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| <b>Unit 6.1</b>             | <b>Area, Surface Area, and Volume</b>  |
| <b>Target Standards</b>     | <p><b>6.G.A.1</b> Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</p> <p><b>6.G.A.2</b> Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas <math>V = lwh</math> and <math>V = bh</math> to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p> <p><b>6.G.A.3</b> Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p> <p><b>6.G.A.4</b> Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</p> |
| <b>Supporting Standards</b> | <p><b>6.EE.A.2.A</b> Write expressions that record operations with numbers and with letters standing for numbers.</p> <p><b>6.EE.A.2.C</b> Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). <i>For example, use the formulas <math>V = s^3</math> and <math>A = 6s^2</math> to find the volume and surface area of a cube with sides of length <math>s = \frac{1}{2}</math>.</i></p> <p><b>6.EE.A.4</b> Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). <i>For example, the expressions <math>y + y + y</math> and <math>3y</math> are equivalent because they name the same number regardless of which number <math>y</math> stands for.</i></p> <p><b>6.EE.B.6</b> Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p>  |

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| <p><b><i>The story before this unit (including prior knowledge)</i></b></p> | <p>Students will already be familiar with area of rectangles. They will have seen the partition of shapes into parts of equal areas. They will have graphed points in the first quadrant of the coordinate plane and applied this technique to solve real-world and mathematical problems.</p> <p>They will already understand the concept of volume and be familiar with volume of rectangular prisms with whole number side lengths. They will have used unit cubes to pack - without gaps or overlaps - a rectangular prism to find its volume. They will have found that the volume would be the same if they multiplied the side edges, or multiplying the area of the base by the height. They will have related volume to multiplication and addition operations. They will have applied the formulas <math>V = l \times w \times h</math> and <math>V = B \times h</math> for rectangular prisms with whole number dimensions to solve real-world and mathematical problems. They will have learned how to multiply fractions and mixed numbers. They will also have seen that volumes of solids, similar to the composition and decomposition of areas of shapes, can also be composed of smaller non-overlapping solids, thus recognizing volume as additive.</p>   |
| <p><b><i>The part of the story happening in this unit</i></b></p>           | <p>Students will connect their knowledge of area of rectangles to area of triangles, special quadrilaterals and composite shapes by decomposing them into pieces whose area they can determine. They will have an understanding of the area formulas for rectangles and triangles. They will extend this understanding to finding the surface area of any composite figure that has triangular and/or rectangular surfaces. They will extend their work with the coordinate plane by graphing points in all four quadrants; they do this by drawing vertices of a polygon and connecting them to find the area of that polygon. They will draw nets of three-dimensional figures to represent the surface area and calculating it.</p> <p>They will continue to use the strategy of packing unit cubes to find volume of a rectangular prism, but the prism now has fractional side lengths. They may also use drawings or interactive software to simulate the packing-with-unit-cubes process. They are reminded that volume is additive - the adding of layers of cubes with each layer being the same as the bottom layer. This will strengthen their understanding of the volume formula <math>V = B \times h</math>. They will work with all these strategies in the context of solving real-world and mathematical problems.</p> |
| <p><b><i>The story after this unit</i></b></p>                              | <p>In grade 7, students will extend their understanding of area to find the areas of circles and the surface area of other solids. Specific to circles, they will informally derive the area formula of a circle from the work of cutting the circle into wedges and rearranging them into a parallelogram. They will understand the relationship between the circumference and area of a circle. They will construct geometric figures - mainly various triangles - with given constraints. The importance of being able to visualize geometric shapes is evident in students being able to draw cross-sections (two-dimensional slices) of three-dimensional figures such as rectangular prisms and rectangular pyramids. Students find volume of other solids, except cylinders, by applying their understanding of volume of rectangular prisms. They will also use floor plans and scale drawings to find the actual area of shapes.</p>   |

