IAL A level
Statistics 2
Continuous Random
QP

1.Jan 2025-2

2.Jan 2025-4

4. (i) The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} \dfrac{x^3}{20} & 1 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Using algebraic integration find $E(X^2)$
You must show your working.

(3)

Given that $E(X) = 2.42$

(b) find the value of $\text{Var}(X)$ correct to 3 significant figures.

(2)

(ii) A random sample of size 10 is taken from a continuous random variable Y

(a) Find the probability that at least 7 of these values are **smaller** than the upper quartile of Y

(3)

(b) Find the probability that less than or equal to 5 of these values are **larger** than the upper quartile of Y

(2)

3.Jan 2025-7

7. A continuous random variable X has probability density function defined as

$$f(x) = \begin{cases} k(x-3)^2 & 2 \leqslant x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $y = f(x)$

(2)

(b) Hence write down the mode of X

(1)

(c) Using algebraic integration and showing your working clearly

(i) show that $k = \dfrac{3}{28}$

(4)

(ii) verify that the upper quartile of X lies between 5.71 and 5.72

(3)

4.Jan 2024-4

4. The continuous random variable G has probability density function $\mathrm{f}(g)$ given by

$$\mathrm{f}(g)=\begin{cases}\dfrac{1}{15}(g+3) & -1<g\leqslant 2\\[2mm] \dfrac{3}{20} & 2<g\leqslant 4\\[2mm] 0 & \text{otherwise}\end{cases}$$

(a) Sketch the graph of $\mathrm{f}(g)$

(2)

(b) Find $\mathrm{P}\left((1\leqslant 2G\leqslant 6)\,\middle|\,G\leqslant 2\right)$

(4)

The continuous random variable H is such that $\mathrm{E}(H)=12$ and $\mathrm{Var}(H)=2.4$

(c) Find $\mathrm{E}\left(2H^2+3G+3\right)$
Show your working clearly.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

5. Jan 2024-7

7. A continuous random variable X has cumulative distribution function $\mathrm{F}(x)$ given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ k(ax + bx^3 - x^4 - 4) & 1 \leqslant x \leqslant 2 \\ 1 & x > 2 \end{cases}$$

where a, b and k are non-zero constants.

Given that the mode of X is 1.5

(a) show that $b = 3$

(3)

(b) Hence show that $a = 2$

(1)

(c) Show that the median of X lies between 1.4 and 1.5

(4)

6.June 2024-2

2 The continuous random variable H has cumulative distribution function given by

$$\mathrm{F}(h) = \begin{cases} 0 & h \leqslant 0 \\ \dfrac{h^2}{48} & 0 < h \leqslant 4 \\ \dfrac{h}{6} - \dfrac{1}{3} & 4 < h \leqslant 5 \\ \dfrac{3}{10}h - \dfrac{h^2}{75} - \dfrac{2}{3} & 5 < h \leqslant d \\ 1 & h > d \end{cases}$$

where d is a constant.

(a) Show that $2d^2 - 45d + 250 = 0$

(2)

(b) Find $\mathrm{P}(H < 1.5 \mid 1 < H < 4.5)$

(4)

(c) Find the probability density function $\mathrm{f}(h)$
You may leave the limits of h in terms of d where necessary.

(3)

7.June 2024-6

6 **In this question solutions relying entirely on calculator technology are not acceptable.**

The continuous random variable X has the following probability density function

$$f(x) = \begin{cases} a + bx & -1 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where a and b are constants.

