*These summary notes are valid for examination in 2023 and 2024.*

# H2 Physics (Syllabus 9749)

# Section I: Measurement

## Topic 1: Measurement

**Physical quantities and SI units**

- Base quantities and their SI units:
    - Mass – kilogram (kg)
    - Length – metre (m)
    - Time – second (s)
    - Current – ampere (A)
    - Temperature – kelvin (K)
    - Amount of substance – mole (mol)
- A derived unit can be expressed as a combination of products or quotients of any of the base units.
- Using SI base units to check the homogeneity of physical equations:
    - A physical equation is homogeneous if all the terms in the equation have the same units
    - If the equation involves addition or subtraction of quantities, the quantities that are added or subtracted must also have the same units
    - Equations that are valid must be homogeneous, but equations that are homogeneous are not necessarily valid
- Conventions for labelling graph axes and table columns:
    - The table headers and graph axes must show the quantity and its corresponding unit (e.g. $L$ / m)
    - The numerical labels for each axis should have the same number of decimal places
- Prefixes and symbols to indicate decimal sub-multiples or multiples of both base and derived units:
    - Pico (p) – $10^{-12}$
    - Nano (n) – $10^{-9}$
    - Micro (μ) – $10^{-6}$
    - Milli (m) – $10^{-3}$
    - Centi (c) – $10^{-2}$
    - Deci (d) – $10^{-1}$
    - Kilo (k) – $10^{3}$
    - Mega (M) – $10^{6}$
    - Giga (G) – $10^{9}$
    - Tera (T) – $10^{12}$
- Estimates of physical quantities are made to judge the plausibility of any given quantity and estimate the sizes of further quantities, and are generally expressed to one significant figure.

**Scalars and vectors**

- Distinction between scalar and vector quantities:
  - A scalar quantity is a quantity that has magnitude only
  - A vector quantity is a quantity that has both magnitude and direction
  - Examples of scalar quantities include mass, time, length, volume, temperature, density, speed, energy, pressure and current
  - Examples of vector quantities include displacement, velocity, acceleration, force and momentum
- Addition and subtraction of coplanar vectors:
  - Parallel vectors can be added or subtracted using simple addition or subtraction
  - Non-parallel vectors can be added using either the parallelogram or triangle of vectors
  - Resultant vectors can be computed using scale drawing, Pythagoras theorem, sine and cosine rule or resolution and recombination of vectors
- Representation of a vector as two perpendicular components:
  - $|\boldsymbol{R_x}| = |\boldsymbol{R}| \cos\theta$
  - $|\boldsymbol{R_y}| = |\boldsymbol{R}| \sin\theta$
  - $\theta$ is the angle between $\boldsymbol{R_x}$ and $\boldsymbol{R}$

**Errors and uncertainties**

- Distinction between systematic errors and random errors:
  - A systematic error is an error that has a constant magnitude and is either always positive or always negative
  - A random error is an error that has a varying magnitude and has an equal chance of being negative or positive
  - A zero error is an example of a systematic error
- Distinction between precision and accuracy:
  - Precision refers to how close individual measurements are to one another, without reference to any true value
  - Accuracy refers to how close a measured value is to the true value
- Assessing the uncertainty in a derived quantity:
  - When quantities are added or subtracted, the actual uncertainties of each quantity are added, i.e. $z = x \pm y \Rightarrow \Delta z = \Delta x + \Delta y$
  - When quantities are multiplied or divided, the fractional / percentage uncertainty of the quantity to be calculated is the sum of the fractional / percentage uncertainties of each individual quantity, i.e. $z = xy$ or $z = x / y \Rightarrow \Delta z / z = \Delta x / x + \Delta y / y$
  - If a quantity is multiplied by a constant, its actual uncertainty is also multiplied by the constant, i.e. $z = kx \Rightarrow \Delta z = k \Delta x$
  - If a quantity is raised to the nth power, its fractional uncertainty is given by the fractional / percentage uncertainty of the quantity multiplied by the absolute value of the index, i.e. $z = x^n \Rightarrow \Delta z / z = |n| (\Delta x / x)$
  - For quantities calculated using other mathematical functions, the error can be estimated using numerical substitution, taking the difference between the

largest or smallest possible value and the calculated value, whichever is greater

# Section II: Newtonian Mechanics

## Topic 2: Kinematics

**Rectilinear motion**

- Displacement, speed, velocity and acceleration:
  - The displacement of a body is its distance from a defined reference point in a defined direction
  - The speed of a body is the rate of change of its distance travelled
  - The velocity of a body is the rate of change of its displacement
  - The acceleration of a body is the rate of change of its velocity
  - Speed is a scalar quantity while displacement, velocity and acceleration are vector quantities
- Distance, displacement, speed, velocity and acceleration can be represented using distance-time, displacement-time, speed-time, velocity-time and acceleration-time graphs respectively.
- Properties of displacement-time and velocity-time graphs:
  - The gradient of a displacement-time graph is the velocity of the body
  - The gradient of a velocity-time graph is the acceleration of the body
  - The area under a velocity-time graph is the change in displacement of the body
  - In cases of non-uniform acceleration, the velocity-time graph is a curve (i.e. the gradient is not constant)
- Derivation of equations which represent uniformly accelerated motion in a straight line:
  - From the definition of acceleration, $a = (v - u) / t \Rightarrow v = u + at$
  - Displacement is the area under the velocity-time graph, thus $s = \frac{1}{2} (v - u) t + ut$
  - From the definition of acceleration, $v - u = at$, hence $s = ut + \frac{1}{2}at^2$
  - Using the trapezoid formula for calculating area, $s = (v + u) t / 2$
  - From the definition of acceleration, $t = (v - u) / a$, hence $v^2 = u^2 + 2as$
- Solving problems using equations which represent uniformly accelerated motion in a straight line:
  - The equations can be used to solve problems such as the motion of bodies falling in a uniform gravitational field without air resistance
  - Sketch a diagram showing the path of the object, and select one direction as positive
  - Select appropriate initial and final points for calculation, and determine the values of $u$, $v$, $a$, $s$ and $t$
  - Use the equations to solve the problem
- Motion of bodies falling in a uniform gravitational field with air resistance:

    - At low speed with no turbulence, drag force is approximately proportional to speed
    - At higher speeds with turbulence, drag force is approximately proportional to the square of the speed
    - With air resistance, a body's acceleration would be less than *g* when it is falling downward, causing it to take a longer time to reach the ground
    - After falling for some time, the air resistance becomes equal to the weight of the body and the body falls at terminal velocity

**<u>Non-linear motion</u>**

- Motion due to a uniform velocity in one direction and uniform acceleration in a perpendicular direction:
    - The equations for uniformly accelerated motion in a straight line apply to such scenarios when the displacement and velocity of the body are resolved in the direction of acceleration and the direction of uniform velocity
    - From these equations, the body's displacement in the direction of acceleration is a quadratic function of time while that in the direction of uniform velocity is a linear function of time
    - The body's velocity in the direction of acceleration is a linear function of time
    - Thus, the trajectory of the body is parabolic
    - If the acceleration is caused by gravity alone, the motion is known as projectile motion

# Topic 3: Dynamics

**<u>Newton's laws of motion</u>**

- Newton's laws of motion:
    - Newton's First Law states that in the absence of a resultant force acting on a body, a body at rest will remain at rest and a body in motion will continue in motion at constant velocity
    - Newton's Second Law states that the rate of change of momentum of a body is proportional to the resultant force acting on the body, and that the momentum change takes place in the direction of the resultant force
    - Newton's Third Law states that when a body A exerts a force on a body B, body B exerts an equal and opposite force on body A
    - Newton's laws of motion can be applied to solve problems involving forces and motion
- Mass is the property of a body which resists a change in motion (inertia) – the more mass a body has, the harder it is to start it moving from rest, bring it to rest or change its direction of motion.
- Concept of weight:
    - The weight of a body is the gravitational force experienced by a mass in a gravitational field
    - The apparent weight of a body is the force exerted on the body by the body that is freely supporting it

    - The weight of a body is the product of its mass and the gravitational field strength, i.e. $W = mg$

**Linear momentum and its conservation**

- The linear momentum of a body is defined as the product of the mass and the linear velocity of the body, and is a vector quantity in the direction of the body's velocity.
- The impulse of a force is defined as the product of the net force and the time of impact, and is equal to the change in momentum of the body caused by the impact.
- The resultant force on a body is equal to the rate of change of momentum of the body, and the momentum change takes place in the direction of the resultant force.
- The relationship $F = ma$, where resultant force and acceleration are always in the same direction, can be used to solve problems involving forces acting on an object.
- The principle of conservation of momentum states that when bodies in a system interact, the total momentum remains constant provided no resultant external force acts on the system.
- Application of the principle of conservation of momentum to solve collision problems:
    - A (perfectly) elastic collision is one in which both kinetic energy and momentum is conserved
    - An inelastic collision is one in which momentum is conserved but kinetic energy is not conserved
    - A completely inelastic collision is a form of inelastic collision in which the bodies coalesce after the collision and kinetic energy loss is maximum
    - Collision problems can be solved by applying the appropriate conservation laws
- For a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.
- Whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place, converted to other forms of energy such as internal energy or potential energy of deformation.

# Topic 4: Forces

**Types of force**

- Hooke's Law states that the applied force is proportional to the amount of extension or compression it produces in an elastic object, i.e. $F = kx$, where $k$ is the force constant – it can be used to solve problems involving compression or extension of an elastic object.
- Forces on a mass, charge and current-carrying conductor in gravitational, electric and magnetic fields:
    - Two masses experience a gravitational force in the direction of their centre of mass, in the direction of the gravitational field
    - Two like charges experience a repulsive electric force while two opposite charges experience an attractive electric force towards each other, in the direction of the electric field (positive charge) or in the direction opposite to the electric field (negative charge)

    - A current-carrying conductor experiences an electromagnetic force that is perpendicular to both the direction of the magnetic field and the direction of current in the conductor, in accordance with Fleming's left-hand rule
- Normal contact forces, frictional forces and viscous forces:
    - A normal contact force is a force that two surfaces exert on each other, perpendicular to the surfaces in contact
    - A frictional force is a force that two rough surfaces exert on each other when there is motion or tendency to motion between them, opposite to the direction of motion or impending motion and parallel to the surfaces
    - A viscous force is a force exerted on a body moving through a fluid, opposite to its direction of motion relative to the fluid
    - Air resistance is a type of viscous force experienced by a body moving in air

**Centre of gravity**

- The weight of a body may be taken as acting at a single point known as its centre of gravity.

**Turning effects of forces**

- Moment of a force and the torque of a couple:
    - The moment of a force about a point is defined as the product of the force and the perpendicular distance from the line of action of the force to that point
    - The torque of a couple is defined as the product of either force and the perpendicular distance between the forces
    - The moment of a force and the torque of a couple can be applied to solve problems involving rotational motion
- A couple is a pair of forces which tends to produce rotation only, having no translational effect.
- The principle of moments states that if a body is in rotational equilibrium, the sum of clockwise moments about any point must be equal to the sum of anticlockwise moments about the same point – the principle is used to solve problems involving bodies in static equilibrium.

