.wordpress.com/2013/03/13/day-100-integration-first-stab-guesser-and-checker/>

<http://bowmanimal180.wordpress.com/2013/03/11/day-98-why-do-we-need-integrals/>

<http://www.abstractmath.org/Word%20Press/?p=8501>

<http://www.math.hmc.edu/funfacts/ffiles/10001.2-5.shtml> (sum of cubes)

Resources for the course

- Boelkins' text and videos:
<http://opencalculus.wordpress.com/2013/12/30/active-calculus-v-12-30-13-updates/>
- An infinitesimal text: <http://www.math.wisc.edu/~keisler/calc.html>
- Bowman Dickson: bowmandickson.com
- Shawn C's good ideas: http://shawncornally.com/wordpress/?page_id=306
- And Sam's links: <http://samjshah.com/worksheets-projects/>
- big picture:
<http://castingoutnines.wordpress.com/2010/08/08/calculus-and-conceptual-frameworks/>
- <http://bowmandickson.com/2012/05/13/and-the-calculus-final-projects-begin/>
-

Not sure when to use:

- Videos: <http://www.cosmolearning.com/topics/calculus/>
-
- <http://bowmandickson.com/2012/10/20/teaching-through-concrete-examples-the-intermediate-value-theorem/#comment-1544>
- <http://www.intmath.com/differentiation/derivative-graphs.php> (derivative applets)
- <http://bowmanimal180.wordpress.com/2012/12/02/day-46-coloring-in-calculus-classes/>
- graphing:
<http://games.commonsc.gc.cuny.edu/2012/09/19/calculus-art-on-the-wall-game-teena-carroll-saint-norbert-college/>
- Review with Taboo cards: <http://drawingonmath.blogspot.com/2011/11/taboo-review.html>

Before we Start

<http://chronicle.com/blognetwork/castingoutnines/2013/06/25/what-math-topic-do-engineering-faculty-rate-as-the-most-important/>

Unit 1. Exploring Rate of Change & Slopes of curvy lines (with informal limit exploration) (in [Active Calculus](#), it's understanding the derivative) 3 weeks

I want to dig into this from lots of perspectives (position, velocity, acceleration; graphs; slope and rate of change; tables of values; finding the tangent; ...). And I want students to get a bit of grounding in the wonderful history of calculus as we start up. [Active Calculus](#) has lots of good explorations for this unit.

Chapter zero: History

- 3 phases: ancient (Greeks and circles), 1600s (Newton & Leibniz), 1800s (logical foundations)
- <http://www.quadrivium.info/mathhistory/DennisKreinovichRump.pdf>
- Calc history webquest: <http://web.archive.org/web/20070117173301/http://staff.jccc.net/jcrabtre/mission.html>
- <http://apcentral.collegeboard.com/apc/members/features/2015.html>
-

Morris Kline, *Calculus: An Intuitive and Physical Approach*, pages 1 to 6: Scientists in 17th century (1600s) were studying

1. **Motion**: planetary & projectile
2. **tangents** to curves, for projectiles (will it hit head on?) & for lenses (telescopes & microscopes)
3. **Optimizing** (angle of cannon for max range, planet's max and min distance from sun)
4. Lengths of curves, areas, volumes, also center of gravity (mostly delayed to calc II)

Chapter One: Exploring the Idea of the Derivative

- Graphing Story thoughts and videos: <http://castingoutnines.wordpress.com/2010/01/11/simplifying-calculus-by-assuming-linearity/>
- <http://samjshah.com/2012/11/01/what-does-it-mean-to-be-going-58-mph-at-203pm/>
- As intro: <http://juliatsygan.blogspot.com/2012/03/intro-to-calculus-graphing-stories-way.html>
- Ask students: What does the word 'speed' mean?
- Dog chasing ball: <http://www.youtube.com/watch?v=gy7uR-7tmJE>
- Guy running on hill: <http://www.youtube.com/watch?v=bVDwAOqbc6s>
- "What is slope?" Describing graphs: <http://studyofchange.wordpress.com/2011/09/17/week-1-practicing-engagement/#comment-18>
- slope, needs goegebra: <http://sweenymath.blogspot.com/2010/11/geogebra-derivatives-with-limits.html>
- Find tangent to $y=x^3+x^2$ at (1,2): <http://juliatsygan.blogspot.com/2012/04/tangents-and-normals-peer-scaffolding.html?showComment=1334929132924#c3860963918167197651>

