Online Supplementary Index: Optimizing Coordination Mechanisms for a Sustainable Ethanol-Blended Fuel Supply Chain in India's Transportation Sector

Table – A1: Literature Review on Supply Chain Coordination Mechanisms: CDM vs. DDM, RS mechanism, and CA mechanism (Appendix A)

Sources	EBP Program	SC Coordination Mechanism	RS	CA	Centralized/ Decentralized	Carbon Emission	Objectives	Applications
Fragnelli & Iandolino, 2004	No	The paper discusses a CA problem for urban waste collection, using game theory to distribute costs among municipalities.	No	CA is a central theme, with methods like the Shapley value and Owen value used to distribute infrastructure and maintenance costs fairly among municipalities.	No	The environmental impact, specifically carbon emissions, is an indirect focus, with the coordination aiming to optimize resource use and reduce waste.	The objective is to achieve a fair distribution of costs related to urban waste collection and disposal while minimizing environmental impact.	Applications include urban waste management and the optimization of cost-sharing mechanisms among municipalities in consortiums.
Saltari & Travaglini, 2011	No	This paper discusses how environmental policies impact the abatement investment	No	CA is discussed in the context of abatement investments	No	The focus is on the reduction of pollution through	The objective is to optimize abatement investment	Applications include investment decision-makin

		decisions of firms under uncertainty.		where the form of investment cost functions influences decisions.		abatement investments influenced by environmental policies.	while considering the irreversibility of capital and ecological uncertainty.	g in firms for pollution control and abatement under uncertainty.
Hoen et al., 2014	No	The paper explores game theory applications for supply chain coordination, specifically in joint replenishment scenarios.	RS is discussed in the context of cooperative strategies where cost savings are shared among participants.	CA is central to the paper, focusing on how costs are distributed among supply chain partners to optimize the total cost.	No	Carbon emissions are indirectly considered through the efficiency improvements in the supply chain, which could reduce overall emissions.	The objective is to develop efficient and cooperative supply chain strategies to minimize costs and improve overall supply chain performance.	Applications include supply chain management, particularly in joint replenishment and cooperative strategies among multiple partners.
Bai et al., 2017	No	This study focuses on coordinating sustainable supply chains using revenue and promotional cost-sharing contracts versus two-part tariff	Detailed analysis of revenue and promotional cost-sharing contracts to align	CA between the manufacturer and retailer is a primary focus, especially in managing the costs associated	No	The paper emphasizes carbon emission reduction, particularly under carbon	The objective is to achieve perfect coordination in the supply chain while reducing carbon	Applications include sustainable supply chain management for products with

		contracts for deteriorating items.	incentives and promote sustainability.	with deteriorating items and sustainable practices.		cap-and-trade regulation, and its impact on supply chain decisions.	emissions and optimizing the handling of deteriorating items.	deteriorating characteristics, such as perishables, under strict carbon regulations.
Song & Gao, 2018	No	The paper explores green supply chain management using RS contracts. It presents a game-theoretic model comparing different RS contracts and their impacts on the supply chain.	The study focuses on retailer-led and bargaining RS contracts, showing how these contracts can improve the greening level of products and overall supply chain profitability.	CA is addressed indirectly through the sharing of revenues, where both manufacturers and retailers share costs associated with greening efforts based on their revenue share.	No	The paper highlights the reduction of environmental impacts through improved greening levels of products, driven by the coordination mechanisms in the supply chain.	The objective is to enhance cooperation between manufacturers and retailers to improve the greening level of products and increase overall supply chain profitability.	Applications include green supply chain management, particularly in industries where product greening is a competitive advantage, such as electronics or automotive.
Fan & Hao, 2020	No	Coordination of energy production between renewable energy sources and traditional	RS is addressed through models that ensure fair	CA uses cooperative game theory, particularly the	No	Significant focus on reducing carbon	The objective is to maximize the efficiency of energy	The application is in the energy sector, specifically in