**Equilibrium of forces**

- When there is no resultant force and no resultant torque, a system is in equilibrium – this means that their lines of action must intersect at a common point.
- Forces in static equilibrium can be represented by a closed vector triangle.

**Upthrust**

- Derivation of the equation $p = \rho gh$:
    - Pressure $p$ is defined as the normal force acting per unit area, thus pressure $p$ on an area $A$ below the fluid surface is $p = W / A$, where $W$ is equal to the weight of the fluid column above $A$
    - Since $W = mg$, $p = mg / A$
    - Density $\rho$ is defined as the mass per unit volume, thus $\rho = m / V \Rightarrow m = \rho V$
    - Thus $p = \rho Vg / A$
    - Hence $p = \rho gh$, where $h$ is the height of the fluid column

- The equation $p = \rho gh$ can be used to solve problems involving fluid pressure.
- Upthrust is an upward force exerted on an object submerged or floating in a fluid due to the difference in pressure exerted by the fluid on the upper and lower surfaces of the object.
- Upthrust is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object.
- Upthrust can be calculated by determining the weight of fluid displaced by the object, i.e. $U = m_{\text{fluid}}g$ or $U = \rho_{\text{fluid}}Vg$.
- For an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object – this principle can be used to solve problems involving floating objects.

# Topic 5: Work, Energy and Power

**Work**

- The work done by a force on a body is defined as the product of the force and the displacement of the body in the direction of the force.
- Calculation of work done:
    - The work done by a constant force is given by $W = Fs\cos\theta$, where $\theta$ is the angle between the force vector and the displacement vector
    - The work done by a varying force is given by the area under the force-displacement graph
    - The work done by an expanding gas is given by $W = p\Delta V$, where $p$ is the external pressure and $\Delta V$ is the change in volume of the gas

**Energy conversion and conservation**

- Examples of energy in different forms, its conversion and conservation, and the principle of energy conservation:
    - Kinetic energy – a sprinting student
    - Potential energy – a satellite in orbit around a planet
    - Internal energy – a body at any temperature above absolute zero
    - Photon energy – a quanta of light from a laser pointer
    - Mass energy – anything with mass
    - Nuclear binding energy – bound nucleons in an atom
    - Ionisation energy – an electron in a hydrogen atom being liberated
    - Work function – electron release from a gold plate in vacuum due to the photoelectric effect
    - Electromotive force – a mobile charger or power bank
    - Energy conversion – kinetic energy is converted to gravitational potential energy when a ball is being thrown upwards
    - Energy conservation – the total energy of the ball remains constant provided there is no air resistance
    - The principle of energy conservation states that energy can neither be created nor destroyed but it can be converted from one form to another, and the total energy in an isolated system remains constant

    - The principle of energy conservation can be applied to solve problems involving energy conversion

**Efficiency**

- Implications of energy losses in practical devices and the concept of efficiency:
    - For all practical devices, the useful energy output is always less than the useful energy input as a result of energy loss to the environment
    - The efficiency $\eta$ of a device is the ratio of useful energy output to the energy input, i.e. $\eta$ = *useful energy output / energy input*
    - The concept of efficiency can be used to solve problems involving practical devices

**Potential energy and kinetic energy**

- Derivation of the equation $E_k = ½mv^2$:
    - The kinetic energy $E_k$ of a body is the work done to accelerate it from rest to a speed $v$, i.e. $E_k = Fs = mas$
    - From the equations for uniformly accelerated motion in a straight line, $v^2 = u^2 + 2as$
    - Since the body was accelerated from rest, $u = 0 \Rightarrow as = ½v^2$
    - Hence $E_k = ½mv^2$
- The equation $E_k = ½mv^2$ can be used to solve problems involving moving bodies.
- Distinction between gravitational potential energy, electric potential energy and elastic potential energy:
    - Gravitational potential energy is the energy possessed by a body due to its mass and its position in a gravitational field
    - Electric potential energy is the energy possessed by a body due to its charge and its position in an electric field
    - Elastic potential energy is the energy possessed by a body due to its deformation
- Elastic potential energy in a deformed material:
    - The elastic potential energy in a deformed material is the work done to deform the material by an extension $x$ from its natural length
    - Since work done is the area under the force-displacement graph, the elastic potential energy of the deformed material is the area under the force-extension graph
- In a uniform field, the magnitude of the force acting on an object is equal to the change in potential energy of the object per unit distance travelled, i.e. $F = \Delta U / \Delta s$ – this relationship can be used to solve problems involving force and potential energy.
- Derivation of the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface:
    - When a body is being lifted a displacement $h$, the gain in gravitational potential energy $E_p$ of the body is equivalent to the work done by an external force $F$
    - From the definition of work done, $E_p = W = Fh$
    - Since the body of mass $m$ is lifted without a change in velocity, by Newton's Second Law, $mg - F = 0 \Rightarrow F = mg$

    - Hence $E_p = mgh$
- The equation $E_p = mgh$ can be used to solve problems involving gravitational potential energy changes near the Earth's surface.

**Power**

- Definition of power and derivation of power as the product of a force and velocity in the direction of the force:
    - Power is defined as the work done per unit time, i.e. $P = \Delta W / \Delta t$
    - Instantaneous power at a point in time is given by $P = \mathrm{d}W / \mathrm{d}t = \mathrm{d}(Fs) / \mathrm{d}t$, where $F$ is constant
    - Thus $P = F\, \mathrm{d}s / \mathrm{d}t = Fv$ (since $\mathrm{d}F / \mathrm{d}t = 0$)
    - Hence, instantaneous power is given by the product of the force and the velocity in the direction of the force

# Topic 6: Motion in a Circle

**Kinematics of uniform circular motion**

- Angular displacement can be expressed in radians, where one radian is the angle subtended at the centre of the circle by an arc equal in length to the radius of the circle.
- Concept of angular velocity:
    - Angular velocity is the rate of change of angular displacement of a body, i.e. $\omega = \Delta\theta / \Delta t$
    - $\omega = 2\pi / T = 2\pi f$, where $T$ is the period of rotation and $f$ is the frequency of rotation
    - All points on a rotating body have the same angular velocity
    - This concept can be used to solve problems involving rotational motion
- $v = r\omega$, where $r$ is the radius of circular motion, can be used to convert angular velocity $\omega$ to linear velocity $v$ and vice versa.

**Centripetal acceleration**

- Motion in a curved path due to a perpendicular force and centripetal acceleration in the case of uniform motion in a circle:
    - By Newton's Second Law, when a force acts on a body perpendicular to its direction of motion, it causes an acceleration in the same direction
    - Since there is no component of acceleration in the direction of motion, the body's direction of motion continuously changes but its speed does not change
    - In the case of uniform motion in a circle, centripetal acceleration is the acceleration towards the centre of the circle and perpendicular to the linear velocity of the body
- Centripetal acceleration $a = r\omega^2$ and $a = v^2 / r$ can be used to solve problems.

**Centripetal force**

- Centripetal force $F = mr\omega^2$ and $F = mv^2 / r$ can be used to solve problems.

# Topic 7: Gravitational Field

**Gravitational field**

- Concept of a gravitational field and definition of gravitational field strength at a point:
    - A gravitational field is a region of space in which a mass experiences a force due to the presence of another mass, and is an example of a field of force
    - The gravitational field strength $g$ at a point in a gravitational field is defined as the gravitational force per unit mass acting on a test mass placed at that point, i.e. $g = F / m$ or $F = mg$
- Analogy between certain qualitative and quantitative aspects of gravitational and electric fields:
    - Gravitational force results from interaction between masses, while electric force results from interaction between charges
    - Both gravitational and electric fields are inverse square law fields

**Gravitational force between point masses**

- Newton's Law of Gravitation states that every particle of matter in the universe attracts every other particle with a gravitational force $F$ that is directly proportional to the product of the masses of the particles $m_1m_2$ and inversely proportional to the square of the distance $r$ between them, i.e. $F = Gm_1m_2 / r^2$, where $G$ is the gravitational constant which has the value $6.67 \times 10^{-11}$ N m$^2$ kg$^{-2}$.

**Gravitational field of a point mass**

- Derivation of the equation $g = GM / r^2$ for the gravitational field strength of a point mass:
    - By Newton's Law of Gravitation, the gravitational force $F$ exerted on a mass $m$ by a mass $M$ is $F = GMm / r^2$
    - From the definition of gravitational field strength, $g = F / m$
    - Hence $g = GM / r^2$
- The equation $g = GM / r^2$ for the gravitational field strength of a point mass can be applied to solve problems.

**Gravitational field near to the surface of the Earth**

- Near the surface of the Earth, gravitational field strength is approximately constant and is equal to the acceleration of free fall, 9.81 m s$^{-2}$.

**Gravitational potential**

- The gravitational potential at a point in a gravitational field is defined as the work done per unit mass by an external agent to bring a small test mass from infinity to that point without any change in kinetic energy of the test mass.
- The equation $\phi = -GM / r$ for the gravitational potential in the field of a point mass can be used to solve problems.

**Circular orbits**

- For a mass *m* orbiting at a certain distance *r* from the centre of a mass *M*, the gravitational force provides the required centripetal acceleration for the mass *m* to orbit about mass *M*, i.e. $GMm / r^2 = mv^2 / r = mr\omega^2$ – this relation can be used to analyse circular orbits in inverse square law fields.
- Geostationary orbits and their application:
    - Geostationary satellites are satellites with orbits such that the satellites are always positioned over the same geographical spot on Earth
    - The plane of orbit of the satellite must be the same as that of the equator, as the gravitational force exerted by the Earth on the satellite is always directed towards the centre of the Earth
    - All geostationary satellites have the same orbital radius, period and velocity
    - Geostationary satellites are useful because they allow a fixed antenna on Earth to maintain a constant line of communication with the satellite

# Section III: Thermal Physics

## Topic 8: Temperature and Ideal Gases

**Thermal equilibrium**

- Regions of equal temperature are in thermal equilibrium, and no net exchange of thermal energy occurs when they are placed in thermal contact.

**Temperature scales**

- How empirical evidence leads to the gas laws and to the idea of an absolute scale of temperature:
    - It was observed that a change in pressure of a fixed mass of gas at constant volume is caused by a change in temperature of the gas
    - Hence, the Pressure Law states that the pressure of a fixed amount of gas at constant volume is directly proportional to its absolute temperature
    - It was observed that a change in volume occupied by a fixed mass of gas at constant pressure is caused by a change in temperature of the gas
    - Hence, Charles' Law states that the volume of a fixed amount of gas at constant pressure is directly proportional to its absolute temperature
    - It was observed that a change in pressure of a fixed mass of gas at constant temperature is caused by a change in volume of the gas
    - Hence, Boyle's Law states that the pressure of a fixed amount of gas at constant temperature is inversely proportional to its volume
    - It was found that the temperature of all gases converge to the same value of −273.15°C, when the graph of pressure against temperature is extrapolated to 0 Pa
    - It was also found that the temperature of all gases converge to the same value of −273.15°C when the graph of volume against temperature is extrapolated to 0 $m^3$

- These findings led to the idea of an absolute scale of temperature – a thermodynamic scale that is independent of the property of any particular substance
    - The scale has an absolute zero, the temperature at which a substance will have the smallest possible internal energy
- Temperatures measured in degrees Celsius can be converted to kelvin using $T$ / K = $T$ / °C + 273.15.

