

LESSON

17

RoboTrig

Forward Kinematics

From Angles to Position

Name: _____

Team: _____

Date: _____

HOOK

FK stands for Forward Kinematics. Forward means you already know both joint angles. The question is: **where does the tip end up?**

Every robot controller runs FK continuously — thousands of times per second. Before you can program a robot to reach a target, you need to predict the tip position from any pair of angles.

Your prediction before any formula:

Set $\theta_1 = 45^\circ$ and $\theta_2 = 30^\circ$ on your robot. Where do you think the tip will land? Sketch a quick guess below, then label your estimated (x, y).

ROBOT FIRST — Predict, Then Measure

STEP 1

Set $\theta_1 = 45^\circ$ and $\theta_2 = 30^\circ$. Measure where the tip lands. Record below.

Measurement	Your value
θ_1 (base joint)	45°
θ_2 (elbow joint)	30°
Measured x	
Measured y	

STEP 2

Choose two more angle pairs. Predict (x, y) before moving the robot, then measure. Record all four columns.

θ_1	θ_2	Predicted (x, y)	Measured (x, y)
30°	60°		
90°	45°		
Your choice			

STEP 3

How close were your predictions to the measurements? What made it hard to predict without a formula?

NAME IT — The FK Formula

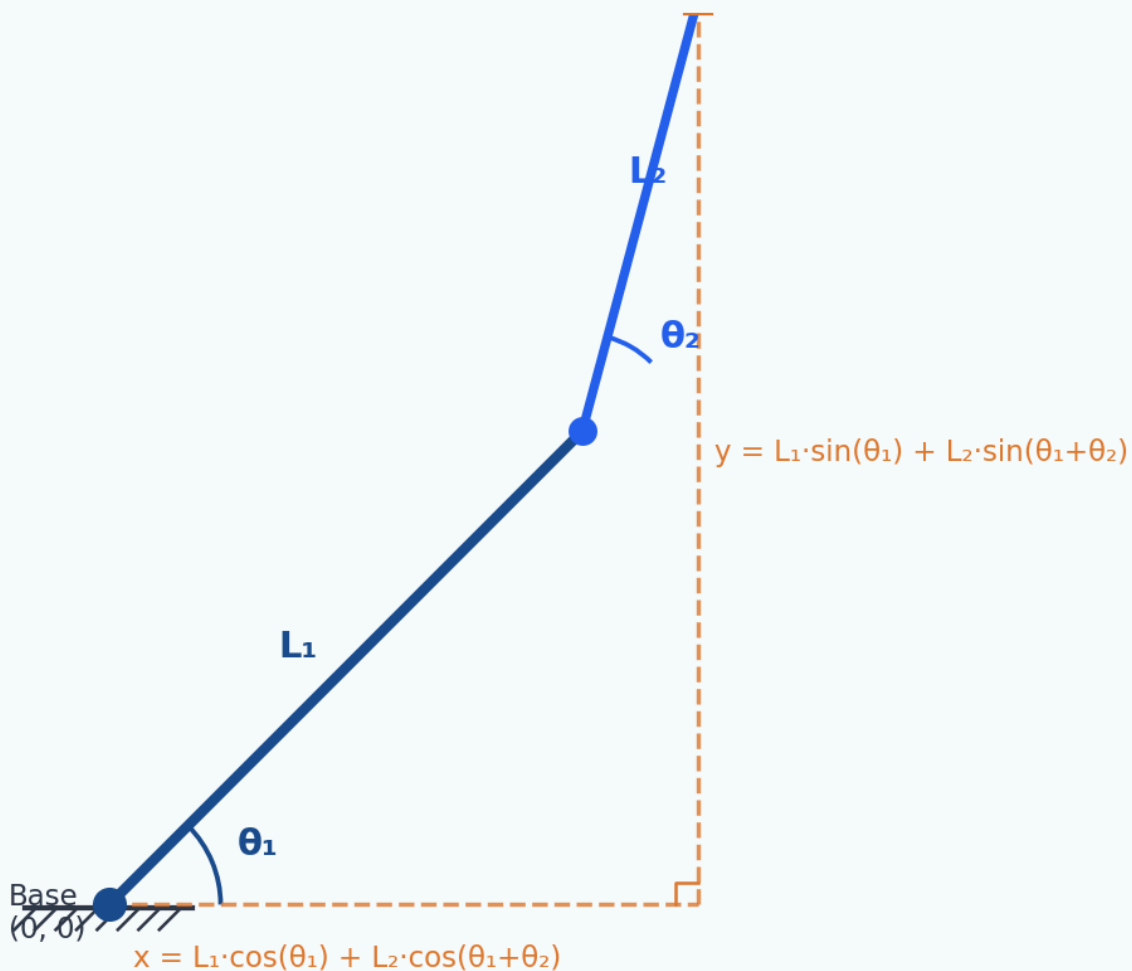
$$x = L_1 \cdot \cos(\theta_1) + L_2 \cdot \cos(\theta_1 + \theta_2)$$