- newton's method, using python: see 27 min in on this video:
<https://www.youtube.com/watch?v=Qe4SD6UNseE>
- The belly flop mentioned in [a comment to Robert's post](#) sounds good, except that many of my urban students may not have experience with diving.
- **Review: function composition:**
<http://www.epsilon-delta.org/2012/12/function-composition.html?showComment=1356017962664#c9171172638617822036>
- <http://www.epsilon-delta.org/2012/08/i-heart-these-two-calc-problems.html> (#1 good here)
- **Video:**
http://www.youtube.com/watch?v=Fzrkq1r-sAI&list=PL9bljQJDwfGuXQHUS5Jkmum_CFILoCZX- (goes with active calc)

Sept 9: Now I want some review activities for folks who don't do well on the test.

1. Christopher suggests finite differences:

Many students are now coming through K-12 experiences that have emphasized patterns of change in algebraic tables. When we write equations describing these patterns of change, we get the "not-quite derivative". E.g. a table that counts by 1 for $y=x^2$ will yield the pattern $y_1=2x+1$ (or $2x-1$, depending on how you assign the differences to the inputs). If we make the interval smaller (e.g. counting by $\frac{1}{2}$ or $\frac{1}{4}$), and adjust appropriately in ways that correspond to the ways we work with slopes, we get an expression that better approximates the derivative. We get a sequence of functions that converges (of course) to the derivative $y_1=2x$.

2. [Paul has his students measure wind-up cars' speeds](#)
- 3.

Chapter Two: Graphing and derivatives of polynomials

Binomial Thm for power rule:

<http://numberwarrior.wordpress.com/2010/05/23/qbert-teaches-the-binomial-theorem/>

<https://app.box.com/s/7h4u6spbp2qoc7eip04k>

Chapter Three: Modeling periodic functions (includes product & quotient properties)

Product rule: <http://101studiostreet.com/wordpress/?p=204> (Shawn uses $1/x * \sin x$ to model guitar string)

<http://untilnextstop.blogspot.com/2013/11/product-rule-via-geometry.html>

Kate: "[What is 1 radian?](#)" and also [pi whats?](#)

My handout for der of sine and cosine versus an infinitesimals approach, using the "complete triangle":

<http://thephysicsvirtuosi.com/posts/trigonometric-derivatives.html>

<http://www.epsilon-delta.org/2013/06/loves-me-loves-me-not-using.html> (trig)

http://www.youtube.com/watch?v=DSQc4x2_7UE (human powered ferris wheel)

<http://gottwurfelt.com/2013/08/17/revisiting-pythagoras-goes-linear/>

Mini-Chapter Four: Optimizing

- dating pool: <http://shawncornally.com/wordpress/?p=2971&cpage=1#comment-12908>
- screencasts:
<http://castingoutnines.wordpress.com/2010/11/12/this-week-in-screencasting-optimization-palooza/>
- <http://gitchosblog.wordpress.com/2012/11/18/calculus-big-ideas-and-a-visit-from-mr-feynman/comment-page-1/#comment-62> (optimization, light)
- packaging:
<http://bowmandickson.com/2012/06/04/calculus-final-project-spotlight-packaging-consultants/>
- <http://samjshah.com/2012/05/31/a-calculus-optimization-poster-project/>
- <http://thescamdog.wordpress.com/2010/09/12/the-peanut-butter-rant/>
- minimize walking dist in pkg lot:
<http://www.scribd.com/doc/85538687/2012-02-29-an-Introduction-to-Optimization>
- max corral size, Pat has fun:
<http://pballew.blogspot.com/2010/11/repeat-of-isoperimetric-discovery.html>
- <https://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/maxmindirectory/> (lots of optimizing problems)

[Test Two here, for my courses.]

Chapter Five: Exponential Growth and Decay, and all the rest (Compositions, Implicit Functions, Related Rates, ...)