**Equation of state**

- The equation of state of an ideal gas, $pV = nRT$, where $n$ is the amount of gas in moles and $R$ is the molar gas constant which has the value 8.31 J $K^{-1}$ $mol^{-1}$, can be used to calculate the amount, pressure, volume or temperature of an ideal gas.
- One mole of any substance contains $6.02 \times 10^{23}$ particles – the Avogadro number $N_A$ = $6.02 \times 10^{23}$ $mol^{-1}$ can be used to calculate the number of particles in a substance given the number of moles or vice versa.

**Kinetic theory of gases**

- Basic assumptions of the kinetic theory of gases:
    - Any gas is made up of a large number of particles in constant and random motion
    - The particles make perfectly elastic collisions among themselves and with the wall of the container
    - The volume of each particle is negligible compared to the volume of the gas
    - The forces between the particles are negligible except during collision
    - The duration of collisions is negligible compared to the time interval between collisions
- How molecular movement causes the pressure exerted by a gas and derivation of the relationship $pV = \frac{1}{3} Nm\langle c^2 \rangle$:
    - The movement of molecules within a container results in the continual bombardment by the particles on its walls
    - This gives rise to a force per unit area on the walls, which is the pressure of the gas.
    - Consider a gas containing $N$ molecules, each of mass $m$, in a cubic container of side $L$
    - Consider a molecule having a component of velocity $c_x$ travelling towards a wall, with which it collides elastically
    - The change in momentum of the molecule $\Delta p = -mc_x - mc_x = -2mc_x$
    - As the volume of each molecule is negligible compared to that of the gas, and the forces between molecules are negligible except during collision, it is assumed that the molecule travels back and forth between the walls without disturbance
    - The time taken to come back to the wall, $\Delta t = 2L / c_x$
    - By Newton's Second Law, the average force exerted by the wall on the molecule $F_{WM} = \Delta p / \Delta t$
    - By Newton's Third Law, the average force exerted by the molecule on the wall, $F_{MW} = -F_{WM} = -\Delta p / \Delta t = mc_x^2 / L$

- The total force on the wall by $N$ molecules, $\Sigma F_{MW} = m\Sigma(c_x^2) / L$
- The pressure exerted on the wall of area $L^2$, $p = \Sigma F_{MW} / L^2 = m\Sigma(c_x^2) / L^3 = m\Sigma(c_x^2) / V$, where $V$ is the volume of the container
- Let $<c_x^2>$ be the mean square speed in the $x$-direction, i.e. $<c_x^2> = \Sigma(c_x^2) / N \Rightarrow \Sigma(c_x^2) = N<c_x^2>$
- In 3 dimensions, the velocities of the molecules have components in the $x$-, $y$- and $z$-directions
- By Pythagoras' Theorem, $c^2 = c_x^2 + c_y^2 + c_z^2 \Rightarrow <c^2> = <c_x^2> + <c_y^2> + <c_z^2>$
- Since the number of particles is large and they are in random motion, $<c_x^2> = <c_y^2> = <c_z^2> = ⅓ <c^2>$
- Thus $p = Nm<c_x^2> / V = ⅓ Nm<c^2> / V$
- Hence $pV = ⅓ Nm<c^2>$

**Kinetic energy of a molecule**

- The mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature (i.e. $½ m<c^2> = 3/2 kT$, where $k$ is the Boltzmann constant which has the value $1.38 \times 10^{-23}$ J K$^{-1}$) – this relationship can be used to solve problems involving the kinetic energy of molecules.

# Topic 9: First Law of Thermodynamics

**Specific heat capacity and specific latent heat**

- Definition and concepts of specific heat capacity and specific latent heat:
    - Specific heat capacity is defined as the quantity of heat per unit mass required to change a substance by a unit temperature
    - Specific latent heat of fusion is defined as the quantity of heat per unit mass required to change a substance from solid state to liquid state without any change in temperature
    - Specific latent heat of vaporisation is defined as the quantity of heat per unit mass required to change a substance from liquid state to gaseous state without any change in temperature
    - Specific heat capacity and specific latent heat are properties of a substance that are independent of its mass and can be used to determine the state and temperature of an object

**Internal energy**

- Internal energy is determined by the state of the system and can be expressed as the sum of a random distribution of microscopic kinetic and potential energies associated with the molecules of a system, i.e. $U = E_k + E_p$.
- A rise in temperature of a body can be attributed to an increase in the kinetic energy of its molecules and hence an increase in its internal energy.

**First law of thermodynamics**

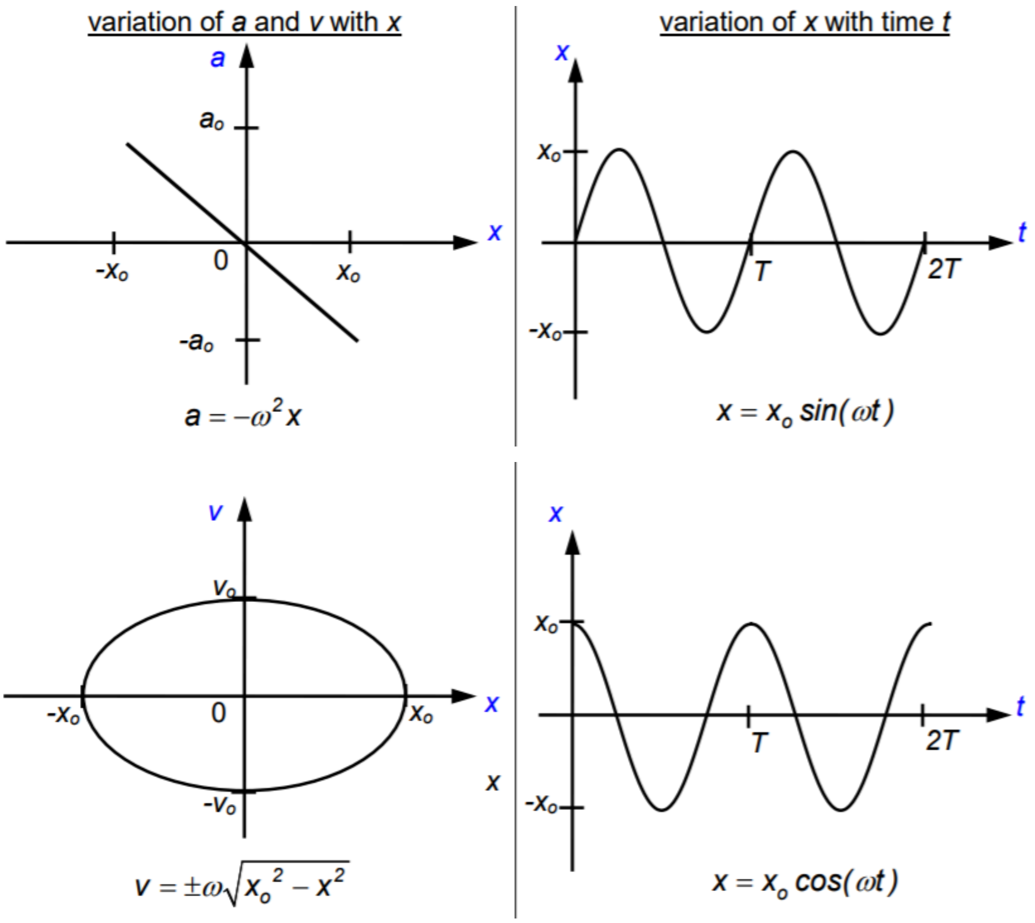
- The First Law of Thermodynamics states that in a closed system, the increase in its internal energy is equal to the sum of the heat supplied to the system and the work done on the system, i.e. $\Delta U = Q + W$ – this law can be used to solve problems such as those involving an ideal gas.

# Section IV: Oscillation and Waves

## Topic 10: Oscillations

**Simple harmonic motion**

- Simple examples of free oscillations:
    - A simple pendulum oscillates to and fro under the influence of gravity only
    - An object attached to the end of a spring oscillates back and forth under the influence of the elastic force only
- Investigation of the motion of an oscillator using experimental and graphical methods:
    - Attach a spring to a mass at one end and a retort stand on the other, and attach a pen to the mass
    - Place the setup against a moving sheet of paper and start the mass oscillating
    - The setup will produce a graph of displacement of the mass against time on the moving sheet of paper
- Amplitude, period, frequency, angular frequency and phase difference:
    - Amplitude is the maximum displacement of the oscillator from its fixed position
    - Period is the time taken for the oscillator to complete one oscillation
    - Frequency is the number of oscillations performed per unit time
    - Angular frequency is the rate of change of phase of the oscillator
    - Phase difference is the difference in phase between two oscillators oscillating with the same period
    - The period of an oscillator is given by *period* = 1 / *frequency* = 2π / *angular frequency*
- The equation $a = -\omega^2 x$, where $a$, $\omega$ and $x$ are the acceleration, angular frequency and displacement of the oscillator respectively, is the defining equation of simple harmonic motion – the acceleration is always opposite to that of displacement and proportional to it
- The equation $x = x_0 \sin \omega t$, where $x_0$ is the amplitude of the oscillation, is a solution to the equation $a = -\omega^2 x$ and can be used to calculate the displacement of an oscillating body at a certain point in time.
- The equations $v = v_0 \cos \omega t$ and $v = \pm\omega\sqrt{(x_0^2 - x^2)}$, where $v$ and $v_0$ are the velocity and maximum velocity of the oscillator respectively, can be used to find its velocity given the time from the start of the oscillation or its displacement respectively.
- Changes in displacement, velocity and acceleration during simple harmonic motion:



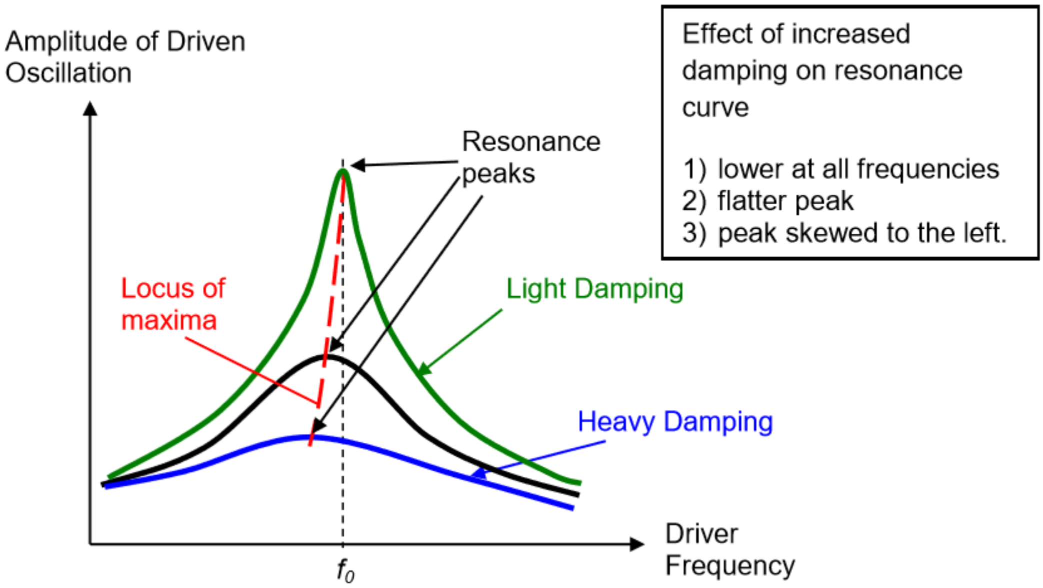

**Energy in simple harmonic motion**

- Interchange between kinetic and potential energy during simple harmonic motion:
  - The total mechanical energy of an oscillator in simple harmonic motion remains constant
  - The kinetic energy of the oscillator is maximum and potential energy is minimum when velocity is maximum, and vice versa
  - The minimum potential energy is usually taken to be zero
  - The kinetic and potential energy of the oscillator each reaches maximum value twice during one oscillation

**Damped and forced oscillations, resonance**

- Practical examples of damped oscillations:
  - Damping results in a decrease in amplitude of an oscillation with time due to resistive forces acting on it, resulting in a loss of energy
  - The amplitude of an object oscillating in air or water decreases gradually with time but its period remains almost the same – it is said to be underdamped (lightly damped)
  - A car suspension system and the needle of a measuring instrument does not oscillate but returns to equilibrium in the shortest possible time – it is said to be critically damped
  - A swing door takes a long time to return to its equilibrium position without oscillating – it is said to be overdamped (heavily damped)

- A car suspension system consists of a shock absorber inside a coil spring at each wheel
  - The shock absorber contains a piston oscillating in a chamber filled with oil which is squeezed through the piston and the chamber to cause a damping of its oscillations
  - Such a system is critically damped to ensure a smooth ride
- Practical examples of forced oscillations and resonance:
  - A block moving up and down in water with waves incident on it is said to undergo a forced oscillation with water waves as the driver and the block as the oscillator, which results in a net transfer of energy from the water waves to the block
  - Resonance is the phenomenon where the amplitude of the oscillator reaches a maximum when the driving frequency is equal to the natural frequency of the oscillator, resulting in maximum transference of energy to the oscillator
  - An acoustically resonant object such as a string under tension vibrates at certain frequencies directly related to its mass, length and tension
  - Minimal impedance is achieved when the impressed voltage has a certain resonant frequency
- How the amplitude of a forced oscillation changes with driving frequency near the natural frequency of the system, and the factors which determine the frequency response and sharpness of the resonance:




  - Lighter damping increases the sharpness of the resonance
  - Heavier damping lowers the resonance frequency
- There are some circumstances in which resonance is useful, such as in magnetic resonance imaging (MRI), and other circumstances in which resonance should be avoided, such as in bridge design to prevent collapse due to resonant oscillations.

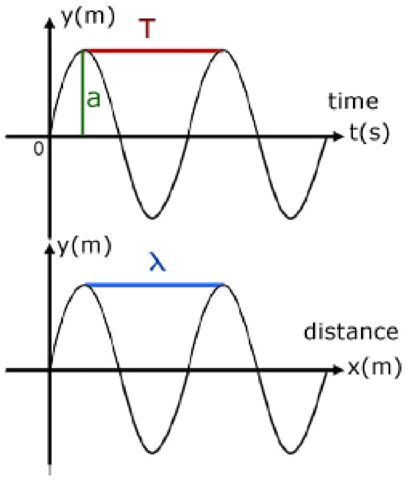
# Topic 11: Wave Motion

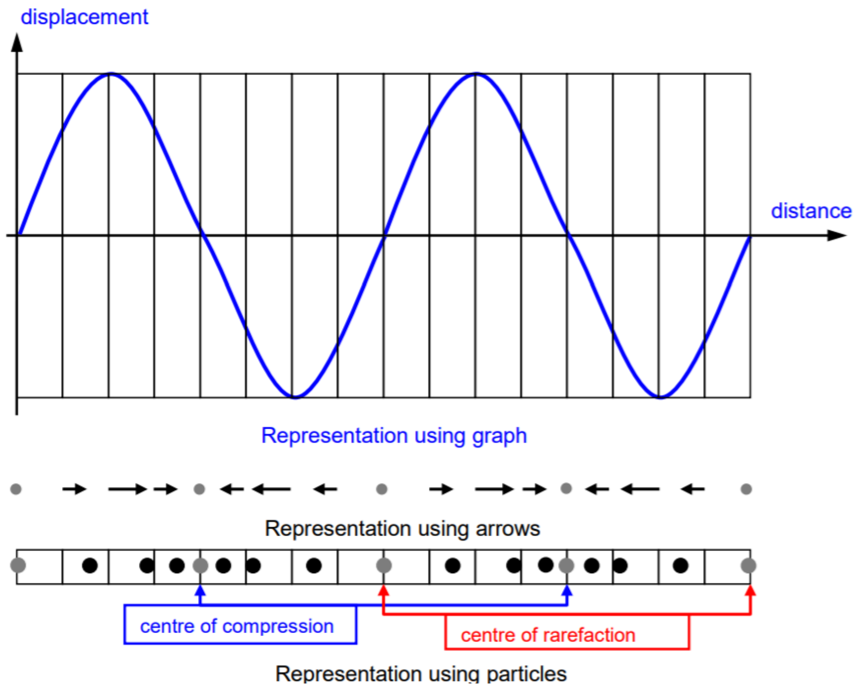
**Progressive waves**

- Displacement, amplitude, period, frequency, phase difference, wavelength and speed:
    - Displacement is the position vector of a wave particle relative to its equilibrium position at a particular instant
    - Amplitude is the maximum distance of a wave particle from its equilibrium position
    - Period is the time taken for one complete wave to pass a given point
    - Frequency is the number of oscillations per unit time of a point on the wave
    - Phase difference is a measure of how much one wave is out of step with another or how much one particle in the wave is out of step with another particle in the same wave
    - Wavelength is the distance between two successive points on the wave that are in phase
    - Speed is the distance travelled by the wave per unit time
- Deduction of the equation $v = f\lambda$:
    - From the definition of speed, speed $v = d / t$ where $d$ is the distance travelled by the wave within a time $t$
    - From the definition of frequency, frequency $f = 1 / t$ where $t$ is the period of the wave $\Rightarrow t = 1 / f$
    - From the definition of wavelength, wavelength $\lambda = d$ since the wave travels one wavelength in one period
    - Hence $v = \lambda / (1 / f) \Rightarrow v = f\lambda$
- The equation $v = f\lambda$ can be used to find one of these quantities given the other two.
- There is a net energy transfer from the source outwards due to a progressive wave along its direction of propagation.
- The relationship $intensity \propto (amplitude)^2$ can be used to solve problems involving the intensity of a wave.
- A wave from a point source and travelling without loss of energy obeys an inverse square law, i.e. $intensity \propto 1 / (distance)^2$, since the power of the wave has been distributed over a larger surface area, and this concept can be applied to solve problems involving the intensity of a wave at different distances from the source.

**Transverse and longitudinal waves**

- Graphical representations of transverse and longitudinal waves:
    - For transverse waves, the positive axis usually refers to the upward displacement of a particle
    - For longitudinal waves, the positive axis usually refers to the displacement of the particle to the right
    - Period and amplitude can be obtained from a displacement-time graph
    - Wavelength and amplitude can be obtained from a displacement-distance graph




**Polarisation**

- Polarisation is a phenomenon associated with transverse waves only.
- Malus' Law states that when completely plane polarised light is incident on an analyser, the intensity of the light transmitted by the analyser is directly proportional to the square of the cosine of the angle between the transmission axes of the analyser and polariser, i.e. $intensity \propto \cos^2\theta$ – this law, along with the relationship $intensity \propto (amplitude)^2$ can be used to calculate the amplitude and intensity of a plane polarised electromagnetic wave after transmission through a polarising filter.

**Determination of frequency and wavelength of sound waves**

- Determination of the frequency of sound using a calibrated oscilloscope:
  - Place a microphone in the path of the sound wave so that it converts the sound energy into electrical energy such that the amplitude of the microphone's voltage output is proportional to the amplitude of the sound wave

- Since the x-axis of the oscilloscope is the time base, the oscilloscope displays the voltage-time graph of the microphone output, which will give the displacement-time graph of the sound wave
- The period of the sound wave is given by the product of the length of one wave cycle and the time-base setting, and the frequency of sound can be calculated using $f = 1 / T$

- Determination of the wavelength of sound using stationary waves:
  - The wavelength of sound can be measured using a resonance tube or a reflector and microphone connected to a c.r.o.
  - For the former method, immerse a resonance tube in water, and place the sound source on top of the tube
  - Slowly raise the tube vertically out of the water until the loudest note is first heard – this is where resonance occurs and stationary waves are formed
  - Measure the length $L_1$ from the water level to the top of the tube
  - Raise the tube further until the next loud sound is heard and measure the length $L_2$ from the water level to the top of the tube
  - The wavelength of sound is given by $\lambda = 2 (L_2 - L_1)$
  - Alternatively, place a microphone between the source and a reflector and connect it to a c.r.o.
  - By moving the microphone slowly forward and backward along the path of the wave, the amplitude shown on the c.r.o. varies from minimum and maximum showing the positions of nodes and antinodes respectively
  - The wavelength of the sound is twice the distance between successive nodes or successive antinodes

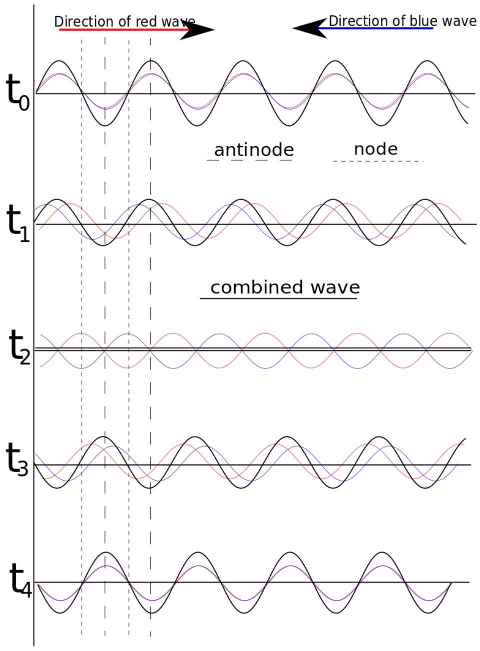
# Topic 12: Superposition

**Principle of superposition**

- Principle of superposition in simple applications:
  - The principle of superposition states that when waves of the same type meet, the total displacement at any point is the vector sum of their individual displacements at that point, and each wave proceeds as though no other waves exist
  - For example, when two wave pulses travel in opposite directions through a rope, the resultant displacement of the rope is always equal to the sum of the displacements produced by each pulse
- The terms interference, coherence, phase difference and path difference:
  - Interference is the overall change in intensity as a result of the superposition of wavetrains from a finite number of coherent sources
  - Coherent sources are sources that have a constant phase difference
  - Phase difference is a measure of how much one wave is out of step with another
  - Path difference is the extra length that the wave from one source must travel, compared to the wave from the other source, in order to reach the point of observation