		power plants using game theory models.	distribution of profits among stakeholders involved in energy production.	Shapley value and Owen value, to ensure fair distribution of costs related to infrastructure and maintenance.		emissions by optimizing the integration of renewable energy into the power grid.	production while minimizing carbon emissions, contributing to sustainable energy goals.	the optimization of energy production and the integration of renewable energy sources into the grid.
Saravanan et al., 2020	Detailed examination of taxation and subsidy policies affecting ethanol blending in India and developing countries like Brazil, China, South Africa.	Conceptually examined the policy mechanisms, including taxation and subsidies, to encourage ethanol production and blending.	No	No	No	Emission reduction targets aligned with national policies and international commitments.	To reduce crude oil imports, promote renewable energy, and meet national and international environmental goals.	Application primarily in the energy and transportation sectors.
Qin et al., 2021	No	This paper discusses supply chain coordination under uncertainty, using robust	RS is explored as a mechanism to align	The paper examines cost-sharing models where	No	Carbon emission reduction is central, with a	The objective is to optimize supply chain decisions under	Applications are relevant to any industry facing stringent

		optimization approaches for managing risks associated with carbon emissions.	incentives between supply chain partners, especially in managing the costs and benefits associated with carbon emission reduction.	the costs of carbon emission reduction are distributed among supply chain partners based on their contributions to emissions.		focus on robust optimization to manage the risks associated with achieving emission targets.	uncertainty, ensuring compliance with carbon emission regulations while maintaining profitability.	carbon emission regulations, such as manufacturing, energy, and logistics.
Hou et al., 2022	No	The paper explores supply chain coordination under uncertain demand and emission reduction targets using game theory models.	RS is discussed in the context of coordinating decisions between the manufacturer and retailer to achieve emission reduction targets.	CA is central, focusing on how costs related to emissions reduction are shared between the supply chain partners.	No	The paper places significant emphasis on achieving emission reduction targets under uncertainty, examining different contract types for effective coordination.	The objective is to optimize the supply chain's overall profit while meeting emission reduction targets and managing uncertainty.	Applications are found in industries where managing supply chain emissions is crucial, such as manufacturing and retail.

Bangjun et al., 2023	No	The Stackelberg model is employed to facilitate coordination between traditional and renewable energy producers within the framework of the renewable energy quota system.	RS models are designed to motivate renewable energy producers to engage in the green certificate trading system by distributing profits from traditional energy producers.	CA mechanisms involve traditional energy producers sharing the expenses of carbon emission reduction efforts with renewable energy producers.	A comparison was made between CDM and DDM coordination approaches for traditional and renewable energy producers.	Carbon emission reduction remains a key focus, with centralized decision-making achieving greater reductions compared to decentralized approaches.	The objective is to optimize overall profit of the supply chain while ensuring compliance with renewable energy quota and reducing carbon emissions.	Applications involve integrating renewable energy into traditional coal power supply chains through mechanisms like green certificate trading and compliance with renewable energy quotas.
Azad et al., 2024	Discussion on the national biofuel policy of India and its impact on ethanol production using a case study approach.	Discussed about the coordination mechanisms involving government policy support and incentives for the production and blending of biofuels.	No	No	No	Reduction in carbon emissions through increased use of biofuels, supported by policy measures.	To achieve energy security, reduce carbon emissions, and support rural development through biofuel production.	Applications include transportation, rural energy supply, and as an alternative to fossil fuels.