- <http://games.common.gc.cuny.edu/2012/09/19/calculus-art-on-the-wall-game-teen-a-car-roll-saint-norbert-college/>
- Rvw Topics, Logs: <http://www.intmath.com/blog/logarithms-a-visual-introduction/4526>
- Exp vs Poly Der, comic: <http://abstrusegoose.com/26> (not helpful, just silly)
- <http://bowmanimal180.files.wordpress.com/2013/02/wp1d-photo-feb-3-2013-609-pm.jpg> (review with In derivative)
- <http://bowmanimal180.wordpress.com/2013/01/29/day-70-a-manual-derivative-for-ex/>
- Graphing, inflection point with infection spread: <http://bowmandickson.com/2012/01/15/introducing-inflection-with-infection/>
- www.epsilon-delta.org/2013/09/derivatives-of-exponentials-and-logs.html
- related rates: <http://matharguments180.blogspot.com/2014/11/315-pizza-maker.htm>
-

Related Rates

- drawing pumpkins: <http://bowmandickson.com/2012/03/29/drawing-in-math-class/>
- <http://www.johndcook.com/blog/2012/12/22/spotting-sensitivity-in-an-equation/>
- speed trap: <http://realteachingmeansrealllearning.blogspot.com/2012/05/calculus-speed-trap.html>
- beakers: <http://realteachingmeansrealllearning.blogspot.com/2012/03/rate-of-change-of-beaker.html>
- <http://www.math.utah.edu/~petersen/1210/RelatedRatesAnswers.pdf> (10 problems)
- <http://bowmandickson.com/2011/11/26/relating-those-rates/> (geogebra applets)
- <https://docs.google.com/file/d/0B9GuwbUfAT6MYUxSeURIZWdfLXM/edit>
-

If addressing mean value thm, project on bees and marginal value:

<http://courses.ncssm.edu/math/apcalcprojects/econ/>

[This would make test three, which students find the hardest. I haven't yet gotten them comfortable with implicit differentiation.]

Chapter Six: Finding Areas and Getting Serious with those Limits

Exploring Limits informally:

- $60/x$ (Don Cohen), ... anything from book yet?
- Summary: <http://mathteachermambo.blogspot.com/2012/04/limit-does-not-exist.html>
- <http://opinionator.blogs.nytimes.com/2010/04/04/take-it-to-the-limit/>
- <http://www.epsilon-delta.org/2012/08/i-heart-these-two-calc-problems.html> (#2 good here)
- <http://bowmandickson.com/2012/11/24/whiteboarding-mode-simultaneous-show-and-tell/> (creating a graph that has certain limit properties)
- <http://mrhodotnet.blogspot.com/2010/10/comic-guide-to-limits-part-1-when.html>

Henri: One way to introduce limits is through iterating linear functions.

For example, say we want to iterate $y = .5x + 8$. Start with your birthday as your first input. Divide by 2 and add 8. Take the result as your next input. Repeat. You will approach a limit. Not only that, but it's the same limit I would approach if I started with my birthday.

See <<http://www.mathedpage.org/iterating/>> for a bunch of lessons based on this.

Intermediate Value Theorem

- <http://bowmandickson.com/2012/10/20/teaching-through-concrete-examples-the-intermediate-value-theorem/#comment-1514>
- <https://www.box.com/s/bgyyw8o9hm386xc3f36h>
- <http://www.epsilon-delta.org/2013/09/continuity-and-ivt.html>
-

Riemann Sums

- <http://threeacts.mrmeyer.com/pyramidofpennies/act1/actone.mov> (sum of squares) and we should do other sums
- <http://users.tru.eastlink.ca/~brsears/math/oldprob.htm#s32> (sum of squares)
- <http://bowmandickson.com/2013/01/24/an-anchor-problem-for-riemann-sums/>
- <http://samjshah.com/2010/03/17/riemann-sum-set-up/>
- Demo: <http://www.abstractmath.org/Word%20Press/?p=4362>
- <http://opinionator.blogs.nytimes.com/2010/04/18/it-slices-it-dices/>
- why does $1+4+9+\dots+n^2 = \dots$?
<http://wyoinnovation.blogspot.com/2010/02/12-22-32-42-52-n2.html>
-