## UNIT FLOW SUMMARY

| <b>UNIT 6.1 (19-27 days)</b> | <b>Area, Surface Area, and Volume</b>                      |
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| <b>Section 0 (1 day)</b>     | <b>Diagnostic Pre-Unit Assessment</b>                      |
| <b>Section 1 (1-2 days)</b>  | <b>Real-World Application (Launch)</b>                     |
| <b>Section 2 (1 day)</b>     | <b>Comparing Areas</b>                                     |
| <b>Section 3 (2-3 days)</b>  | <b>Finding Area of Right Triangles and Other Triangles</b> |
| <b>Section 4 (2-3 days)</b>  | <b>Finding Area of Polygons and Special Quadrilaterals</b> |
| <b>Section 5 (2-3 days)</b>  | <b>Real-World Application</b>                              |
| <b>Section 6 (1 day)</b>     | <b>Mid-Unit Assessment - Areas of Polygons</b>             |
| <b>Section 7 (2-3 days)</b>  | <b>Drawing Polygons in the Coordinate Plane</b>            |
| <b>Section 8 (3-4 days)</b>  | <b>Finding Volume of Rectangular Prisms</b>                |
| <b>Section 9 (2-3 days)</b>  | <b>Drawing Nets to Find Surface Area</b>                   |
| <b>Section 10 (1-2 days)</b> | <b>Real-World Application (Culminating Activity)</b>       |
| <b>Section 11 (1 day)</b>    | <b>End of Unit Assessment</b>                              |

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| <b>Section 0: 1 day</b>  | <b>Diagnostic Pre-Unit Assessment</b>  |
| <b>Mathematical Goal</b> | Determine students' skill in finding perimeter and area of rectangles and shapes composed of rectangles. Check their skill in multiplying fractions and mixed numbers. Determine their skill in finding the volume of rectangular prisms with whole number side lengths. This will be important in establishing their understanding of composing and decomposing that will lead them toward the understanding of why the formulas for area and volume are what they are. |

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| <b>Key Diagnostic Targets</b><br><br><i>What do we want to know if students can do before starting the unit?</i> | Assess students' ability to <ul style="list-style-type: none"> <li><input type="checkbox"/> find area of a rectangle when it's drawn on grid paper and when it's not on grid paper but with dimensions given <b>3.MD.C.7</b></li> <li><input type="checkbox"/> sketch a rectangle with the same area as one shown but with a different perimeter <b>3.MD.D.8</b></li> <li><input type="checkbox"/> multiply fractions and mixed numbers <b>5.NF.6</b></li> <li><input type="checkbox"/> find the volume of a rectangular prism with whole number side lengths <b>5.MD.5.a, 5.MD.5.b</b></li> </ul> |
| <b>Sample Target Items</b>   | <a href="#"><u>6.1 Diagnostic Pre-Unit Assessment</u></a>  |

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| <b>Section 1</b>                                  | <b>Real-World Application (Launch)</b>  |
| <b>Mathematical Goal</b>                          | To begin the unit, students will explore a real-world topic related to surface area and/or volume. This activity should neither assume nor require significant prior knowledge of the mathematics involved, but should provide students of varying abilities an opportunity to contextualize mathematical skills and concepts that they will explore more formally later in the unit. The launch activity will help motivate the rest of the unit, and will reinforce the truism that mathematics is an important tool for analyzing and understanding the world around us. |
| <b>Any of these tasks might be a nice launch!</b> | <b>6.G.1 Task 1:</b> Covering the Patio: Determine the area of a parallelogram and shapes within it.<br><b>6.G.1 Task 2:</b> Building a Collage: Determine the minimum and maximum dimensions and the areas of triangles.<br><b>6.G.2 Task 1:</b> Packing Shoe Crates: Find the volume of shoe boxes and the crates in which the boxes are packed.<br><b>6.G.2 Task 2:</b> More than a Cubic Foot?: Determine the volume of rectangular prisms.   |

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| <b>Section 2: 1 day</b>  | <b>Comparing Areas</b>   |
| <b>Mathematical Goal</b>   | Compare areas of shapes by composing smaller shapes into the larger shape and decomposing the larger shape into smaller ones.  |
| <b>Narrative overview of section</b><br>(and how the standards are achieved) | Students begin the unit working with familiar shapes of pattern blocks from elementary grades, relating the given area of one shape to another <b>3.G.A.2, 6.G.A.1</b> . The ability to see that it takes 3 green triangles to compose 1 red trapezoid or that 1 yellow hexagon decomposes into 3 blue rhombuses helps them understand that area is additive, which later helps them find and justify the area formulas <b>MP3</b> . (They may also use <a href="#">pattern block grid paper</a> or <a href="#">virtual pattern blocks</a> <b>MP5</b> ). In addition to working with area, students operate on fractions, mainly adding of |