(a) Show that $4a + 4b = 1$

(3)

Given that $E(X^2) = \frac{17}{5}$

(b) (i) find an equation in terms of a only

(5)

(ii) hence show that $b = 0.1$

(2)

(c) Sketch the probability density function f(x) of X

(2)

(d) Find the value of k for which $P(X \geqslant k) = 0.8$

(4)

8.Oct 2024-5

5. The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}(3-x) & 1 \leqslant x \leqslant 2 \\ \frac{1}{4} & 2 < x \leqslant 3 \\ \frac{1}{4}(x-2) & 3 < x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

The cumulative distribution function of X is $F(x)$

(a) Show that $F(x) = \frac{1}{4}\left(3x - \frac{x^2}{2}\right) - \frac{5}{8}$ for $1 \leqslant x \leqslant 2$

(2)

(b) Find $F(x)$ for all values of x

(5)

(c) Find $P(1.2 < X < 3.1)$

(2)

9.Oct 2024-7

7. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} ax & 0 \leqslant x \leqslant 4 \\ bx + c & 4 < x \leqslant 8 \\ 0 & \text{otherwise} \end{cases}$$

where a, b and c are constants.

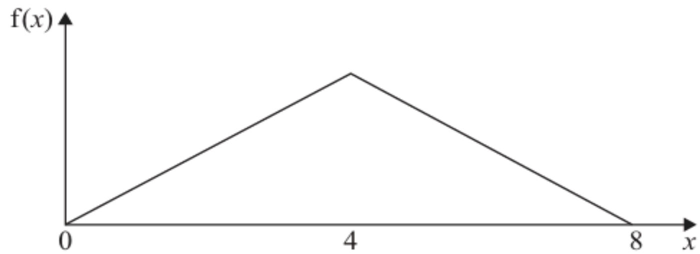


Figure 1

Figure 1 shows the graph of the probability density function $f(x)$

The graph consists of two straight line segments of equal length joined at the point where $x = 4$

(a) Show that $a = \frac{1}{16}$

(1)

(b) Hence find

(i) the value of b

(ii) the value of c

(3)

(c) Using algebraic integration, show that $\text{Var}(X) = \frac{8}{3}$

(6)

(d) Find, to 2 decimal places, the lower quartile and the upper quartile of X

(3)

A statistician claims that

$$P(-\sigma < X - \mu < \sigma) > 0.5$$

where μ and σ are the mean and standard deviation of X

(e) Show that the statistician's claim is correct.

(2)

10.Jan 2023-2

2. A bag contains a large number of coins. It only contains 20p and 50p coins. A random sample of 3 coins is taken from the bag.

(a) List all the possible combinations of 3 coins that might be taken.

(2)

Let $\bar{X}$ represent the mean value of the 3 coins taken.

Part of the sampling distribution of $\bar{X}$ is given below.

$\bar{x}$	20	a	b	50
$P(\bar{X} = \bar{x})$	$\frac{4913}{8000}$	c	d	$\frac{27}{8000}$

(b) Write down the value of a and the value of b

(1)

The probability of taking a 20p coin at random from the bag is p

The probability of taking a 50p coin at random from the bag is q

(c) Find the value of p and the value of q

(2)

(d) Hence, find the value of c and the value of d

(3)

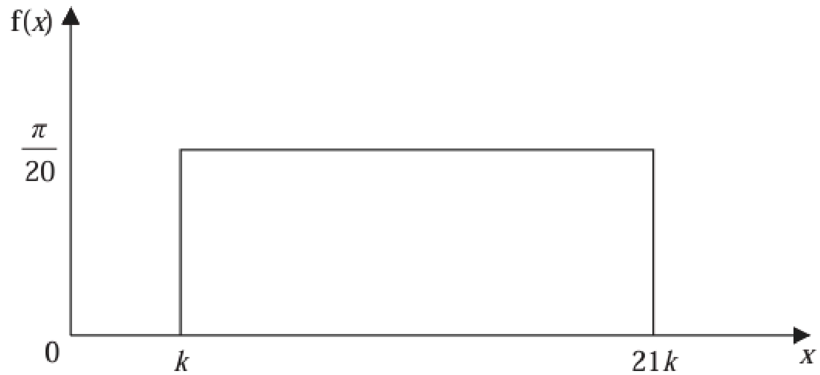
Let M represent the mode of the 3 coins taken at random from the bag.

(e) Find the sampling distribution of M

(3)

11.Jan 2023-4

4. The continuous random variable X has probability density function $\mathrm{f}(x)$, shown in the diagram, where k is a constant.