Azad et al., 2024	Ethanol blending with petrol, focusing on environmental impacts and CO2 emission reduction.	Policy proposals for coordination between sugar mills, ethanol plants, and cogeneration systems to optimize production and reduce emissions.	No	No	No	Life cycle analysis of GHG emissions for different blending cases, demonstrating significant emission reductions.	To meet the 20% ethanol blending target by 2025-26, reduce GHG emissions, and ensure energy security.	Implemented in the transportation sector to decrease reliance on fossil fuels and address the effects of climate change.
Govindan & Popiuc, 2014	No	The paper discusses reverse supply chain coordination through RS contracts, specifically in the personal computer industry.	RS contracts are used to align incentives and improve the overall profitability of the reverse supply chain.	Costs are allocated based on the functions within the reverse supply chain, such as remanufacturing and recycling.	No	The reduction of electronic waste through effective recycling and remanufacturin g processes indirectly reduces carbon emissions.	The main objective is to improve the efficiency and profitability of the reverse supply chain while ensuring environmental sustainability.	Applications are primarily in the electronics industry, specifically for managing the lifecycle of personal computers.
Cheng & Wang (2023)	No	Coordination explored through trade-in programs in closed-loop	Includes trade-in reimbursement rates as part of	Considers cost-sharing mechanisms like	Explores both manufacturer-l ed (centralized) and retailer-led	Focuses on carbon caps, trading, and subsidies to	To optimize decision-making under carbon policies,	Relevant to sustainable supply chains in electronics

		supply chains under carbon constraints.	the coordination.	green credit policies.	(decentralized) frameworks.	reduce emissions.	enhance resource recovery, and coordinate environmental and economic goals.	and automotive industries.
Zhu et al., 2021	No	Analyzes impact of cash subsidy (CS) and carbon regulation (CR) on NEV supply chains	No	discusses cost allocation between manufacturers and retailers in NEV supply chains under CS and CR	Compares DDM, where the manufacturer and retailer act independently to maximize their own profits, and CDM where the supply chain is optimized as a whole.	Studies the effects of CS and CR on total carbon emissions in NEV industry	Evaluate the effectiveness of CS and CR in increasing NEV adoption and reducing carbon emissions	NEV policy design, sustainable supply chain management, government interventions in clean energy
Panda (2014)	No	Analyzes manufacturer-retailer supply chains using revenue-sharing	Revenue-sharing contracts effectively resolve channel	No	Manufacturer-Stackelberg game explored for	No	To integrate CSR and channel coordination,	Applicable to CSR-driven supply chains in general

		contracts for CSR-focused coordination.	conflicts and achieve coordination.		decentralized decision-making.		balancing profits and consumer surplus.	industries like retail and manufacturing.
Kumar et al. (2021)	No	Dyadic supply chain coordination modeled for CSR motives with cost-learning capabilities over two periods.	Revenue-sharing contracts shown to resolve channel conflicts under CSR contexts.	No	Investigates multi-period coordination within a socially responsible framework.	No	To model socially responsible supply chains while incorporating cost-learning and resolving conflicts.	Applicable to CSR-oriented industries focusing on long-term supply chain sustainability.
Huang et al. (2010)	No	Strategic supply chain optimization explored with multi-stage models for bioethanol production.	No	No	Centralized decision-making for supply chain optimization.	Focuses on reducing greenhouse gas emissions through efficient biofuel systems.	To minimize supply chain costs and optimize bioethanol production to meet projected demand.	Applicable to renewable energy supply chains and sustainable transportation systems.
This Study	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Appendix-B: Decentralized decision making

We verify the KKT conditions for both binding and non-binding cases to ensure correctness. This includes:

- Necessary Conditions (first-order optimality, feasibility, dual feasibility, and complementary slackness).
- Sufficient Conditions (concavity of the objective function to confirm a global maximum).

Problem Formulation

The petrol company maximizes its profit given by:

$$\max_{p_r} \Pi_r = \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)\left(a - bp_r + \gamma\mu - \delta S\right)$$

Constraint:

$$p_r \geq p_m$$

To enforce this constraint, we introduce the Lagrange multiplier λ and define the Lagrangian function:

$$L\left(p_r, \lambda\right) = \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)\left(a - bp_r + \gamma\mu - \delta S\right) + \lambda\left(p_r - p_m\right)$$

where $\lambda \geq 0$ is the Lagrange multiplier associated with the constraint $p_r \geq p_m$.

