YEAR 11 - MATHEMATICS

Preliminary Topic 15 - Further Trigonometric Identities

MATHEMATICS EXTENSION

LEARNING PLAN			
Learning Intentions Student is able to:	**Learning Experiences** Implications, considerations and implementations:	**Success Criteria** I can:	**Resources**
Derive and use the sum and difference expansions for the trigonometric functions $sin\ (A \pm B)$, $cos\ (A \pm B)$ and $tan\ (A \pm B)$ – $sin\ (A \pm B) = sin\ A\ cos\ B \pm cos\ A\ sin\ B$ – $cos\ (A \pm B) = cos\ A\ cos\ B \pm sin\ A\ sin\ B$ – $tan\ (A \pm B) = \frac{tan\ A \pm tan\ B}{1 \pm tan\ A\ tan\ B}$	▪ This subtopic needs to be taught in the context of both degrees and circular measure (radians). ▪ Find the exact value of $tan\ 75°$. ▪ Find the exact value of $cos\ \frac{\pi}{8}$. ▪ If $cos\ \theta\ = -\frac{3}{5}$ and $0 < \theta < \pi$, determine the exact value of $tan\ \theta$.	**Derive** and use the sum and difference expansions for the trigonometric functions $sin\ (A \pm B)$, $cos\ (A \pm B)$ and $tan\ (A \pm B)$	
derive and use the double angle formulae for $sin\ 2A$,$cos\ 2A$ and $tan\ 2A$ – $sin\ 2A = 2\ sin\ A \cos cos\ A$	The double angle formulae for $cos\ 2A$, $sin\ 2A$ and $tan\ 2A$ should be obtained explicitly as particular	derive and use the double angle formulae for $sin\ 2A$,$cos\ 2A$ and $tan\ 2A$	2016-3, 2013-8, 2009-3c

– $\cos 2A = \cos^2 A - \sin^2 A$ $= 2\cos^2 A - 1$ $= 1 - 2\sin^2 A$ – $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$	cases of the sum and difference formulae. **Note:** Expressions involving cos3A, for example, are in Year 12 Extension 1 topic T3.		
derive and use expressions for $\sin A$, $\cos A$ and $\tan A$ in terms of t where $t = \tan\frac{A}{2}$ (the t-formulae) – $\sin A = \frac{2t}{1+t^2}$ – $\cos A = \frac{1-t^2}{1+t^2}$ – $\tan A = \frac{2t}{1-t^2}$	▪ Denoting $\tan\frac{A}{2}$ by t, the addition formula for the tangent gives $\tan A = \frac{2t}{1-t^2}$ $(t\neq\pm1)$. The expressions for $\cos A$ and $\sin A$ in terms of t should also be derived.	derive and use expressions for $\sin A$, $\cos A$ and $\tan A$ in terms of t where $t = \tan\frac{A}{2}$ (the t-formulae)	2007-2a, 2005-4b
derive and use the formulae for trigonometric products as sums and differences for $\cos A\cos B$, $\sin A\sin B$, $\sin A\cos B$ and $\cos A\sin B$ (ACMSM047) – $\cos A\cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$ – $\sin A\sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$ – $\sin A\cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$ – $\cos A\sin B = \frac{1}{2}[\sin(A+B)-\sin(A-B)]$	.		

Practical application problems

- students investigate mathematically the superposition of waves. For example, when two waves of similar frequency are combined, the graph of the result can be interpreted as a wave with amplitude modified by another wave. In sound waves, this is heard as ‘beats’ and is used in tuning musical instruments.

  For example, graphing software could be used to draw the functions

  $f(t) = 5(\cos cos\ 3t - \cos cos\ 3.1t)$ and $g(t) = 10(\sin sin\ 3.05t)\ \sin sin\ (0.05t)$ to show that they are equivalent, and trigonometric identities then used to establish the underlying result: $\cos cos\ \alpha - \cos cos\ \beta = -2\sin sin\ \frac{1}{2}(\alpha+\beta)\ \sin sin\ \frac{1}{2}(\alpha-\beta)$