(a) Find $\mathrm{P}(X < 10k)$

(1)

(b) Show that $k = \dfrac{1}{\pi}$

(2)

(c) Find, in terms of π, the values of

(i) $\mathrm{E}(X)$

(ii) $\mathrm{Var}(X)$

(3)

Circles are drawn with area A, where

$$A = \pi\left(X + \frac{2}{\pi}\right)^2$$

(d) Find $\mathrm{E}(A)$

(4)

12.Jan 2023-6

6. The continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ ax + bx^2 & 0 \leqslant x \leqslant k \\ 1 & x > k \end{cases}$$

where a, b and k are positive constants.

(a) Show that $ak = 1 - bk^2$

(1)

Using part (a) and given that $\mathrm{E}(X) = \frac{6}{5}$

(b) show that $5bk^3 = 36 - 15k$

(6)

Using part (a) and given that $\mathrm{E}(X) = \frac{6}{5}$ and $\mathrm{Var}(X) = \frac{22}{75}$

(c) show that $5bk^4 = 52 - 10k^2$

(5)

Given that $k < 3$

(d) find the value of k

(4)

(e) Hence find the value of a and the value of b

(2)

13.June 2023-3

3. The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{48}\left(x^2 - 8x + c\right) & 2 \leqslant x \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $c = 31$

(3)

(b) Find $P(2 < X < 3)$

(2)

(c) State whether the lower quartile of X is less than 3, equal to 3 or greater than 3
Give a reason for your answer.

(1)

Kei does the following to work out the mode of X

$$f'(x) = \frac{1}{48}(2x - 8)$$

$$0 = \frac{1}{48}(2x - 8)$$

$$x = 4$$

Hence the mode of X is 4

Kei's answer for the mode is incorrect.

(d) Explain why Kei's method does not give the correct value for the mode.

(1)

(e) Find the mode of X
Give a reason for your answer.

(2)

14.June 2023-5

5. A continuous random variable Y has cumulative distribution function given by

$$F(y) = \begin{cases} 0 & y < 3 \\ \frac{1}{16}\left(y^2 - 6y + a\right) & 3 \leqslant y \leqslant 5 \\ \frac{1}{12}\left(y + b\right) & 5 < y \leqslant 9 \\ \frac{1}{12}\left(100y - 5y^2 + c\right) & 9 < y \leqslant 10 \\ 1 & y > 10 \end{cases}$$

where a, b and c are constants.

(a) Find the value of a and the value of c

(4)

(b) Find the value of b

(2)

(c) Find $P(6 < Y \leqslant 9)$
Show your working clearly.

(3)

(d) Specify the probability density function, $f(y)$, for $5 < y \leqslant 9$

(1)

Using the information

$$\int_3^5 (6y - 5)\,f(y)\,dy + \int_9^{10} (6y - 5)\,f(y)\,dy = 26.5$$

(e) find $E(6Y - 5)$
You should make your method clear.

(4)

15.Oct 2023-2

2. The continuous random variable X has probability density function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x) = \begin{cases} ax^3 & 0 \leqslant x \leqslant 4 \\ bx + c & 4 < x \leqslant d \\ 0 & \text{otherwise} \end{cases}$$

where a, b, c and d are constants such that

- $bx + c = ax^3$ at $x = 4$
- $bx + c$ is a straight line segment with end coordinates $(4, 64a)$ and $(d, 0)$

(a) State the mode of X

(1)

Given that the mode of X is equal to the median of X

(b) use algebraic integration to show that $a = \dfrac{1}{128}$

(2)

(c) Find the value of d

(2)

(d) Hence find the value of b and the value of c

(3)

16.Oct 2023-6

6. The continuous random variable Y has cumulative distribution function given by

$$\mathrm{F}(y) = \begin{cases} 0 & y < 0 \\ \dfrac{1}{21}y^2 & 0 \leqslant y \leqslant k \\ \dfrac{2}{15}\left(6y - \dfrac{y^2}{2}\right) - \dfrac{7}{5} & k < y \leqslant 6 \\ 1 & y > 6 \end{cases}$$

(a) Find $\mathrm{P}\left(Y < \frac{1}{4}k \mid Y < k\right)$

(2)

(b) Find the value of k

(4)

(c) Use algebraic calculus to find $\mathrm{E}(Y)$

(6)

17.Jan 2022-2

2 The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < -k \\ \dfrac{x+k}{4k} & -k \leqslant x \leqslant 3k \\ 1 & x > 3k \end{cases}$$

where k is a positive constant.