Necessary Conditions (KKT Conditions)

The necessary conditions for optimality are:

0.1 Stationarity Condition (First-Order Condition)

$$\frac{\partial L}{\partial p_r} = \left(a - bp_r + \gamma\mu - \delta S\right) - \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)b + \lambda = 0$$

0.2 Primal Feasibility

$$p_r \geq p_m$$

0.3 Dual Feasibility

$$\lambda \geq 0$$

0.4 Complementary Slackness

$$\lambda\left(p_r - p_m\right) = 0$$

1. Case 1: Non-Binding Constraint ($\lambda = 0$)

If the constraint does not bind (i.e., $p_r > p_m$), then the Lagrange multiplier is zero ($\lambda = 0$), and the stationarity condition simplifies to:

$$\left(a - bp_r + \gamma\mu - \delta S\right) - \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)b = 0$$

Rearranging:

$$bp_r - \left((1 - \alpha)c_r + \alpha p_m\right)b - Gb - a + \gamma\mu - \delta S = 0$$

Solving for p_r :

$$p_r^* = \frac{a+\left((1-\alpha)c_r+\alpha p_m\right)b+Gb-\gamma\mu+\delta S}{b}$$

Verification of Necessary Conditions for Non-Binding Case

- Primal Feasibility: If $p_r^* \geq p_m$, then the solution is valid, and the constraint is inactive.
- Dual Feasibility: Since $\lambda = 0$, it satisfies $\lambda \geq 0$.
- Complementary Slackness: Since $\lambda = 0$, the condition $\lambda\left(p_r - p_m\right) = 0$ holds.

Conclusion: If $p_r^* \geq p_m$, then the optimal solution is $p_r = p_r^*$, and the constraint does not bind.

2. Case 2: Binding Constraint ($\lambda > 0$)

If the constraint binds (i.e., $p_r = p_m$), then the complementary slackness condition forces:

$$\lambda > 0,\ p_r = p_m$$

Substituting $p_r = p_m$ into the stationarity condition:

$$\left(a - bp_m + \gamma\mu - \delta S\right) - \left(p_m - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)b + \lambda = 0$$

Rearrange:

$$\lambda = - Gb - \left((1 - \alpha)c_r + \alpha p_m\right)b + bp_m + \delta S - \gamma\mu - a$$

For the solution to be valid, we require:

$$- Gb - \left((1 - \alpha)c_r + \alpha p_m\right)b + bp_m + \delta S - \gamma\mu - a \geq 0$$

Verification of Necessary Conditions for Binding Case

- Primal Feasibility: $p_r = p_m$ satisfies $p_r \geq p_m$.

- Dual Feasibility: $\lambda \geq 0$ must hold.
- Complementary Slackness: Since $p_r = p_m$, the condition $\lambda(p_r - p_m) = 0$ holds.

Conclusion: If the inequality holds, the optimal solution is $p_r = p_m$, and the constraint is binding.

3. Sufficient Conditions for Optimality

To confirm that the KKT conditions are sufficient for a global maximum, we check the concavity of the objective function.

The second derivative of Π_r with respect to p_r is:

$$\frac{\partial^2 \Pi_r}{\partial p_r^2} = -2b$$

Since $b > 0$, the second derivative is negative, confirming that Π_r is concave in p_r. Therefore, any solution that satisfies the KKT conditions is a global maximum.

Conclusion: The concavity condition confirms that our derived solutions are optimal.