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|  | fractions <b>5.NF.A.1</b> . Students need to see fractions more frequently in their math work that arise naturally from context. They identify and write equivalent expressions <b>6.EE.A.4</b> .   |
| <b>Sample Activity 2.1</b><br><br><b>6.G.A.1</b><br><br><b>MP1, MP2, MP3</b> | <p><a href="#"><i>Bubble Wrap</i></a>, Dan Meyer</p> <p><b>WHAT:</b> This activity serves as a hook lesson with a purposefully open-ended prompt/video with multiple entry points. Students can explore area in a “meaningful” context and begin to make sense of area by comparing various sizes of squares of bubble wrap. Though the area of a square itself is a much earlier concept <b>3.G.2</b>, putting the question asking in the hands of the students affords them the opportunity to explore area and extend the problem as far as they can. This activity also introduces students to the idea that the area of the big shape is found by a collection of smaller shapes (bubbles in this case) <b>6.G.A.1</b></p> <p><b>WHY:</b> Starting with a hook lesson like this allows students a real-life context with which to access the material and begin to feel engaged in the topic. It also provides an opportunity for students to reason abstractly and quantitatively <b>MP2</b> before the side lengths are given and to make sense of the problem and persevere in solving it <b>MP1</b>. Students will finally need to construct a viable argument <b>MP3</b> for why the solution they have come up with works.</p> |

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| <b>Section 3: 2-3 days</b>   | <b>Finding Area of Right Triangles and Other Triangles</b>   |
| <b>Mathematical Goal</b>   | Find the formula for the area of triangles and understand why the formula works.   |
| <b>Narrative overview of section</b><br>(and how the standards are achieved) | <p>Students understand the definition of a polygon and know that a triangle has the fewest number of sides possible for a polygon. They know a triangle’s three corners are called <i>vertices</i>, and any of the triangle’s three sides can be considered the <i>base</i> of the triangle. They learn that the <i>height</i> of a triangle is the perpendicular <b>4.G.A.2</b> distance from the vertex opposite the base to the base. They recall that area is the number of squares needed to cover a plane. Students will find areas of right triangles and other triangles drawn on grid paper by directly counting unit squares or by the “surround and subtract” method. They will find the relationship between the area of a triangle and the area of the smallest rectangle surrounding it <b>MP7</b>. By investigating the bases and heights of different triangles, students will be able to find and understand the formula for the area of a triangle as <math>A = \frac{1}{2}b \times h</math>, where <math>b</math> is the base and <math>h</math> is the height <b>6.G.A.1</b>. The “<math>\frac{1}{2}</math>” in the area formula shows that the area of a triangle is half of the smallest rectangle surrounding it.</p> |
| <b>Sample Activity 3.1</b>   | <a href="#"><i>Base and Height</i></a> , Illustrative Mathematics  |

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| <p><b>6.G.A.1</b></p> <p><b>MP6, MP7</b></p>   | <p><b>WHAT:</b> Students correctly identify the base and height for each triangle <b>6.G.A.1</b>. The lesson addresses a common mistake students make that a base can only be the side that the triangle rests on, or a height is simply another side length. They learn that every triangle has three base-height pairs. They also learn precise vocabulary related to triangles <b>MP6</b>.</p> <p><b>WHY:</b> The purpose of this activity is for students to have a firm grasp of the base-height relationship. By having them identify and draw in base-height pairs, students make use of structure that will guide them toward understanding the area formula for triangles <b>6.G.A.1 MP7</b>.</p>  |
| <p><b>Sample Activity 3.2</b></p> <p><b>6.G.A.1</b></p> <p><b>MP7</b></p>                | <p><a href="#"><i>Same Base and Height, Variation 1</i></a>, Illustrative Mathematics</p> <p><b>WHAT:</b> Students find the areas of triangles drawn on grid paper that have the same base and height. They do this by counting unit squares or by the “surround and subtract” method. They learn to draw in the height for each rectangle, perhaps be introduced to the term <i>altitude</i> and how this altitude may lie inside or outside of a triangle, or it can be one of the sides.</p> <p><b>WHY:</b> This task helps students focus on the base and height components in the area formula, which are important skills in finding the areas of right and other triangles <b>6.G.A.1</b>. By comparing and contrasting the different triangles, they are looking for and making use of structure <b>MP7</b> to recognize the significance of base and height in computing area.</p> |
| <p><b>Sample Activity 3.3</b></p> <p><b>6.G.A.1</b></p> <p><b>MP2, MP4, MP5, MP6</b></p> | <p><a href="#"><i>Triangle Explorer</i></a>, Shodor</p> <p><b>WHAT:</b> Students are given random triangles on a coordinate grid and asked to determine the area of the triangles. This is an on-line application that provides three levels of difficulty and the option of getting hints.</p> <p><b>WHY:</b> This activity provides practice and feedback on determining areas of triangles placed on a grid in a variety of orientations. The levels and hints make the activity appropriate for a range of students.</p>  |
| <p><b>Sample Activity 3.4</b></p> <p><b>6.G.A.1</b></p> <p><b>MP2, MP4, MP5, MP6</b></p> | <p><a href="#"><i>Sierpinski's Carpet</i></a>, Illustrative Mathematics</p> <p><b>WHAT:</b> The purpose of this task is to help give students an opportunity to work with area in a more challenging context and attempt to write expressions based on increasingly complicated area situations. <b>6.G.A.1</b> The task assumes that students have seen and evaluated numeric expressions that involve whole-number exponents. Even so, students might not think to use exponential notation to represent the areas.</p>   |