(a) Specify fully, in terms of k, the probability density function of X

(2)

(b) Write down, in terms of k, the value of $E(X)$

(1)

(c) Show that $\text{Var}(X) = \dfrac{4}{3}k^2$

(2)

(d) Find, in terms of k, the value of $E(3X^2)$

(3)

18.Jan 2022-4

4 The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}k(x-1) & 1 \leqslant x \leqslant 3 \\ k & 3 < x \leqslant 6 \\ \frac{1}{4}k(10-x) & 6 < x \leqslant 10 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a) Sketch $f(x)$ for all values of x

(2)

(b) Show that $k = \frac{1}{6}$

(2)

(c) Specify fully the cumulative distribution function $F(x)$ of X

(7)

Given that $E(X) = \frac{61}{12}$

(d) find $P(X > E(X))$

(2)

(e) Describe the skewness of the distribution, giving a reason for your answer.

(2)

19.June 2022-2

2. The time, in minutes, spent waiting for a call to a call centre to be answered is modelled by the random variable T with probability density function

$$f(t) = \begin{cases} \frac{1}{192}\left(t^3 - 48t + 128\right) & 0 \leqslant t \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find, in minutes and seconds, the mean waiting time.

(3)

(b) Show that $P(1 < T < 3) = \frac{7}{16}$

(3)

A supervisor randomly selects 256 calls to the call centre.

(c) Use a suitable approximation to find the probability that more than 125 of these calls take between 1 and 3 minutes to be answered.

(5)

20.June 2022-6

6. The continuous random variable X has probability density function

$$f(x) = \begin{cases} 0.1x & 0 \leqslant x < 2 \\ kx(8-x) & 2 \leqslant x < 4 \\ a & 4 \leqslant x < 6 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are constants.

It is known that $\mathrm{P}(X < 4) = \dfrac{31}{45}$

(a) Find the exact value of k

(4)

(b) (i) Find the exact value of a

(ii) Find the exact value of $\mathrm{P}(0 \leqslant X \leqslant 5.5)$

(3)

(c) Specify fully the cumulative distribution function of X

(6)

21.Oct 2022-2

2. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4} & -\frac{1}{2} \leqslant x < \frac{1}{2} \\ 2x - \frac{3}{4} & \frac{1}{2} \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a) Sketch the graph of $f(x)$

(2)

(b) By forming and solving an equation in k, show that $k = 1.25$

(4)

(c) Use calculus to find $E(X)$

(4)

(d) Calculate the interquartile range of X

(5)

22.Oct 2022-5

5. The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 3 \\ \frac{1}{6}(x-3)^2 & 3 \leqslant x < 4 \\ \frac{x}{3} - \frac{7}{6} & 4 \leqslant x < c \\ 1 - \frac{1}{6}(d-x)^2 & c \leqslant x < 7 \\ 1 & x \geqslant 7 \end{cases}$$

where c and d are constants.

(a) Show that $c = 6$

(4)

(b) Find $P(X > 3.5)$

(2)

(c) Find $P(X > 4.5 \mid 3.5 < X < 5.5)$

(3)

23.Jan 2021-2

2. The distance, in metres, a novice tightrope artist, walking on a wire, walks before falling is modelled by the random variable W with cumulative distribution function

$$\mathrm{F}(w) = \begin{cases} 0 & w < 0 \\ \dfrac{1}{3}\left(w - \dfrac{w^4}{256}\right) & 0 \leqslant w \leqslant 4 \\ 1 & w > 4 \end{cases}$$

(a) Find the probability that a novice tightrope artist, walking on the wire, walks at least 3.5 metres before falling.