4. Summary of the Verified KKT Conditions

Condition	Non-Binding Case ($\lambda = 0$)	Binding Case ($\lambda > 0$)
Optimal p_r	$p_r = \frac{a+\left((1-\alpha)c_r+\alpha p_m\right)b+Gb-\gamma\mu+\delta S}{b}$	$p_r = p_m$

Primal Feasibility	Holds if $p_r^* \geq p_m$	Holds since $p_r = p_m$
Dual Feasibility	$\lambda = 0$ satisfies $\lambda \geq 0$	$\lambda \geq 0$ must hold
Complementary Slackness	$\lambda(p_r - p_m) = 0$ holds	$\lambda(p_r - p_m) = 0$ holds
Concavity Verification	$-2b < 0$ ensures global max	$-2b < 0$ ensures global max

Final Conclusion: The KKT conditions hold for both binding and non-binding cases, ensuring that our solutions are globally optimal.

Appendix C: Proof of Concavity of the Profit Function Π_m

To ensure that the optimal solution derived for the ethanol company's pricing decision is globally optimal, we verify the concavity of the profit function Π_m. A function is concave if its Hessian matrix is negative definite, which requires checking the secondorder derivatives.

In this section, we derive the Hessian matrix for Π_m and establish conditions under which it remains negative definite, thereby proving concavity.

2. Profit Function of the Ethanol Supplier

The profit function for the ethanol supplier is given by:

$$\Pi_m = (p_m - c_m + s_b)\cdot(\alpha Q) - \epsilon\mu^2$$

where:

$$Q = a - bp_m + \gamma\mu - \delta S$$

Substituting Q into Π_m :

$$\Pi_m = (p_m - c_m + s_b)\cdot\alpha(a - bp_m + \gamma\mu - \delta S) - \epsilon\mu^2$$

Expanding:

$$\Pi_m = \alpha(p_m - c_m + s_b)(a - bp_m + \gamma\mu - \delta S) - \epsilon\mu^2$$

$$\Pi_m =- \alpha bp_m^2 + \alpha(a + \gamma\mu - \delta S)p_m + \alpha(s_b - c_m)a - \alpha b(s_b - c_m)p_m + \alpha(s_b - c_m)(\gamma\mu - \delta S) - \epsilon\mu^2$$

Rewriting:

$$\Pi_m =- \alpha bp_m^2 + [\alpha(a + \gamma\mu - \delta S) - \alpha b(s_b - c_m)]p_m + \alpha(s_b - c_m)a + \alpha(s_b - c_m)(\gamma\mu - \delta S) - \epsilon\mu^2$$

3. First-Order Partial Derivatives

To construct the Hessian matrix, we compute the first-order partial derivatives.

Partial derivative with respect to p_m :

$$\frac{\partial\Pi_m}{\partial p_m} =- 2\alpha bp_m + \alpha(a + \gamma\mu - \delta S) - \alpha b(s_b - c_m)$$

Partial derivative with respect to :

$$\frac{\partial\Pi_m}{\partial\mu} = \alpha\gamma(p_m - c_m + s_b) - 2\epsilon\mu$$

4. Second-Order Partial Derivatives

Now, we compute the second-order derivatives required for the Hessian matrix.

Second derivative with respect to p_m :

$$\frac{\partial^2 \Pi_m}{\partial p_m^2} =- 2\alpha b$$

Second derivative with respect to :

$$\frac{\partial^2 \Pi_m}{\partial \mu^2} =- 2\epsilon$$

Mixed second derivative (cross-partial derivatives):

$$\frac{\partial^2 \Pi_m}{\partial p_m \partial \mu} = \frac{\partial^2 \Pi_m}{\partial \mu \partial p_m} = \alpha\gamma$$

5. Constructing the Hessian Matrix

The Hessian matrix H is given by:

$$H = \left[\frac{\partial^2 \Pi_m}{\partial p_m^m} \ \frac{\partial^2 \Pi_m}{\partial p_m \partial \mu} \ \frac{\partial^2 m_m}{\partial \mu \partial p_m} \ \frac{\partial^2 \Pi_m}{\partial \mu^2} \right] = [- 2\alpha b \ \alpha\gamma \ \alpha\gamma \ - 2\epsilon]$$

For H to be negative definite, the following conditions must hold:

- The leading principal minor must be negative:

$$H_{11} =- 2\alpha b < 0$$

- The determinant of H must be positive:

$$det(H) = (-2\alpha b)(-2\epsilon) - (\alpha\gamma)^2, 4\alpha b\epsilon - \alpha^2\gamma^2 > 0, \ 4b\epsilon > \alpha\gamma^2$$

Concavity of Π_m

The necessary condition for Π_m to be concave is that the Hessian matrix H is negative definite. We have shown that:

- $H_{11} < 0$ is always true for $b > 0$.
- The determinant condition $4b\epsilon > \alpha\gamma^2$ ensures that H remains negative definite.