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|  | <p><b>WHY:</b> This activity provides students the opportunity to write a number as a numeric expression rather than evaluating the expression, which is an important facet of <b>MP7</b>, Look for and make use of structure. This task naturally engages students in <b>MP8</b>, Look for and express regularity in repeated reasoning.</p> |
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| <b>Section 4: 2-3 days</b>   | <b>Finding Area of Polygons and Special Quadrilaterals</b>  |
| <b>Mathematical Goal</b>   | Understand how a polygon may be composed of rectangles or decomposed into triangles. Apply the area formula of rectangles to the area of parallelograms. Apply the area formula of triangles to the area of trapezoids.   |
| <b>Narrative overview of section</b><br>(and how the standards are achieved)                   | <p>Students will move from finding areas of right triangles and other triangles to finding areas of other composite figures <b>6.G.A.1</b>. They decompose polygons into smaller rectangles and/or triangles because they are now familiar with calculating areas of both these figures. They learn that a rectangle is a special parallelogram that has the same descriptors of base, height, and side length as those of a triangle. (They will have learned perpendicular and parallel lines from grade 4 <b>4.G.A.2</b>.) They find the area formula of a parallelogram and see how it relates to the area of a triangle and the area of a rectangle. They may explore areas of trapezoids, rhombuses, and kites through decomposition <b>MP7</b>. Students may continue to do their area investigation on grid or dot paper, and they may use NCTM's Illuminations <a href="#">Isometric Drawing Tool</a> for dynamic explorations that will allow them to shift, rotate, color, decompose, and view in 2-dimension or 3-dimension <b>MP5</b>. This additional exploration through certain rigid transformations, in addition to finding the area of figures, allows students to see figures in different orientations, such as seeing a right triangle or a rectangle resting only on one of its vertices. They use these strategies to solve real-world and mathematical problems.</p> |
| <b>Sample Activity 4.1</b><br><br><b>6.G.A.1</b><br><br><b>6.EE.A.4</b><br><br><b>MP1, MP2</b> | <p><i><a href="#">Finding Areas of Polygons</a></i>, Illustrative Mathematics</p> <p><b>WHAT:</b> Students find the area of a polygon using various strategies, from counting the number of square units inside a polygon to decomposing and composing each shape into triangles and rectangles. They may also use the “surround-and-subtract” strategy to find area.</p> <p><b>WHY:</b> Students work on a sequence of area problems that shows the advantage of increasingly abstract strategies in preparation for developing general area formulas for parallelograms <b>6.G.A.1</b>. Asking them to solve it two ways <b>MP1</b> encourages students to both expand their strategies for finding area (by reasoning abstractly and quantitatively <b>MP2</b>) and to check their work.</p>   |

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| <b>Sample Activity 4.2</b><br><b>6.G.A.1</b><br><br><b>MP1, MP3,MP4</b> | <a href="#"><i>Wallpaper Decomposition</i></a> , Illustrative Mathematics<br><br><b>WHAT:</b> The purpose of this task is for students to use both composition and decomposition to determine the areas of given shapes in a real world context <b>6.G.A.1</b> <b>MP4</b> (wallpapering a room). Students must decompose an irregular polygon two different ways and determine the areas of the shapes in order to answer the questions.<br><br><b>WHY:</b> Placed here, this task serves as an opportunity to apply and put to practice some of the strategies that students worked at in the previous section. It also gives students the opportunity to construct a viable argument about the areas of shapes and reason within a context that may have more than one correct answer <b>MP1, MP3</b> .   |
| <b>Sample Activity 4.3</b><br><b>6.G.A.1</b><br><br><b>MP7, MP8</b>     | <a href="#"><i>Areas of Special Quadrilaterals</i></a> , Illustrative Mathematics<br><br><b>WHAT:</b> The purpose of this task is for students to use both composition and decomposition to determine the area formulas of special quadrilaterals <b>6.G.A.1</b> . Students decompose polygons in two different ways and determine the areas of the shapes in order to understand the geometric structure of these special quadrilaterals and relate their area to rectangles of the same area.<br><br><b>WHY:</b> After having practiced finding areas of polygons in a variety of contexts using composition and decomposition to make use of the geometric structure <b>MP7</b> to relate special quadrilaterals to rectangles of the same area. Student can then apply repeated reasoning <b>MP8</b> to develop the general formulas for the area of parallelograms and trapezoids. |