Anti-derivatives

- <http://www.doingmathematics.com/2/post/2012/11/my-anti-teaching-version-of-antiderivatives.html> (anti-derivatives stationary bike)

- Practice power rule:
<https://docs.google.com/file/d/0B2gP86lVzGj9Q0IPbEF1TDg1MTQ/edit>
-

Fundamental Theorem

- Sums of cubes: <http://www.jamestanton.com/?p=820>

Area

<http://girlsangle.wordpress.com/2013/07/31/why-does-the-area-under-one-hump-of-a-sine-curve-exactly-equal-2/>

Average

<http://www.epsilon-delta.org/2013/06/intro-to-average-value.html>

Volume

- <http://www.intmath.com/blog/volume-of-a-pendant/7629> (volume of rev)
- <http://pedagogicalpredicaments.blogspot.com/2010/03/volumes-in-calculus-part-1-cross.html>
- <http://signednumbers.blogspot.com/2010/03/help-and-critique-needed-gsp-file.html>
- <http://bowmandickson.com/2012/04/27/musings-about-volume/>
- <http://www.epsilon-delta.org/2013/07/made4math-volumes-in-calculus.html>
-

- Kobe Bryant jump possible?
<http://realteachingmeansrealllearning.blogspot.com/2011/12/calculus-and-kobe-bryant.html>

Bead: <http://pballew.blogspot.com/2010/10/surprisingly-constant.html>

gini (inequity measure):

- <http://courses.ncssm.edu/math/apcalcprojects/econ/>
- <http://samjshah.com/2012/06/07/wealth-inequality-a-calculus-investigation/>

Problem-Solving

Warmup: "The warm up for class goes like. What number is directly in the middle of 10 and 20? Share your strategy for how you figure it out? How can you check to make sure that number you found is precisely in the middle?"

<http://teachbrianteach.wordpress.com/2012/06/09/something-that-seems-to-be-working/>

Safe speed: <http://kalamitykat.com/2011/09/15/gettin-real-with-centripetal/#comment-2893>

Pile of paper art: <http://mrhodotnet.blogspot.com/2011/08/bianca-changs-paper-craft.html>

Projects with too much prep for me:

Geogebra (maybe not too much...)

Area btwn curves: <http://bowmandickson.com/2012/05/15/integration-drawing-projects-12/>

Glow Sticks (what topic?): <http://shawncornally.com/wordpress/?p=2687>

Egg Bungee: <http://teachingninja.wordpress.com/2010/05/21/egg-bungee/>

Free (or inexpensive) Textbooks

- collection at [AIM](#)
- <http://www.whitman.edu/mathematics/calculus/> (Guichard)
- http://faculty.gvsu.edu/boelkinm/Home/Open_Calculus_files/Chapter1%287.24.12%29.pdf (I may use this one extensively. An activity book is also available for free from the author by email request.)
- <http://ocw.mit.edu/resources/res-18-001-calculus-online-textbook-spring-2005/textbook/> (Strang)
- http://scidiv.bellevuecollege.edu/dh/Calculus_all/Calculus_all.html (Hoffman)
- https://docs.google.com/viewer?url=http%3A%2F%2Fdjm.cc%2Flibrary%2FCalculus_Made_Easy_Thompson.pdf
- <http://www.math.upenn.edu/~ghrist/FLCT/>
- <https://www.math.duke.edu/education/ccp/>
- <http://www.e-booksdirectory.com/details.php?ebook=11>
- <http://www.math.smith.edu/Local/cicintro/>
- <http://www.centerofmath.org/textbooks/calc1/index.html> (Massey text, \$20 pdf, \$30 print)
- <http://www.mathlogarithms.com/> (sample pages free, text inexpensive)
- Exeter problems (intense hs level, some of math4 is calculus):
http://www.exeter.edu/academics/72_6539.aspx
-

Course Videos:

<http://ocw.mit.edu/high-school/courses/highlights-of-calculus/highlights-of-calculus-5-videos/big-picture-of-calculus/>

Alternatives to graphing calculators:

<http://www.intmath.com/blog/graphs-using-free-math-software/4115>

http://phet.colorado.edu/sims/calculus-grapher/calculus-grapher_en.html

Old Discussion

January 22, 2013: This document has all the links I've found so far, and is organized the way I'm teaching Calc I. Resources, by unit, start on the 3rd page.