(2)

A random sample of 30 novice tightrope artists is taken.

(b) Find the probability that more than 1 of these novice tightrope artists, walking on the wire, walks at least 3.5 metres before falling.

(3)

Given $\mathrm{E}(W) = 1.6$

(c) use algebraic integration to find $\mathrm{Var}(W)$

(5)

24.Jan 2021-4

4. A continuous random variable X has probability density function

$$f(x) = \begin{cases} k(a-x)^2 & 0 \leqslant x \leqslant a \\ 0 & \text{otherwise} \end{cases}$$

where k and a are constants.

(a) Show that $ka^3 = 3$

(3)

Given that $E(X) = 1.5$

(b) use algebraic integration to show that $a = 6$

(4)

(c) Verify that the median of X is 1.2 to one decimal place.

(3)

25.Jan 2021-5

5. A piece of wood AB is 3 metres long. The wood is cut at random at a point C and the random variable W represents the length of the piece of wood AC.

(a) Find the probability that the length of the piece of wood AC is more than 1.8 metres.
(2)

The two pieces of wood AC and CB form the two shortest sides of a right-angled triangle. The random variable X represents the length of the longest side of the right-angled triangle.

(b) Show that $X^2 = 2W^2 - 6W + 9$
(2)

[You may assume for random variables S, T and U and for constants a and b that if $S = aT + bU$ then $\mathrm{E}(S) = a\mathrm{E}(T) + b\mathrm{E}(U)$]

(c) Find $\mathrm{E}(X^2)$
(6)

(d) Find $\mathrm{P}(X^2 > 5)$
(4)

26.June 2021-3

3. The continuous random variable Y has the following probability density function

$$f(y) = \begin{cases} \frac{6}{25}(y-1) & 1 \leqslant y < 2 \\ \frac{3}{50}(4y^2 - y^3) & 2 \leqslant y < 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f(y)$ **(2)**

(b) Find the mode of Y **(3)**

(c) Use algebraic integration to calculate $E(Y^2)$ **(4)**

Given that $E(Y) = 2.696$

(d) find $\text{Var}(Y)$ **(2)**

(e) Find the value of y for which $P(Y \geqslant y) = 0.9$
Give your answer to 3 significant figures. **(4)**

27.June 2021-5

5. A game uses two turntables, one red and one yellow. Each turntable has a point marked on the circumference that is lined up with an arrow at the start of the game. Jim spins both turntables and measures the distance, in metres, each point is from the arrow, around the circumference in an anticlockwise direction when the turntables stop spinning.

The continuous random variable Y represents the distance, in metres, the point is from the arrow for the yellow turntable. The cumulative distribution function of Y is given by $\mathrm{F}(y)$ where

$$\mathrm{F}(y) = \begin{cases} 0 & y < 0 \\ 1 - (\alpha + \beta y^2) & 0 \leqslant y \leqslant 5 \\ 1 & y > 5 \end{cases}$$

(a) Explain why (i) $\alpha = 1$

(ii) $\beta = -\dfrac{1}{25}$

(2)

(b) Find the probability density function of Y

(2)

The continuous random variable R represents the distance, in metres, the point is from the arrow for the red turntable. The distribution of R is modelled by a continuous uniform distribution over the interval $[d, 3d]$

Given that $\mathrm{P}\left(R > \dfrac{11}{5}\right) = \mathrm{P}\left(Y > \dfrac{5}{3}\right)$

(c) find the value of d

(3)

In the game each turntable is spun 3 times. The distance between the point and the arrow is determined for each spin. To win a prize, at least 5 of the distances the point is from the arrow when a turntable is spun must be less than $\frac{11}{5}$ m

Jo plays the game once.

(d) Calculate the probability of Jo winning a prize.

(4)

28.Oct 2021-3

3. A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ 4ax^2 & 0 \leqslant x \leqslant 1 \\ a\left(bx^3 - x^4 + 1\right) & 1 < x \leqslant 3 \\ 1 & x > 3 \end{cases}$$

where a and b are positive constants.