| Section 5  | Real-World Application   |
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| <b>Mathematical Goal</b>                           | Now that students have developed a formal understanding of some of the topics related to area, surface area and/or volume, they will apply what they've learned to explore another real-world topic. This activity should allow students to synthesize the procedural skills and conceptual understandings that they've developed thus far, and reiterate the important role that mathematics can play in thinking critically about the world. |
| <b>CCSS:</b><br><b>6.G.A.2</b><br><br><b>CCMP:</b> | <a href="#"><i>Tricks of the Tray'd</i></a> , Mathalicious<br><br><b>Description:</b><br><br>It's not always easy to get students to make healthy choices in the lunchroom, but one possible solution might be   |



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| <b>MP3, MP7</b> | right under their noses: the cafeteria tray. By giving more visual real estate – but not more actual space – to the most enticing items, students might feel like they’re getting more of the foods they love, without going overboard on calories. In this lesson, students calculate volumes of rectangular prisms and use that information to design an appealing and well-balanced tray. |
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| <b>Section 6: 1 day</b>   | <b>Mid-Unit Assessment - Areas of Polygons</b>   |
| <b>Unit Assessment Targets</b><br><i>(What are important mathematical targets for students to demonstrate what they have accomplished in this unit)</i> | Assess students’ ability to <ul style="list-style-type: none"> <li><input type="checkbox"/> find area of right triangles and other triangles</li> <li><input type="checkbox"/> find area of polygons and special quadrilaterals</li> <li><input type="checkbox"/> solve real-world problems</li> </ul> |
| <b>Sample Assessment Items</b>  | <a href="#">Mid-Unit Assessment</a>  |

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| <b>Section 7: 2-3 days</b>   | <b>Drawing Polygons in the Coordinate Plane</b>   |
| <b>Mathematical Goals</b>  | Draw polygons in the coordinate plane given coordinates for the vertices.   |
| <b>Narrative overview of section</b><br>(and how the standards are achieved)     | Students extend their work from grade 5 of graphing (x, y) points in the coordinate plane in all four quadrants <b>6.G.A.3</b> . They recognize that when two points have the same first coordinate <b>MP7</b> , such as (3, -1) and (3, 5), then connecting these 2 points gives a vertical segment of length between -1 and 5, or 6. If two points have the same second coordinate, such as (-3, 6) and (7, 6), then the segment joined is horizontal and has a length between -3 and 7, or 10. Given 3 vertices of a rectangle to graph, students locate the 4th vertex and find the area (and perimeter) of the rectangle. Students apply these strategies in real-world context to solve problems. |
| <b>Sample Activity 7.1</b><br><br><b>6.G.A.1, 6.G.A.3</b><br><br><b>MP1, MP6</b> | <a href="#">Polygons in the Coordinate Plane</a> , Illustrative Mathematics<br><br><b>WHAT:</b> Students plot points in the coordinate plane and find the area of each polygon formed by joining its set of vertices.<br><br><b>WHY:</b> The purpose of this task is for students to practice plotting points in the coordinate plane <b>6.G.A.3</b> and finding the areas of polygons. This task assumes that students already understand how to find areas of polygons by decomposing them into rectangles and triangles <b>6.G.A.1</b> .   |