Note:

If $4b\epsilon > \alpha\gamma^2$ holds, then the profit function Π_m is concave with respect to p_m and μ. Since Π_m is concave, any local maximum is also a global maximum, ensuring that our derived solutions are optimal.

Decentralized Decision-Making Model

In this section, we derive the optimal pricing and emission reduction strategies in a decentralized decision-making model, where the ethanol supplier (leader) and the petrol company (follower) operate independently.

This formulation follows a Stackelberg competition framework, where:

- The ethanol company (leader) sets the wholesale price p_m and determines the emission reduction level μ.
- The petrol company (follower) sets the retail price p_r in response to p_m.

The goal is to compute the optimal retail price p_r^*, wholesale price p_m^*, and emission reduction level μ^* for the decentralized scenario.

5 Step-by-Step Calculation of Optimal Values

5.1 Step 1: Deriving the Optimal Retail Price p_r^*

The petrol company's profit function is:

$$\Pi_r = \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)Q$$

where the demand function is:

$$Q = a - bp_r + \gamma\mu - \delta S$$

Differentiate Π_r with Respect to p_r

To find the optimal retail price p_r^*, we take the first-order derivative:

$$\frac{\partial \Pi_r}{\partial p_r} = Q + \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)(-b) = 0$$

Solving for p_r^*:

$$p_r = \frac{\left(-2b\left(c_m - c_r - sb\right)\alpha + \left(-2G - 2c_r\right)b + 6\delta S - 6a\right)\varepsilon + \left(\left(c_m - c_r - sb\right)\alpha + G + c_r\right)\gamma^2}{-8b\varepsilon + \gamma^2}$$

This is the optimal retail price under decentralized decision-making.

5.2 Step 2: Deriving the Optimal Wholesale Price p_m^* and Emission Reduction Level μ^*

The ethanol company's profit function is:

$$\Pi_m = \left(p_m - c_m + s_b\right)\alpha Q - \varepsilon\mu^2$$

where the demand function remains:

$$Q = a - bp_r + \gamma\mu - \delta S$$

Differentiate Π_m with Respect to μ

$$\frac{\partial \Pi_m}{\partial \mu} = \alpha\gamma\left(p_m - c_m + s_b\right) - 2\varepsilon\mu = 0$$

Since the profit function of the ethanol company is a concave function, the profits of the ethanol company can be maximized within $\left(p_m^*, \mu^*\right)$. By calculation, the optimal decision-making of the supply chain between petrol and ethanol companies under decentralized decision-making is:

Retail Price of EBP (p_r)

$$p_r = \frac{\left(-2b\left(c_m-c_r-sb\right)\alpha+\left(-2G-2c_r\right)b+6\delta S-6a\right)\varepsilon+\left(\left(c_m-c_r-sb\right)\alpha+G+c_r\right)\gamma^2}{-8b\varepsilon+\gamma^2}$$

Wholesale Price of Ethanol (p_m)

$$p_m = \frac{\left(-4b\left(c_m+c_r-sb\right)\alpha+\left(4G+4c_r\right)b+4\delta S-4a\right)\varepsilon+\gamma^2\alpha\left(c_m-sb\right)}{\left(-8b\varepsilon+\gamma^2\right)\alpha}$$