**Stationary waves**

- Experiments which demonstrate stationary waves using microwaves, stretched strings and air columns:
  - A microwave transmitter continually generates transverse progressive microwaves which are reflected back by a metal sheet
  - The reflected waves superpose with incident waves to form stationary waves in the space between the transmitter and the reflector
  - A microwave receiver detects the positions where vibrations are maximum (antinodes) and minimum (nodes)
  - A vibrator continually generates transverse progressive waves on a stretched string which are reflected at the fixed end of the string
  - The reflected waves superpose with the incident waves to form stationary waves on the string
  - When the driving frequency matches one of the resonant frequencies of the string, stationary waves are produced and the amplitude of oscillation is the largest
  - For each mode of vibration, it must meet the boundary conditions at both ends – a displacement node at each end
  - A vibrating tuning fork or a speaker attached to a signal generator produces sound waves in a tube
  - At the end of the tube, sound waves are reflected whether the end is closed or open
  - When sound waves satisfy the boundary conditions of the tube, resonance occurs, stationary waves are produced and loud sounds are heard as a result of the superposition of reflected waves with incident waves
  - At a closed end, sound waves reflect with a reversal of phase, resulting in the layer of air next to the closed end being a node of the stationary wave
  - At an open end, sound waves reflect without a reversal of phase, resulting in the layer next to the closed end being an antinode of the stationary wave
- Formation of a stationary wave and identification of nodes and antinodes:



    - Stationary waves can only be formed when the two superposing progressive waves are of equal amplitude, frequency and speed and are travelling in opposite directions
    - Nodes are points where particles do not oscillate as component waves superpose destructively
    - Antinodes are points where particles oscillate with maximum amplitude as component waves superpose constructively

**Diffraction**

- Diffraction is the spreading of waves when they pass through an opening or around an obstacle.
- Experiments which demonstrate diffraction:
    - When water waves in a ripple tank pass through an opening, they spread on either side of the opening
    - When the width of the opening is larger than the wavelength, little diffraction occurs
    - When the width of the opening is less than the wavelength, there is appreciable diffraction
    - Sound waves of a particular frequency have wavelengths that are similar in dimension to openings such as doorways or windows and spread around them

**Two-source interference**

- Experiments which demonstrate two-source interference using water waves, sound waves, light waves and microwaves:
    - Interference of water waves can be observed when two dippers are attached to the same vibrating bar in a ripple tank
    - The bright and dark bands of the interference pattern indicate the locations of crests and troughs respectively
    - Interference of sound waves can be observed when two loudspeakers are connected in parallel to an audio-frequency oscillator
    - As a microphone is moved across the two loudspeakers, alternate loud and soft sounds are heard depending on whether it is along a line of constructive or destructive interference
    - If the connections to one of the speakers is reversed such that they oscillate in antiphase, the positions of loud and soft sounds will be reversed
    - Interference of light waves can be observed when a monochromatic light source is focused by a lens onto a narrow slit and two other narrow slits are placed a short distance ahead
    - Alternate bright and dark equally-spaced vertical bands will be observed on a screen placed a distance away in front of the slits
    - Interference of microwaves can be observed when two slits are placed in front of a transmitter
    - A receiver detects positions of constructive and destructive interference as it moves round in front of the slits
- Conditions required for two-source interference fringes to be observed:

    - Sources must be coherent
    - The waves cannot be polarised in mutually perpendicular planes to ensure that they cancel each other when they meet in antiphase
    - The amplitudes of the waves from the two sources must be similar so that the contrast between constructive and destructive interference is apparent
- The equation $\lambda = ax / D$, where $a$ is the slit separation, $x$ is the fringe separation and $D$ is the distance between the slits and the screen, may be used to solve problems involving double-slit interference.

**Single slit and multiple slit diffraction**

- The equation $\sin\theta = \lambda / b$, where $\theta$ is the angle subtended by the first minimum and the central maximum and $b$ is the width of the slit, can be used to locate the position of the first minima for single slit diffraction.
- The Rayleigh criterion states that two sources are just resolved if the centre of one diffraction pattern falls at the first minimum of the other one, or when $\theta \approx \lambda / b$, where $\theta$ is the angular separation of the two sources.
- The equation $d \sin\theta = n\lambda$, where $\theta$ is the angle between the $n$th order maximum and $d$ is the slit separation, can be used to locate the positions of the principal maxima produced by a diffraction grating.
- Use of a diffraction grating to determine the wavelength of light:
    - A diffraction grating with a known slit separation $d$ is mounted on a spectrometer and the light is allowed to pass through the diffraction grating
    - The angle $\theta$ between the central maximum and the $n$th maximum is measured using a spectrometer
    - The wavelength of light is given by $\lambda = d \sin\theta / n$

# Section V: Electricity and Magnetism

## Topic 13: Electric Fields

**Concept of an electric field**

- Concept of an electric field and definition of electric field strength at a point:
    - An electric field is a region of space where a charge experiences a force, and is an example of a field of force
    - Electric field strength at a point is defined as the electric force per unit positive charge on a small test charge placed at that point, i.e. $E = F / Q$ or $F = QE$
- Representation of an electric field by means of field lines:
    - A field line indicates the direction of the force a positive charge would experience if it is placed at that point in the field
    - The number of field lines per unit cross-sectional area is proportional to the electric field strength
- Analogy between certain qualitative and quantitative aspects of electric and gravitational fields:
    - Electric force results from interaction between charges, while gravitational force results from interaction between masses

- Both electric and gravitational fields are inverse square law fields

**Electric force between point charges**

- Coulomb's Law states that the electric force $F$ between two point charges is proportional to the product of the charges $Q_1Q_2$ and inversely proportional to the square of their separation $r^2$, i.e. $F = Q_1Q_2 / 4\pi\varepsilon_0 r^2$, where $\varepsilon_0$ is the permittivity of free space which has the value $8.85 \times 10^{-12}$ F m$^{-1}$ or $(1/(36\pi)) \times 10^{-9}$ F m$^{-1}$ and can be taken to be equal to that of air unless otherwise specified.

**Electric field of a point charge**

- $E = Q / 4\pi\varepsilon_0 r^2$ can be used for the electric field strength of a point charge in free space or air.

**Uniform electric fields**

- The electric field strength $E$ of the uniform field between charged parallel plates can be calculated using $E = V / d$, where $V$ is the potential difference between the plates and $d$ is the plate separation.
- The forces on charges in uniform electric fields can be calculated using $F = QE$.
- Effect of a uniform electric field on the motion of charged particles:
    - A charge projected parallel to the electric field will be accelerated parallel to its direction of motion, resulting in a linear trajectory
    - A charge projected perpendicularly or at an angle to the electric field will be accelerated perpendicularly or at an angle to its direction of motion, resulting in a parabolic trajectory

**Electric potential**

- The electric potential at a point in an electric field is the work done per unit positive charge by an external agent in bringing a small test charge from infinity to that point without any change in kinetic energy of the test charge.
- The field strength of the electric field at a point is equal to the negative of the potential gradient at that point.
- The equation $V = Q / 4\pi\varepsilon_0 r$ for the electric potential in the field of a point charge, in free space or air, can be used to solve problems.

# Topic 14: Current of Electricity

**Electric current**

- Electric current is the net rate of flow of electric charge.
- Derivation of the equation $I = nAvq$ for a current-carrying conductor:
    - The current $I$ in a segment of conductor is the rate of flow of charge Q, i.e. $I = Q / t$
    - The total charge is the product of the number of charge carriers $N$ and the charge $q$ carried by each carrier, i.e. $Q = Nq$
    - The number density of charge carriers $n$ is the number of charge carriers per unit volume $V$, i.e. $n = N / V \Rightarrow N = nV = nAx$, where $A$ is the cross-sectional area of the conductor and $x$ is its length

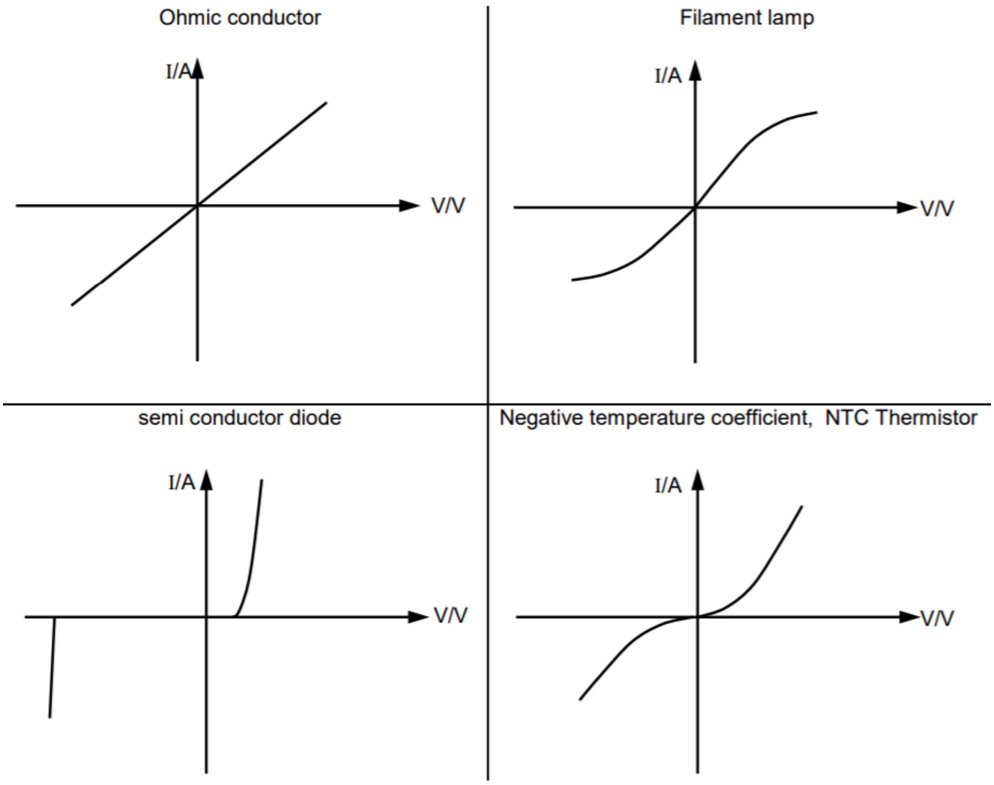
    - The drift velocity $v$ of the charge carriers is the average distance travelled by the charge carriers per unit time, i.e. $v = x / t$
    - Thus $I = Nq / t = nAxq / t$
    - Hence $I = nAvq$
    - This equation can be used to determine the current in a conductor or the drift velocity of the charge carriers
- The equation $Q = It$ can be used to solve problems by relating the current to the rate of flow of charge.