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| <p><b>Sample Activity 7.2</b></p> <p><b>6.G.A.3</b></p> <p><b>MP2, MP4, MP7</b></p> | <p><a href="#"><i>Walking the Block</i></a>, Illustrative Mathematics</p> <p><b>WHAT:</b> The purpose of this task is for students to apply the drawing of polygons and finding distances on a coordinate plane to a real life context <b>6.G.A.3</b>. The task affords students the opportunity to model with mathematics MP4 and reason abstractly and quantitatively <b>MP2</b> as they map out routes and then describe in words their location. draw out routes on a map thereby using techniques of drawing polygons on a coordinate plane in a real life context. Students could also make use of structure <b>MP7</b> to find shortcuts to part (b) and then reason about part (c).</p> <p><b>WHY:</b> Placed here, this activity gives students the opportunity to think about the coordinate plane in a real life modeling context <b>MP4</b>. Though there are no explicit coordinates, the reasoning involved is the same as in the previous tasks and this task provides a coherent progression of thought.</p> |
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| <p><b>Section 8: 3-4 days</b></p>  | <p><b>Finding Volume of Rectangular Prisms</b></p>  |
| <p><b>Mathematical Goals</b></p>   | <p>Find the volume of a right rectangular prism with fractional edge lengths.</p>   |
| <p><b>Narrative overview of section</b><br/>(and how the standards are achieved)</p>       | <p>Students strengthen their understanding of length, area, and volume from earlier grades. They reason that length is one-dimensional, area is two-dimensional, and volume is three-dimensional. In grade 5, students understand that a unit cube is called such because all its edges are 1 unit in length. They learn that a prism is named by its bases - two sides that are parallel and congruent. They notice that the base is always at right angles with the height of the prism (just as they are in a triangle or a parallelogram) <b>MP7</b>. They see a variety of prisms, such as rectangular, pentagonal, hexagonal, and triangular, although they only focus on finding the volume of a rectangular prism. They think about how many cubes a right prism can hold by finding how many cubes it can hold in the bottom layer (base) and multiplying this by how high the prism is. Thus, volume of a right prism can be calculated by multiplying the area of its base by its height, so they are discovering the formula as the shortcut to actual packing the solid with unit cubes <b>MP8</b>. In grade 6, they find the volume of a rectangular prism with fractional edge lengths <b>6.G.A.2</b>. They write and evaluate expressions to find volume and apply this to solve real-world or mathematical problems <b>6.EE.A.2.A</b>, <b>6.EE.A.2.C</b>, <b>6.EE.B.6</b>.</p> |
| <p><b>Sample Activity 8.1</b></p> <p><b>6.G.A.2</b></p> <p><b>6.EE.A.2.C, 6.EE.B.6</b></p> | <p><a href="#"><i>Computing Volume Progression 1</i></a>, <a href="#"><i>Progression 2</i></a>, <a href="#"><i>Progression 3</i></a>, and <a href="#"><i>Progression 4</i></a>, Illustrative Mathematics</p> <p><b>WHAT:</b> This series of tasks gradually increases in complexity, from the concrete problems involving volume to a more abstract understanding of volume. <a href="#"><i>Progression 1</i></a> serves as a review of grade 5's work of finding the volume of a rectangular prism with whole-number edge lengths. <a href="#"><i>Progression 2</i></a> gives a rectangular prism with whole-number edge lengths but asks for the volume of the water if the prism were filled <math>\frac{3}{4}</math> of the way. <a href="#"><i>Progression 3</i></a> asks students</p>   |

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| <p><b>MP1, MP2, MP7</b></p>   | <p>to work backwards to find the height of the rectangular prism if its other two dimensions and the volume are known. <a href="#">Progression 4</a> asks students figure out the volume of a submerged irregular object (piece of rock).</p> <p><b>WHY:</b> The purpose of this series of tasks is to build in a natural way from accessible, concrete problems involving volume to a more abstract understanding of volume. The purpose of <a href="#">Progression 1</a> is to see the relationship between the side-lengths of a cube and its volume. In <a href="#">Progression 2</a>, the task is made more abstract by the removal of the the cube grid line. In <a href="#">Progression 3</a>, students must know that 1 liter is equal to 1,000 cubic centimeters so students get to think about numbers in a different ways <b>MP2</b>. <a href="#">Progression 4</a> makes a science connection based on Archimedes' Principle that the volume of an immersed object is equivalent to the volume of the displaced water. While the stone itself is an irregular solid, relating it to the displaced water in a rectangular tank means that the actual volume calculation is that of a rectangular prism, and therefore, fits in with <b>6.G.A.2</b>.</p>  |
| <p><b>Sample Activity 8.2</b></p> <p><b>6.G.A.3</b></p> <p><b>MP1, MP3, MP6</b></p> | <p><a href="#">Banana Bread</a>, Illustrative Mathematics</p> <p><b>WHAT:</b> This is a real-world task where students have to find volumes of right rectangular prisms in the context of solving a real-world problem <b>6.G.A.3</b>. Given the dimensions of a pan that is too small to bake banana bread in, and the constraint that there must be at least an inch between the top of the batter and the rim of a pan, students must decide as to whether a new pan, whose dimensions are given, is large enough to make the banana bread.</p> <p><b>WHY:</b> The purpose of this task is to provide students with a multi-step problem involving volume to give them a chance to discuss the difference between exact calculations and their meaning in a context. In order to solve this problem, students must analyze givens, constraints, relationships, and goals, in context <b>MP1</b>. They also must provide viable arguments <b>MP3</b> about whether the new pan is appropriate based on their knowledge of mathematics in the context. Students could argue, for example, that whether the new pan is appropriate depends in part on how accurate Leo's estimate for the needed height is. While solving this problem, students must calculate volume accurately and express volume with the degree of precision that is appropriate for this particular context <b>MP6</b>.</p> |

