Roll No…………………….. **Total No. of Printed Pages: 1**

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M. Sc. (Mathematics) (Semester-1st)
REAL ANALYSIS
Subject Code: MMAT1-102
Paper ID: [19220502]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It carries 16 marks. It consists of 4 questions of 4 marks each.
2. Section B consist of 4 questions of 8 marks each. The student has to attempt any 3 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(4 marks each)**

Q1. Show that set of rational numbers is countable.

Q2. Define contraction mapping in a metric space. Explain with example. Prove that contraction mapping is continuous.

Q3. Let (X, d_X) and (Y, d_Y) be two metric spaces and $f : X \to Y$ be a mapping. Then *f* is continuous if and only if inverse image of every closed subset of *Y* is closed subset of *X*.

Q4. Monotonic function on [0,1] is Riemann Stieltjes Integrable. Prove it

Section – B **(8 marks each)**

Q5. Construct connected sets in the real line.

Q6. If $\{A_n\}$ is sequence of nowhere dense sets in a complete metric space (X, d), then $X \neq \bigcup_{n=1}^{\infty} A_n$

Q7. Show that monotonic functions have no discontinuity of the second kind.

Q8. If f is continuous and α is monotonic on [a,b] then prove

$$\int_a^b f d\alpha = \left[f(x)\alpha(x) \right]_a^b - \int_a^b \alpha df$$

Section – C **(10 marks each)**

Q9. (a). Prove that unbounded metric space can be converted into bounded metric space.
(b)Prove that K-cell is compact. (4+6)

Q10. (a). State and prove Cantor's Intersection Theorem.
(b). Prove that continuous image of connected set is connected (6+4)

Q11. State and prove necessary and sufficient condition for Riemann-Stieltjes Integrability.