(a) Show that $b = 4$ **(1)**

(b) Find the exact value of a **(2)**

(c) Find $P(X > 2.25)$ **(2)**

(d) Showing your working clearly,

(i) sketch the probability density function of X

(ii) calculate the mode of X

(5)

29.Oct 2021-6

6. The continuous random variable Y has probability density function $f(y)$ given by

$$f(y) = \begin{cases} \frac{1}{14}(y+2) & -1 < y \leqslant 1 \\ \frac{3}{14} & 1 < y \leqslant 3 \\ \frac{1}{14}(6-y) & 3 < y \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the probability density function $f(y)$

(2)

Given that $E(Y^2) = \frac{131}{21}$

(b) find $\text{Var}(2Y - 3)$

(4)

The cumulative distribution function of Y is $F(y)$

(c) Show that $F(y) = \frac{1}{14}\left(\frac{y^2}{2} + 2y + \frac{3}{2}\right)$ for $-1 < y \leqslant 1$

(2)

(d) Find $F(y)$ for all values of y

(5)

(e) Find the exact value of the 30th percentile of Y

(2)

(f) Find $P(4Y \leqslant 5 \mid Y \leqslant 3)$

(2)

30.Oct 2020-1

1.

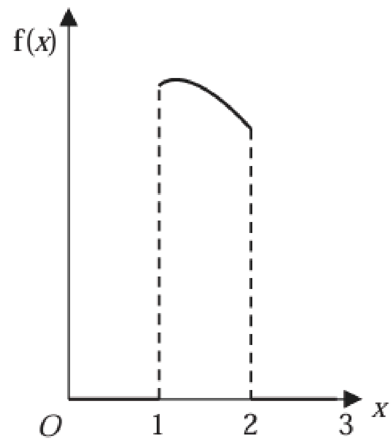


Figure 1

Figure 1 shows a sketch of the probability density function f(x) of the random variable X.

For $1 \leqslant x \leqslant 2$, f($x$) is represented by a curve with equation $f(x) = k\left(\frac{1}{2}x^3 - 3x^2 + ax + 1\right)$ where k and a are constants.

For all other values of x, $f(x) = 0$

(a) Use algebraic integration to show that $k(12a - 33) = 8$

(4)

Given that $a = 5$

(b) calculate the mode of X.

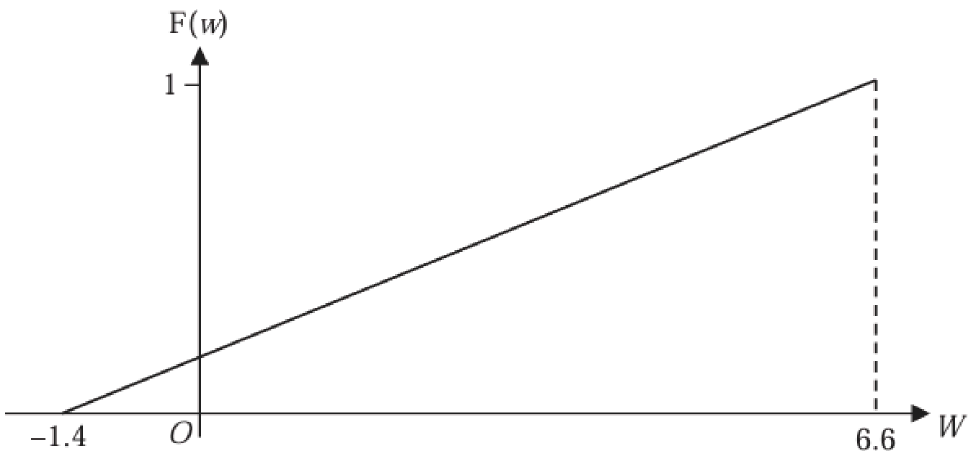
(4)

31-Oct 2020-2

2. In the summer Kylie catches a local steam train to work each day. The published arrival time for the train is 10 am.

The random variable W is the train's actual arrival time minus the published arrival time, in minutes. When the value of W is positive, the train is late.