**Potential difference**

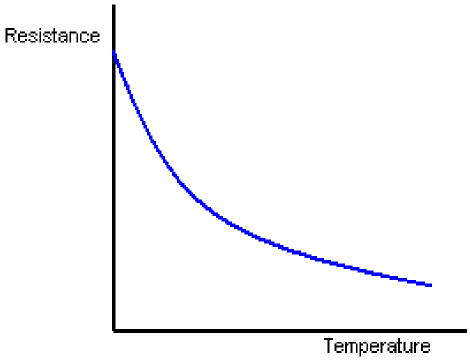
- The equation $V = W / Q$ can be used to solve problems by relating the potential difference across a conductor to the work done to drive a unit charge through it.
- The equations $P = VI$, $P = I^2R$ and $P = V^2 / R$, where $R$ is resistance, can be used to solve problems involving the power dissipated in a resistor.

**Resistance and resistivity**

- The resistance of a circuit component is the ratio of the potential difference across the component to the current passing through it – the equation $V = IR$ can be used to solve problems involving the resistance of a circuit component.
- *I*-*V* characteristics of various electrical components:
    - For an ohmic resistor, its steady-state temperature remains constant as the potential difference across it varies
    - Since the mobility of charge carriers remains constant, the resistance of the ohmic conductor remains constant
    - For a semiconductor diode, resistance is very high in the reverse-bias direction but insignificant in the forward-bias direction
    - At a certain threshold potential in the reverse-bias direction, a Zener breakdown occurs, allowing the diode to conduct an infinitely large current
    - For a filament lamp, the steady-state temperature of the filament increases as the potential difference across it increases
    - This causes conducting electrons to collide more frequently with the lattice ions in the wire, decreasing the mobility of the charge carriers and increasing the resistance of the filament
    - For a negative temperature coefficient (NTC) thermistor, its temperature increases as the potential difference across it increases, resulting in more mobile charge carriers being liberated in it
    - As there is an increase in the amount of mobile charge carriers, the resistance of the thermistor decreases




- Resistance-temperature characteristic of an NTC thermistor:




- The equation $R = \rho l / A$ , where $\rho$, $l$ and $A$ are the resistivity, length and cross-sectional area of the conductor respectively, can be used to solve problems.

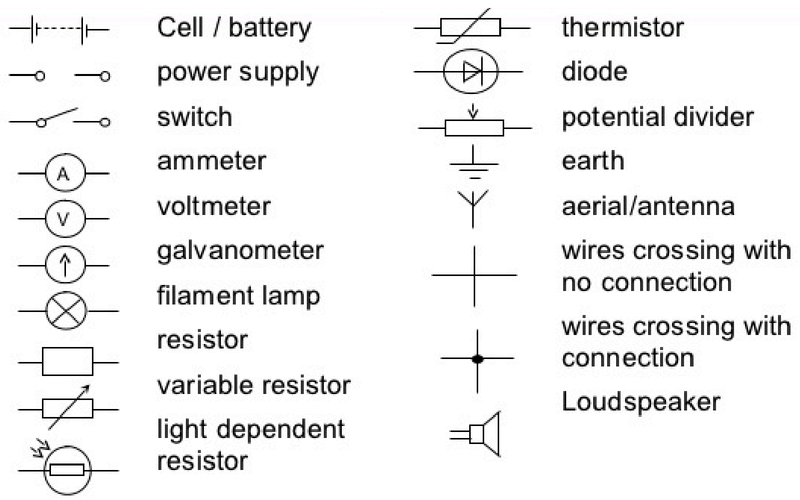
**Electromotive force**

- Distinction between electromotive force (e.m.f.) and potential difference (p.d.):
    - The potential difference between two points is the electrical energy converted to other forms of energy per unit charge passing from one point to another
    - The electromotive force of a source is the energy converted from other forms to electrical energy per unit charge passing through the source
- Effects of the internal resistance of a source of e.m.f. on the terminal potential difference and output power:
    - The terminal potential difference across the cell is lower than the e.m.f. of the cell because some power is dissipated in the internal resistance, causing a potential drop
    - The output power will be lower than the theoretical power output as some energy is dissipated as heat in the internal resistance

# Topic 15: D.C. Circuits

**Circuit symbols and diagrams**

- Circuit symbols:




- Using these symbols, circuit diagrams can be drawn to represent circuits, which contain one or more source(s) together with switches, resistors, ammeters, voltmeters or other components.

**Series and parallel arrangements**

- The formula for the combined resistance of two or more resistors in series, $R_T = R_1 + R_2 + \ldots + R_n$, can be used when there is a common current through the resistors..
- The formula for the combined resistance of two or more resistors in parallel, $1 / R_T = 1 / R_1 + 1 / R_2 + \ldots + 1 / R_n$, can be used if there is a common potential difference across each resistor.
- Problems involving series or parallel circuits for one source of e.m.f. can be solved by rearranging or simplifying the circuit diagram in one of the following two ways:
  - Redraw the circuit so that it is convenient to see resistors arranged in parallel or in series
  - Progressively reduce the number of resistors until the circuit can be simplified to a single resistor

**Potential divider**

- Use of a potential divider circuit as a source of variable p.d.:
  - When two resistors are arranged in series, they have a common current through them, thus the potential difference across each resistor will be directly proportional to its resistance
  - By allowing one of the resistances to vary, the output voltage in a potential divider circuit could be varied
- Use of thermistors and light-dependent resistors in potential divider circuits:
  - The resistance of most thermistors decreases when temperature increases
  - The resistance of a light-dependent resistor decreases when the incident light intensity increases

- A thermistor can be connected in a potential divider circuit to provide a potential difference which is temperature dependent, such as in a fire detection system or an air-conditioner
- A light-dependent resistor can be used in a potential divider circuit to provide a potential difference which is illumination dependent, such as in lighting circuits

**Balanced potentials**

- Principle of the potentiometer as a means of comparing potential differences:
  - A potentiometer is an instrument for measuring an unknown voltage by comparing it with a known reference voltage
  - The principle of the potentiometer is that the potential drop across a segment of a wire with uniform cross-section is directly proportional to its length
  - In a potentiometer circuit, the ends of a uniform resistance wire are connected to the known e.m.f.
  - The unknown e.m.f. is connected in series with a galvanometer and a jockey across a section of the resistance wire
  - At balance point, no current flows through the galvanometer and unknown e.m.f.
  - By measuring the balance length, the value of the potential difference across it and hence the unknown e.m.f. can be determined using the fact that p.d. is directly proportional to length

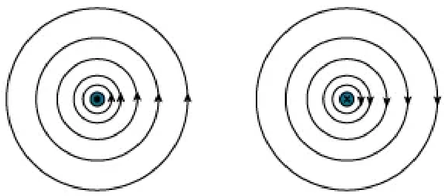
# Topic 16: Electromagnetism

**Concept of a magnetic field**

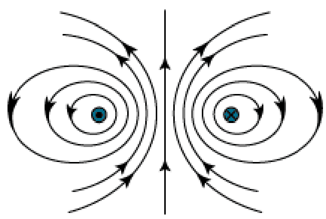
- A magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets, and it is where a moving charge, current-carrying conductor or magnetic material experiences a force.

**Magnetic fields due to currents**

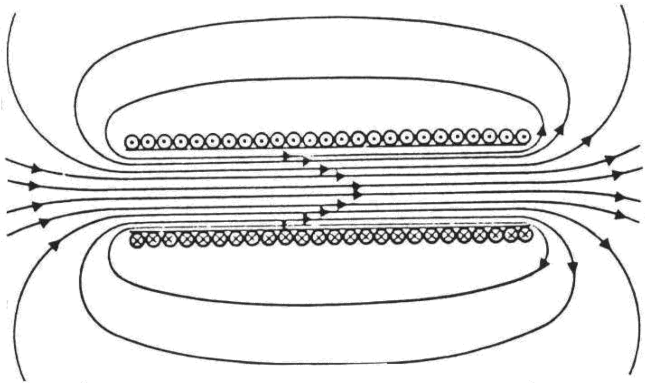
- Flux patterns due to currents in a long straight wire, a flat circular coil and a long solenoid:



long straight wire



flat circular coil



long solenoid

- Flux densities of the fields due to currents in a long straight wire, a flat circular coil and a long solenoid:
    - Long straight wire: $B = \mu_0 I / 2\pi d$, where $d$ is the distance from the wire
    - Flat circular coil: $B = \mu_0 NI / 2r$, where $N$ and $r$ are the number of turns and radius of the coil respectively
    - Long solenoid: $B = \mu_0 nI$, where $n$ is the number of turns per unit length of the solenoid
    - $\mu_0$ is the permeability of free space which has the value $4\pi \times 10^{-7}$ H m$^{-1}$
- The magnetic field due to a solenoid may be influenced by the presence of a ferrous core – it has the ability to align its dipoles by magnetisation and concentrate the field lines, thus strengthening the magnetic field.

**Force on a current-carrying conductor**

- A current-carrying conductor placed in a magnetic field will experience a force if the field is perpendicular to or has a component perpendicular to the direction of current flow.
- The equation $F = BIl \sin\theta$, with directions as interpreted by Fleming's left-hand rule, where $l$ is the length of the conductor and $\theta$ is the angle between the current and the magnetic field, can be used to solve problems involving a current-carrying conductor in a magnetic field.
- Magnetic flux density at a point is defined as the magnetic force per unit length exerted on a long straight conductor carrying a unit current placed perpendicular to the magnetic field, i.e. $B = F / Il$.
- How the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance:
    - Part of a current-carrying rectangular coil is placed inside the magnetic field such that it experiences a downward magnetic force $F$
    - A rider of known mass $m$ is hung at the opposite end of the coil
    - The coil is pivoted at a distance $d_1$ from the part of the coil experiencing a force due to the magnetic field and a distance $d_2$ from the rider such that the coil is in rotational equilibrium
    - The magnetic flux density can be calculated using $B = mgd_2 / Ild_1$ by taking moments about the pivot

**Force between current-carrying conductors**

- Forces between current-carrying conductors:
    - Since a magnetic field is set up when there is a current in a conductor, each conductor lies in the magnetic field created by the current in the other conductor
    - Each conductor thus experiences a magnetic force
    - By Fleming's left-hand rule, when the currents flow in the same direction, there is a force of attraction between them
    - By Fleming's left-hand rule, when the currents flow in opposite directions, there is a repulsive force between them
    - These can be used to predict the direction of the forces acting on the conductors