$$y = L_1 \cdot \sin(\theta_1) + L_2 \cdot \sin(\theta_1 + \theta_2)$$

L_1 = length of first arm segment · L_2 = length of second arm segment

θ_1 = base joint angle · θ_2 = angle of second segment *RELATIVE* to first

Note: $(\theta_1 + \theta_2)$ is the *ABSOLUTE* angle of segment 2 from horizontal. Add both joint angles.

**Worked Example**

$L_1 = 12$ cm, $L_2 = 8$ cm, $\theta_1 = 45^\circ$, $\theta_2 = 30^\circ$

$$x = 12 \cdot \cos(45^\circ) + 8 \cdot \cos(75^\circ)$$

$$= 12(0.707) + 8(0.259) = 8.485 + 2.071 \approx 10.56 \text{ cm}$$

$$y = 12 \cdot \sin(45^\circ) + 8 \cdot \sin(75^\circ)$$

$$= 12(0.707) + 8(0.966) = 8.485 + 7.727 \approx 16.21 \text{ cm}$$

Tip position: approximately (10.56, 16.21)

Verify with your Robot FIRST data:

Use $\theta_1 = 45^\circ$, $\theta_2 = 30^\circ$ and your own L_1 and L_2 . Calculate (x, y) . How close is your result to what you measured on the robot?

PRACTICE — Show Your Work

For each problem: write the formula, substitute all values, then calculate. Round to one decimal place.

P1. $L_1 = 12 \text{ cm}$, $L_2 = 8 \text{ cm}$

$\theta_1 = 45^\circ$, $\theta_2 = 30^\circ$

Find x and y .

P2. $L_1 = 15 \text{ cm}$, $L_2 = 10 \text{ cm}$

$\theta_1 = 0^\circ$, $\theta_2 = 90^\circ$

Find x and y . Does the answer make physical sense?

P3. Use your actual robot arm.

Set $\theta_1 = 60^\circ$, $\theta_2 = 45^\circ$.

Predict (x, y) using FK.

Then move the robot and measure.

How close was the formula?

P4. $L_1 = 10 \text{ cm}$, $L_2 = 10 \text{ cm}$.

What joint angles put the tip at $y = 0$?

(Hint: think about what $\theta_1 + \theta_2$ must equal for $\sin$ to be 0.)

CHALLENGE

$L_1 = 12 \text{ cm}$, $L_2 = 8 \text{ cm}$.

Find a pair of angles (θ_1, θ_2) that puts the tip at $(16, 0)$. Confirm your answer by plugging back into the FK formula.

Then find a SECOND pair of angles that also puts the tip at the same point. Sketch both configurations.

(This is the first hint that IK can have more than one solution.)

REFLECT

Forward kinematics goes from ___ to ___. I use it when ___.

In P2, $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$. What position did you get? Explain in one sentence why that result makes physical sense.

How close was your FK prediction to your robot measurement in P3? Name one thing that could cause a difference.