The cumulative distribution function $F(w)$ is shown in the sketch below.



(a) Specify fully the probability density function $f(w)$ of W.

(2)

(b) Write down the value of $E(W)$

(1)

(c) Calculate α such that $P(\alpha \leqslant W \leqslant 1.6) = 0.35$

(2)

A day is selected at random.

(d) Calculate the probability that on this day the train arrives between 1.2 minutes late and 2.4 minutes late.

(2)

Given that on this day the train was between 1.2 minutes late and 2.4 minutes late,

(e) calculate the probability that it was more than 2 minutes late.

(2)

A random sample of 40 days is taken.

(f) Calculate the probability that for at least 10 of these days the train is between 1.2 minutes late and 2.4 minutes late.

(3)

32.Oct 2020-5

5. The waiting time, T minutes, of a customer to be served in a local post office has probability density function

$$f(t) = \begin{cases} \frac{1}{50}(18 - 2t) & 0 \leqslant t \leqslant 3 \\ \frac{1}{20} & 3 < t \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

Given that the mean number of minutes a customer waits to be served is 1.66

(a) use algebraic integration to find $\text{Var}(T)$, giving your answer to 3 significant figures.
(5)

(b) Find the cumulative distribution function $F(t)$ for all values of t.
(4)

(c) Calculate the probability that a randomly chosen customer's waiting time will be more than 2 minutes.
(2)

(d) Calculate $P([E(T) - 2] < T < [E(T) + 2])$
(2)

33.Jan 2019-3

3. Figure 1 shows an accurate graph of the cumulative distribution function, $F(x)$, for the continuous random variable X

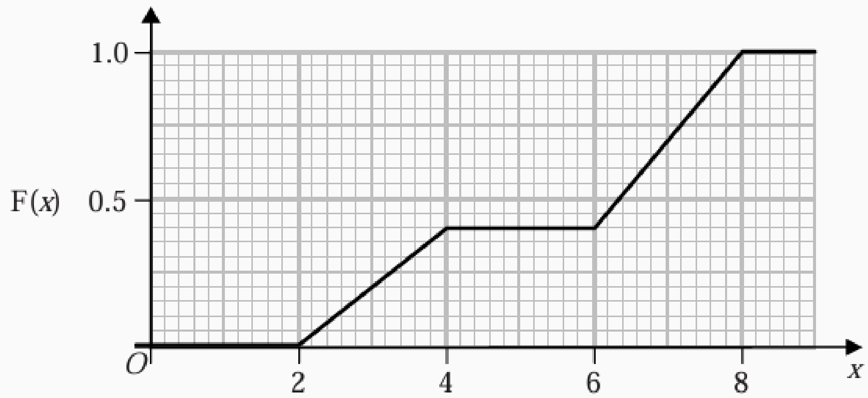


Figure 1

(a) Find $P(3 < X < 7)$

(2)

The probability density function of X is given by

$$f(x) = \begin{cases} a & 2 \leqslant x < 4 \\ b & 4 \leqslant x < 6 \\ c & 6 \leqslant x \leqslant 8 \\ 0 & \text{otherwise} \end{cases}$$

where a, b and c are constants.

(b) Find the value of a, the value of b and the value of c

(3)

(c) Find $E(X)$

(3)

34.Jan 2019-7

7. The continuous random variable X has probability density function

$$f(x) = \begin{cases} c(x+3) & -3 \leqslant x < 0 \\ \frac{5}{36}(3-x) & 0 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where c is a positive constant.

(a) Show that $c = \frac{1}{12}$ **(3)**

(b) (i) Sketch the probability density function.

(ii) Explain why the mode of $X = 0$ **(3)**

(c) Find the cumulative distribution function of X, for all values of x **(4)**

(d) Find, to 3 significant figures, the value of d such that $P(X > d \mid X > 0) = \frac{2}{5}$ **(4)**