Some discussion from earlier in 2012 at bottom.

July 28, 2012 Rationale: I've followed the textbook until now. I don't want to do that; I want to design the course with a flow that makes sense to me. (I know that students come to calc II not really understanding much of the calc I material, so some big changes are in order.)

Problem#1: Limits seem like a crappy way to start. (So does too much review.)

[Shawn Cornally](#): "On the first day of calculus I begin by introducing the limit definition of the derivative. This is a 100% break with tradition. Most calculus classes begin with a review of pre-calc and trigonometry. Let me just come out and say it, that's ridiculous. ... By jumping straight to the limit definition of the derivative I have also circumvented an entire chapter discussing limits in general, continuity, and all sorts of other mathematical abstraction that means nothing to a 17 year-old. ... My goal is to motivate the necessity for learning more math." His day one is kids in office chairs, trying to determine how fast they're going.

My response: Yes! That mathematical abstraction means nothing to a student of any age who hasn't seen ahead to why we might need it. My fall section did great without the review up front and without the limits up front. I sprinkle review throughout, and do limits in my 4th unit (just before Riemann sums and integration).

Issue#2: I give tests that are almost as long as my old chapter tests, but split into sections that each test one topic. Then students can retake by topic, often in my office. I needed to chunk all the material into units that seem sensible to me. I did that, and am happy with it.

Issue#3: I want to start each topic with a compelling project / problem - still looking for more of those.

My Outline

- Unit 1 - Exploring Rate of change & Slopes of curvy lines, using a very loose idea of limits
- Unit 2 - Part 1: Properties of derivatives of polynomials, and graphing; Part 2: Modeling periodic functions (trig derivatives, product and quotient properties), I think I'm moving optimisation here.
- Unit 3 - Exponential Growth, Compositions and Implicit Functions, Optimization, Related Rates
- Unit 4 - Limits, area, anti-derivatives, volume

I have detailed 'unit sheets' for each unit, which list homework problems from our official textbook by Briggs, along with some problems from open source texts. Email me at

mathanthologyeditor on gmail for copies of those.

August 23, 2012 (Christopher Danielson)

I am so totally swamped by my own semester prep that I don't have much time to invest here. I do want to chime in quickly, though.

I am totally with you on the silliness of algebra review and that it is a bad idea to begin with limits. I dig the dive right into slope and thus derivative. On a related note, have you played with finite differences in a calc course? Many students are now coming through K-12 experiences that have emphasized patterns of change in algebraic tables. When we write equations describing these patterns of change, we get the "not-quite derivative". E.g. a table that counts by 1 for $y=x^2$ will yield the pattern $y_1=2x+1$ (or $2x-1$, depending on how you assign the differences to the inputs). If we make the interval smaller (e.g. counting by $\frac{1}{2}$ or $\frac{1}{4}$), and adjust appropriately in ways that correspond to the ways we work with slopes, we get an expression that better approximates the derivative. We get a sequence of functions that converges (of course) to the derivative $y_1=2x$.

I have been playing around with various ways of engaging this line of exploration and am curious whether others have as well.

September 8 (Sue V)

I think I might try this for people who haven't done well on the first test, which I am giving Monday. Since I allow re-testing, I'll want a way to help them improve their understanding. I don't think any of the textbooks I have (Briggs, Boelkins, Guichard) provide a good way for them to ratchet up. Do you have any worksheets for this?

August 1, 2012

I don't know how useful my comments will be, but you mentioned that you were gathering ideas on why calculus is so cool and why we love it (something to that effect). I mostly try to sprinkle little snippets throughout the course as the topics come up and look for realistic ways it's used in life (not the usual textbook fake cheery ways, but genuine, "that's cool" ways). I find these by either a google search on choice words or from scanning 2 or 3 of my calculus texts that seem to gear their content to this in a genuine way. Also in the past on day one I have linked what you can do in calculus vs what you can do with "pre" calculus skills and give a brief description of how it would be useful.

Shireen D (mathteachermambo)