**Force on a moving charge**

- The direction of the force on a charge moving in a magnetic field can be obtained by Fleming's left-hand rule, with the thumb indicating the direction of the force, the index finger indicating the direction of the magnetic field and the middle finger indicating the direction of movement of the charge if it is positive or opposite to its direction of movement if it is negative.
- The equation $F = BQv \sin\theta$ can be used to solve problems involving moving charges in a magnetic field.
- Deflections of beams of charged particles by uniform electric and uniform magnetic fields:
    - In a uniform electric field, a beam of positive charges will be deflected in the direction of the electric field while a beam of negative charges will be deflected in the opposite direction, thus forming a straight or parabolic path
    - In a uniform magnetic field, a beam of charged particles will be deflected in a direction perpendicular to the magnetic field and the velocity of the particles according to Fleming's left-hand rule, thus forming a circular or helical path
    - The magnetic field provides the centripetal force for the moving charges to perform circular or helical motion – this can be used to analyse the deflection of the beam of charged particles
    - In the case of helical motion, the velocity of the charges can be resolved into components perpendicular and parallel to the magnetic field
- How electric and magnetic fields can be used in velocity selection for charged particles:
    - Crossed uniform electric and magnetic fields can be used in velocity selection of charged particles as both fields exert forces in opposite directions on a charge moving through the fields perpendicular to both fields
    - By adjusting the strength of the electric field and the flux density of the magnetic field such that the magnitude of the forces exerted on the charged particle by both fields are equal, the particle can be made to move through the crossed fields undeflected
    - For a beam of charged particles having a range of velocities, only those moving with a certain velocity can be made to emerge at a slit while the rest are blocked – a requirement of many experiments

# Topic 17: Electromagnetic Induction

**Magnetic flux**

- Magnetic flux through an area is defined as the product of the area and the component of the magnetic flux density perpendicular to that area.
- $\Phi = BA$, where $\Phi$ is the magnetic flux through the plane and $B$ is the component of the magnetic flux density normal to its area $A$, can be used to solve problems involving the magnetic flux through a plane.
- Magnetic flux linkage of a coil is defined as the product of the magnetic flux through one loop and the number of turns of wire in the coil, i.e. $N\Phi = NBA$.

**Laws of electromagnetic induction**

- Experiments on electromagnetic induction:
  - When a magnet is moved towards or away from a conducting coil attached to a galvanometer, the galvanometer needle is deflected, but when the magnet is stationary, the galvanometer needle is not deflected – this shows that a changing magnetic field can induce an e.m.f.
  - The direction of the magnetic force on the magnet when it is moved is opposite to that of its motion – this shows that the direction of the induced e.m.f. opposes the change producing it
  - When the speed of the magnet relative to the coil is increased, the deflection of the galvanometer needle increases
  - When a stronger magnet is used instead, the deflection of the galvanometer needle increases
  - These show that the magnitude of the induced e.m.f. is proportional to the rate of change of magnetic flux through the coil
- Solving problems using Faraday's law of electromagnetic induction and Lenz's law:
  - Faraday's Law of Electromagnetic Induction states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage
  - Lenz's Law states that the direction of the induced e.m.f. Is such that its effect opposes the change causing it
  - Lenz's Law means that the polarity of the induced e.m.f. is such that it will produce a current that will set up a magnetic field that opposes the change in magnetic flux linkage if there is a closed loop
  - Thus $\varepsilon = -\Delta N\Phi / \Delta t$, where $\varepsilon$ is the average induced e.m.f.
  - Faraday's Law of Electromagnetic Induction and Lenz's Law can be used to solve problems involving electromagnetic induction by determining the magnitude and direction of the induced e.m.f. respectively
- Simple applications of electromagnetic induction:
  - In an a.c. generator, when the coil is rotated in the magnetic field, there will be a change in magnetic flux linkage due to a change in the orientation of the coil relative to the magnetic field
  - By Faraday's Law of Electromagnetic Induction, there is an induced e.m.f. in the coil along with the conversion of mechanical energy into electrical energy

- In a Faraday disc, a solid disc spins in a uniform magnetic field and there is continuous magnetic flux cutting
- By Faraday's Law of Electromagnetic Induction, there is an induced e.m.f. across the centre and rim of the disc
- In a microphone, sound vibrates the diaphragm and the solenoid, causing a change in its magnetic flux linkage
- By Faraday's Law of Electromagnetic Induction, there is an e.m.f. in the solenoid which can be transmitted and amplified to recreate the sound
- When a solid conductor moves through a magnetic field, there is continuous magnetic flux cutting
- By Faraday's Law of Electromagnetic Induction, there is an induced e.m.f., and hence, induced eddy currents in the conductor, producing a large amount of heat which can be utilised in induction cookers
- According to the principle of energy conservation, this results in a loss of kinetic energy – this 'braking' effect can be utilised in electromagnetic brakes such as those in trains

# Topic 18: Alternating Current

**Characteristics of alternating currents**

- The terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage:
    - Period is the time taken for the completion of one cycle of the alternating current
    - Frequency is the number of complete cycles of the alternating current per unit time
    - Peak value of a current or voltage is the maximum current or potential difference respectively across a component in an alternating current circuit
    - Root-mean-square (r.m.s.) value of an alternating current or voltage is the equivalent value of the steady direct current or voltage respectively which, when passed through the same resistor, produces heat at the same average rate as that of the alternating current or voltage
- Mean power in a resistive load for a sinusoidal alternating current:
    - Since $P = V^2 / R$, the power in the resistive load is proportional to the square of the potential difference across it
    - Since $V_{rms} = V_0 / \sqrt{2}$, where $V_{rms}$ is the r.m.s. p.d. and $V_0$ is the maximum p.d., mean power $\langle P \rangle = V_{rms}^2 / R = (V_0 / \sqrt{2})^2 / R = V_0^2 / (2R) = P_0 / 2$, where $P_0$ is the maximum (peak) power
    - Hence the mean power in a resistive load is half the maximum (peak) power for a sinusoidal alternating current
- An alternating current or an alternating voltage can be represented by an equation of the form $x = x_0 \sin \omega t$ where $\omega$ is the angular frequency of the oscillation and $x$ is the current or voltage.
- Distinction between r.m.s. and peak values:
    - Peak value is the maximum value of the current or voltage while r.m.s. value is the mean value of the square of current or voltage

    - For the sinusoidal case, the relationship $I_{rms} = I_0 / \sqrt{2}$, where $I_{rms}$ is the r.m.s. current and $I_0$ is the maximum current, can be used to solve problems by converting the peak current into r.m.s. current and vice versa

**The transformer**

- Principle of operation of a simple iron-core transformer:
    - An alternating e.m.f. applied across the primary coil produces an alternating current through the coil, setting up a varying magnetic field in the soft-iron core which links with the secondary coil
    - By Faraday's Law of Electromagnetic Induction, an alternating e.m.f. is induced across each turn of wire in the secondary coil, producing an alternating current in the second coil
    - For an ideal transformer, $N_s / N_p = V_s / V_p = I_p / I_s$, where $N_s$ and $N_p$ are the number of turns in the secondary and primary coils respectively, $V_s$ is the e.m.f. induced in the secondary coil, $V_p$ is the e.m.f. applied across the primary coil and $I_p$ and $I_s$ are the currents in the primary and secondary coils respectively, can be used to solve problems involving the turns ratio $N_s / N_p$, voltage and current in each coil

**Rectification with a diode**

- Use of a single diode for the half-wave rectification of an alternating current:
    - Half-wave rectification occurs when a diode is connected in series with an alternating current power source
    - The diode conducts only during half the cycle when current is flowing in the forward-bias direction, causing the potential difference across a component to have a waveform made up of half-waves
    - The result is the production of a fluctuating d.c. from an alternating current source

# Section VI: Modern Physics

## Topic 19: Quantum Physics

**Energy of a photon**

- The particulate nature of electromagnetic radiation:
    - Electromagnetic radiation can be said to be particulate in nature in addition to being waved in nature
    - A beam of light consists of small discrete quanta of electromagnetic energy known as photons
    - Photons transfer either all or none of their energy to another particle instantaneously, contrary to the wave theory which states that the energy transfer is continuous
- The equation $E = hf$, where $h$ is the Planck constant which has the value $6.63 \times 10^{-34}$ J s, can be used to calculate the energy of a photon.

**The photoelectric effect**

- The photoelectric effect provides evidence for the particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for the wave nature.
- The threshold frequency is the minimum frequency of an incident photon that will remove an electron from the surface of the metal and is dependent on the work function of the metal.
- The equation $\frac{1}{2}mv_{max}^2 = eV_s$, where $V_s$ is the stopping potential, can be used to solve problems involving the emitted photoelectrons.
- Explanation of photoelectric phenomena:
    - The photoelectric effect is the phenomenon where electrons are emitted from a metal when electromagnetic radiation of a sufficiently high frequency is incident on it
    - Below the threshold frequency, the photon energy of the incident electromagnetic radiation is less than the work function energy of the metal and is insufficient to remove an electron from the surface of the metal
    - Electrons can only escape if the energy of the incident photon is greater than the work function energy of the metal
- Why the stopping potential is independent of intensity whereas the photoelectric current is proportional to intensity at constant frequency:
    - At a constant frequency, a change in intensity results in a change in the photon arrival rate, but the energy of each photon remains unchanged
    - Since each photoelectron is liberated by one photon, the maximum kinetic energy of the photoelectrons and hence stopping potential remains unchanged
    - On the other hand, the rate of emission of photoelectrons and hence the photoelectric current is proportional to intensity
- The equation $hf = \Phi + \frac{1}{2}mv_{max}^2$ is a statement of the principle of energy conservation – the energy of an incident photon $hf$ is equal to the work function energy of the target metal $\Phi$ and the maximum kinetic energy $\frac{1}{2}mv_{max}^2$ of the ejected photoelectron – and can be used to solve problems involving the interaction of photons and electrons.

**Wave-particle duality**

- Evidence provided by electron diffraction for the wave nature of particles:
    - Electrons are produced at a cathode by thermionic emission with negligible kinetic energy and accelerated at high p.d. through a hole in an anode
    - When the beam of electrons is directed at a thin carbon disc, a diffraction pattern consisting of concentric rings is produced on a fluorescent screen
    - This shows that particles such as electrons have a wave-like nature and can produce a diffraction pattern
    - The regularly arranged atoms of the thin carbon crystal acts as a diffraction grating
- The relation for the de Broglie wavelength $\lambda = h / p$, where $p$ is the momentum of the particle, can be used to find the wavelength of a particle given its momentum.