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| <p><b>Section 9: 2-3 days</b></p>  | <p><b>Drawing Nets to Find Surface Area</b></p>   |
| <p><b>Mathematical Goals</b></p>   | <p>Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.</p>  |
| <p><b>Narrative overview of section</b><br/>(and how the standards are</p> | <p>Students practice visualizing three-dimensional shapes based on two-dimensional representations by analyzing nets and considering how their parts fit together, as well as by drawing and building their own nets. They construct prisms from nets, and deconstruct prisms into their corresponding nets. They use the nets to find the surface area</p> |

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| achieved)   | <b>6.G.A.4.</b> They understand that the properties of 2-dimensional figures are used in solving problems involving 3-dimensional objects <b>MP1</b> . Students apply these strategies in the context of solving real-world and mathematical problems.   |
| <b>Sample Activity 9.1</b><br><br><b>6.G.A.4</b><br><br><b>MP1, MP3, MP4</b>  | <u><a href="#">Designing: Candy Cartons</a></u> , Mathematics Assessment Resource Service (MARS)<br><br><b>WHAT:</b> Students are asked to design a candy box by creating a net from a letter-sized piece of cardboard <b>6.G.A.4 MP4</b> . Students compare and contrast different designs <b>MP1</b> to help decide the advantages of each (fewest cuts, minimum cardboard waste, compact, stacks easily, greatest support, etc.).<br><br><b>WHY:</b> This activity provides a need for conceptualizing and building nets from three-dimensional figures. There is a natural tendency towards finding surface area and volume as it relates to cardboard waste and candy arrangements. Students must select appropriate mathematical methods, consider constraints and variables, specify assumptions, and communicate their reasoning clearly <b>MP3</b> .  |
| <b>Sample Activity 9.2</b><br><br><b>6.G.A.4</b><br><br><b>6.EE.A.2.C, 6.EE.A.4</b><br><br><b>MP1, MP2, MP3, MP4, MP5, MP6</b>                      | <u><a href="#">File Cabinet</a></u> , Andrew Stadel<br><br><b>WHAT:</b> A real-world task that motivates students to use some measure of area (e.g. post-its) to solve a problem involving surface area of a rectangular prism.<br><br><b>WHY:</b> The purpose of this task is to provide a is a real-world problem that challenges students to fully understand surface area as an additive process <b>MP4 6.G.A.4</b> . Most surface area problems involving rectangular prisms can be solved by using the formula, $SA = 2lh + 2lw + 2wh$ . In this task, the formula will not work as one of the sides is not covered with post-its <b>MP1</b> . Students can choose to find the area of each side individually, or alter the formula to account for the missing side. Finally, there are multiple solutions to this task which help students see that area, like all measurements, is referential. While students would likely gravitate towards the use of a common measure of area, the square inch, they could just as easily use a square post-it as their reference <b>MP2</b> . |
| <b>Sample Activity 9.3</b><br><br><b>6.G.A.1, 6.G.A.2</b><br><br><b>6.EE.A.2.A, 6.EE.A.2.C, 6.EE.B.6</b><br><br><b>MP1, MP2, MP3, MP4, MP5, MP6</b> | <u><a href="#">Hotel Snap</a></u> , Fawn Nguyen (via NCTM's Illuminations)<br><br><b>WHAT:</b> Students build a profit yielding hotel using snap cubes. Building costs, rules and regulations, taxes, and income are all variables that students will be required to take into consideration <b>6.EE.A.2.A, 6.EE.A.2.C, 6.EE.B.6</b> .<br><br><b>WHY:</b> The purpose of this task is for students to model with mathematics by relating area to “windows” and volume to “rooms” of a hotel <b>MP4</b> . The task serves as a culminating wrap-up for the unit, and because it’s hands-on, kids are highly engaged and motivated to persevere <b>MP1</b> . While the task of finding area and volume is through direct counting up of the surfaces on the cubes and the cubes themselves <b>6.G.A.1, 6.G.A.2</b> , students need to take these   |

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|  | totals and figure out the associated costs by multiplying each total by the unit cost. There is a penalty involved in the scoring of this task, so attending to precision proves beneficial <b>MP6</b> . This task challenges students to think spatially and critically by engineering their way to the best design that would expose the most surfaces (windows) while keeping the base and height of the hotel minimal <b>MP2</b> . |
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| Section 10  | Real-World Application (Culminating Activity)  |
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| <b>Mathematical Goal</b>  | To conclude this unit, students will apply the skills and concepts they've developed to another real-world topic. This activity should assume an in-depth understanding of area, surface area, and volume, and challenge students to apply their knowledge in new and creative ways. Concluding the unit with a real-world context will help solidify students' conceptual understanding of and fluency with area, surface area, volume, and further reinforce the idea that mathematics is an important tool for exploring and eliciting a deeper understanding of the world. |
| <b>CCSS:</b><br><b>6.RP.3, 6.G.2</b><br><br><b>CCMP:</b><br><b>MP3, MP7</b> | <u><a href="#">Ice Cubed</a></u> , Mathalicious<br><br><b>Description:</b><br>In the past few years, ice cubes have become a big thing. Literally. In glasses all over America, people are chilling their drinks with extra-large ice. But what is it about these heavy-duty crystals that have made them all the rage? In this lesson, students use surface area, volume, and rates to explore the relationship between the size of ice cubes and how good they are at doing their job: chilling.   |

| Section 11: 1 day   | End of Unit Assessment   |
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| <b>Unit Assessment Targets</b><br><i>(What are important mathematical targets for students to demonstrate what they have accomplished in this unit)</i> | Assess students' ability to <ul style="list-style-type: none"> <li><input type="checkbox"/> find area of right triangles and other triangles</li> <li><input type="checkbox"/> find area of polygons and special quadrilaterals</li> <li><input type="checkbox"/> draw polygons in the coordinate plane</li> <li><input type="checkbox"/> find surface area and volume of rectangular prisms with fractional edge lengths</li> <li><input type="checkbox"/> solve real-world problems</li> </ul> |

**Sample Assessment Items**
[Summative Assessment](#)

| Sample Lessons | 6.G.A.1 | 6.G.A.2 | 6.G.A.3 | 6.G.A.4 | 6.EE.A.2.A | 6.EE.A.2.C | 6.EE.A.4 | 6.EE.B.6 | MP 1 | MP 2 | MP 3 | MP 4 | MP 5 | MP 6 | MP 7 | MP 8 |
|----------------|---------|---------|---------|---------|------------|------------|----------|----------|------|------|------|------|------|------|------|------|
| 1.1            |         |         |         |         |            |            |          |          |      |      |      |      |      |      |      |      |
| 2.1            | X       |         |         |         |            |            |          |          | X    | X    | X    |      |      |      |      |      |
| 3.1            | X       |         |         |         |            |            |          |          |      |      |      |      |      | X    | X    |      |
| 3.2            | X       |         |         |         |            |            |          |          |      |      |      |      |      |      | X    |      |
| 3.3            | X       |         |         |         |            |            |          |          |      | X    |      | X    | X    | X    |      |      |
| 3.4            | X       |         |         |         |            |            |          |          |      | X    |      | X    | X    | X    |      |      |
| 4.1            | X       |         |         |         |            |            | X        |          | X    | X    |      |      |      |      |      |      |
| 4.2            | X       |         |         |         |            |            |          |          | X    |      | X    | X    |      |      |      |      |
| 4.3            | X       |         |         |         |            |            |          |          |      |      |      |      |      |      | X    | X    |
| 5.1            |         | X       |         |         |            |            |          |          |      |      | X    |      |      |      | X    |      |
| 6-Mid          |         |         |         |         |            |            |          |          |      |      |      |      |      |      |      |      |
| 7.1            | X       |         | X       |         |            |            |          |          | X    |      |      |      |      | X    |      |      |
| 7.2            |         |         | X       |         |            |            |          |          |      | X    |      | X    |      |      | X    |      |
| 8.1            |         | X       |         |         |            | X          |          | X        | X    | X    |      |      |      |      | X    |      |
| 8.2            |         |         | X       |         |            |            |          |          | X    |      | X    |      |      | X    |      |      |
| 9.1            |         |         |         | X       |            |            |          |          | X    |      | X    | X    |      |      |      |      |
| 9.2            |         |         |         | X       |            | X          | X        |          | X    | X    | X    | X    | X    | X    |      |      |

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| 9.3  | X | X |  |  | X | X |  | X | X | X | X | X | X | X |   |  |
| 10.1 |   | X |  |  |   |   |  |   |   | X |   |   |   |   | X |  |