**Energy levels in atoms**

- Existence of discrete electronic energy levels in isolated atoms and how this leads to the observation of spectral lines:
  - An energy level is the quantised energy corresponding to an allowed state (orbital) of an electron within an atom – an electron is not allowed to possess an amount of energy that is between two energy levels
  - Each type of atom (for example atomic hydrogen) has a characteristic set of energy levels which an electron must have
  - An atom in the excited state is unstable and electrons have a tendency to de-excite to lower energy levels, producing photons of energy equal to the change in energy of the electron
  - Photons of energy equal to the gain in energy of an electron can be absorbed by the electron, exciting it to a higher energy level
  - When the photons emitted are passed through a diffraction grating, the presence or absence of electromagnetic radiation corresponding to the energies of the emitted or absorbed photons can be seen on the screen giving rise to spectral lines

**Line spectra**

- Distinction between emission and absorption line spectra:
  - Emission line spectra are produced when excited electrons de-excite and emitted photons with frequencies corresponding to the energy change radiate in all directions
  - Absorption line spectra are produced when electromagnetic radiation of a continuous range of frequencies is incident on cool atoms resulting in an excitation from ground state and the absorption of photons of certain frequencies
- The relation $hf = E_2 - E_1$, where $E_2 - E_1$ is the energy difference between the initial and final energy levels, can be used to solve problems involving the photons produced from the excitation or de-excitation of electrons in atoms.

**X-ray spectra**

- Origins of the features of a typical X-ray spectrum:
  - X-rays are produced when electrons emitted by thermionic emission from a heated filament are accelerated at high p.d. towards a metal target
  - When high-speed electrons strike the metal target, large decelerations occur producing electromagnetic radiation in the X-ray region of the spectrum
  - Since the magnitudes of the deceleration of the electrons have a continuous distribution, a continuous spectrum of X-rays is produced
  - There exists a minimum cutoff wavelength where all the kinetic energy of an electron is converted into the energy of a single photon – such a photon will possess the maximum possible energy and frequency
  - A characteristic spectrum is formed when accelerated electrons remove electrons in the inner shells of the target atoms

    - The vacancy is quickly filled by electrons from the higher energy levels, producing photons of specific energies due to the discrete electronic energy levels in the atoms
    - The lower the required energy change, the higher the probability that a given electron de-excites to fill a vacancy

**The uncertainty principle**

- The Heisenberg position-momentum uncertainty principle states that it is impossible to measure the exact position and momentum of a body at the same time – it can be expressed in the form $\Delta p \Delta x \gtrsim h$, where $\Delta p$ and $\Delta x$ are the uncertainties in the momentum and position of the particle respectively, and can be applied to solve related problems such as those involving electron diffraction.

# Topic 20: Nuclear Physics

**The nucleus**

- The existence and small size of the atomic nucleus as inferred from the results of the Rutherford α-particle scattering experiment:
    - Most of the α-particles travelled undeflected through the gold film – this shows that an atom consists of mostly empty space and the nucleus is very small compared to the atom
    - A very small number of α-particles were deflected more than 90° – this shows that the nucleus is positively charged and that a gold nucleus has a much larger mass than the α-particles to be able to repel them backwards
- Distinction between nucleon number (mass number) and proton number (atomic number):
    - Nucleon number (mass number) is the total number of nucleons in the nucleus
    - Proton number (atomic number) is the number of protons in the nucleus

**Isotopes**

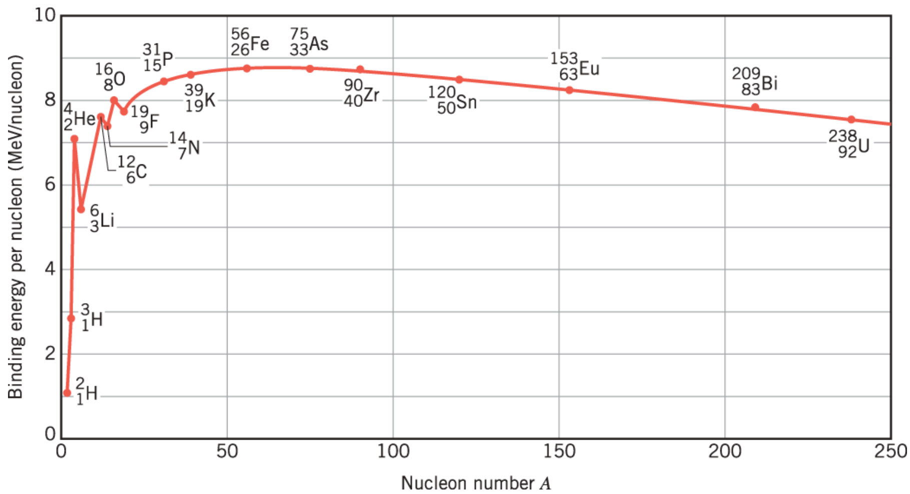
- An element can exist in various isotopic forms each with a different number of neutrons in the nucleus.

**Nuclear processes**

- Usual notation for the representation of nuclides and representation of simple nuclear reactions:
    - Nuclides may be represented by ${}^{A}_{Z}X$ where $A$ is the nucleon number (mass number) of the element, $Z$ is the proton number (atomic number) of the element and X is the chemical symbol of the element
    - Simple nuclear reactions may be represented by nuclear equations of the form ${}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$ where the left hand side shows the reactant nuclides and the right hand side shows the product nuclides
- Nucleon number, charge and mass-energy are all conserved in nuclear processes – this concept can be applied to solve problems involving nuclear reactions.

### Mass defect and nuclear binding energy

- Concept of mass defect:
    - Mass defect is the difference between the total mass of the nucleons of the atom taken separately and the mass of the nucleus
    - It is due to the energy supplied to separate completely the individual nucleons of the nucleus
- The equivalence between energy and mass, as represented by $E = mc^2$, where $c$ is the speed of light in free space which has the value $3.00 \times 10^8$ m s$^{-1}$, can be applied to solve problems involving mass defect and nuclear binding energy.
- Concept of nuclear binding energy and its relation to mass defect:
    - Nuclear binding energy is the energy required to separate the nucleons in the nucleus to infinity
    - It is also the energy released when a nucleus is formed from separated nucleons
    - Binding energy is directly proportional to mass defect and can be calculated using the equivalence between energy and mass
- Variation of binding energy per nucleon with nucleon number:

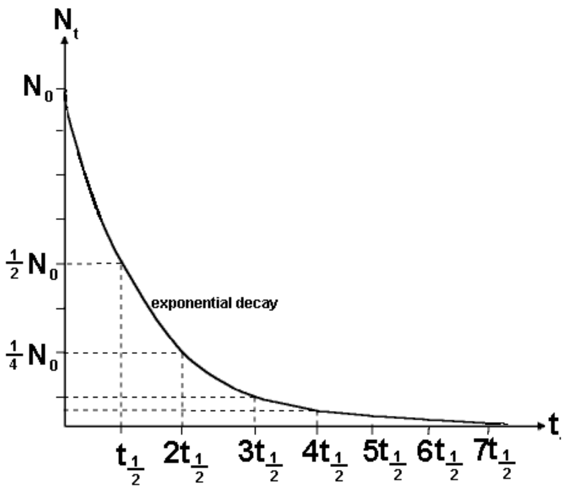



- Relevance of binding energy per nucleon to nuclear fusion and to nuclear fission:
    - For $A < 56$, if nuclei are fused together in a nuclear fusion reaction, the product nucleus will have a greater binding energy per nucleon, causing a loss of mass and a release of energy
    - For $A > 56$, if a nucleus is split into nuclei with lower nucleon number in a nuclear fission reaction, the product nuclei will each have a greater binding energy per nucleon, causing a loss of mass and a release of energy
    - Otherwise, an energy input is required for the nuclear reaction to take place so that energy is conserved
    - The net energy released in a nuclear fusion or nuclear fission process is equal to the difference between the total binding energies of the reactant and product nuclei, plus the input energy

### Radioactive decay

- The spontaneous and random nature of nuclear decay:
    - Nuclear decay is spontaneous – it is not affected by external factors such as temperature or pressure

    - Nuclear decay is random – each nucleus has a constant probability of decay per unit time
- The random nature of radioactive decay can be inferred from fluctuations in count rate.
- Origin and significance of background radiation:
    - Background radiation originates from a variety of sources emitting small quantities of radiation such as food and water, building materials, soil and outer space (in the form of cosmic rays)
    - Background radiation gives rise to a systematic error in the measurement of count rate which has to be eliminated by recording it first and then subtracting it from the count rate obtained in the experiment
- Nature of α, β and γ radiations:
    - α-particles are helium nuclei, β-particles are electrons and γ-rays consist of high-energy photons
    - α-particles are heavy ($4u$) , β-particles are light ($1/1840\ u$) and γ-ray photons have no rest mass
    - α-particles have a charge of $+2e$, β-particles have a charge of $-e$ and γ-ray photons have no charge
    - α-particles produce the strongest ionisation, followed by β-particles and subsequently γ-ray photons
    - α-particles typically move the slowest ($0.1c$) followed by β-particles (up to $0.9c$) and subsequently γ-ray photons ($c$)
    - α radiation is stopped by a piece of paper, β radiation is stopped by a few mm of aluminium and γ radiation is reduced significantly by 10 cm of lead
- How the conservation laws for energy and momentum in β decay were used to predict the existence of the neutrino:
    - It was observed that unlike α decay, β-particles are emitted with a continuous spectrum of speeds and kinetic energies, which seems to violate the principle of energy conservation
    - It was also observed that the nuclear recoil of the daughter nucleus is not in the direction opposite to the momentum of the electron, which seems to violate the principle of conservation of momentum
    - Thus it was predicted that another particle, the neutrino, carrying no charge and almost no mass, is emitted to carry away the missing energy and momentum; the particle was subsequently detected experimentally
- Definition of the terms activity and decay constant:
    - Activity is defined as the rate at which radioactive nuclei decay
    - Decay constant is defined as the probability that a randomly chosen radioactive nucleus will decay in a unit time
    - The equation $A = \lambda N$, where $A$ is the activity of the source, $N$ is the number of radioactive nuclei and $\lambda$ is the decay constant, can be used to solve problems by relating the number of nuclei with activity
- The exponential nature of radioactive decay:
    - Since $A = -dN/dt$, it can be derived from the equation $A = \lambda N$ that the decrease in the number of radioactive nuclei over time is exponential




- The relationship $x = x_0 \exp(-\lambda t)$, where $x$ could represent activity, number of decayed particles or received count rate, can be used to solve problems involving radioactive decay
- Half-life is defined as the time taken for a quantity $x$, where $x$ could represent activity, number of undecayed particles or received count rate, to reduce to half its original value.
- The relation $\lambda = \ln 2 / t_{½}$ , where $t_{½}$ is half-life, can be used to solve problems by relating the half-life of a radioactive source to its decay constant.

**Biological effects of radiation**

- Effects of ionising radiation on living tissues and cells:
  - α- and β-particles are able to ionise atoms by removing electrons from the atom, while γ-rays are able to ionise atoms by means of the photoelectric effect
  - Direct effects include the interaction of radiation with atoms in the DNA molecule or any other critical component of the cell resulting in damage to these components and subsequently cell death or mutation
  - Cell mutation could lead to the rapid and uncontrolled production of defective cells resulting in cancer
  - Indirect effects include the interaction of radiation with water molecules in the cell which may lead to its radiolytic decomposition and the subsequent formation of toxic substances, contributing to the cell's destruction
  - Radiation can also be used to treat cancer since rapidly growing cancer cells are especially susceptible to destruction by radiation