Emission Reduction Level (μ)

$$\mu = \frac{\gamma\left(\alpha c_m b - \alpha b c_r - \alpha s b b + G b + b c_r + \delta S - a\right)}{-8b\varepsilon + \gamma^2}$$

Profit of the Ethanol Company $\left(\pi_m\right)$

$$\pi_m = -\frac{\left(\left(\left(c_m - c_r - s b\right)\alpha + G + c_r\right)b + \delta S - a\right)^2 \varepsilon}{-8b\varepsilon + \gamma^2}$$

Profit of the Petrol Company $\left(\pi_r\right)$

$$\pi_r = \frac{4\left(\left(\left(c_m - c_r - s b\right)\alpha + G + c_r\right)b + \delta S - a\right)^2 b \varepsilon^2}{\left(-8b\varepsilon + \gamma^2\right)^2}$$

Total Supply Chain Profit $\left(\pi_{SC}\right)$

$$\pi_{SC} = -\frac{\left(\left(\left(c_m - c_r - s b\right)\alpha + G + c_r\right)b + \delta S - a\right)^2 \left(-12b\varepsilon + \gamma^2\right)\varepsilon}{\left(-8b\varepsilon + \gamma^2\right)^2}$$

Appendix C1: Proof - Proposition 1 (Incentive Compatibility and Signaling Induced Inefficiency)

In a decentralized EBP supply chain under asymmetric information about ethanol production $cost c_\theta$ and emission reduction cost coefficient ϵ_θ, a separating equilibrium requires:

$$\pi_m\left(p_{mH}, \mu_H; H\right) \geq \pi_m\left(p_{mL}, \mu_L; H\right),\ \pi_m\left(p_{mL}, \mu_L; L\right) \geq \pi_m\left(p_{mH}, \mu_H; L\right)\#(1)$$

To maintain incentive compatibility:
High-efficiency type ($\theta = H$). Selects emission level above first-best (over-signaling), incurring distortion

$$\Delta\mu_H = \mu_H - \mu_H^* > 0,$$

with cost

$$\epsilon_H\left(\mu_H^2 - \left(\mu_H^*\right)^2\right) > 0.$$

Low-efficiency type ($\theta = L$). Selects emission level below first-best (under-signaling), incurring distortion

$$\Delta\mu_L = \mu_L^* - \mu_L > 0,$$

with cost

$$\epsilon_L\left(\left(\mu_L^*\right)^2 - \mu_L^2\right) > 0$$

These signaling distortions cause economic and environmental inefficiencies, reducing overall supply chain profitability and emission performance compared to the first-best equilibrium under full information.

Proposition: Incentive Compatibility Conditions and Signaling-Induced Inefficiency under Bayesian Stackelberg Signaling Game

Step 1: Defining the Problem

Consider a decentralized EBP supply chain with asymmetric information. The ethanol firm privately knows its type θ, characterized by:

$$c_\theta \in \left\{c_H, c_L\right\},\ c_H < c_L,\ \epsilon_\theta \in \left\{\epsilon_H, \epsilon_L\right\},\ \epsilon_H < \epsilon_L$$

All other variables-wholesale price p_m, emission reduction level μ, and market demand Q-are observable.

Two types exist:

High-efficiency ($\theta = H$) : low $costs\left(c_H, \epsilon_H\right)$.

Low-efficiency ($\theta = L$) : high costs (c_L, ϵ_L).

Step 2: Incentive Compatibility (IC) Conditions

The ethanol firm of type θ chooses ($p_{m\theta}, \mu_\theta$) to maximize

$$\pi_m(p_{m\theta}, \mu_\theta; \theta) = (p_{m\theta} - c_\theta + s_b)\alpha Q - \epsilon_\theta \mu_\theta^2$$

For a separating equilibrium (no imitation), the IC constraints are:

$$\pi_m(p_{mH}, \mu_H; H) \geq \pi_m(p_{mL}, \mu_L; H),\ \pi_m(p_{mL}, \mu_L; L) \geq \pi_m(p_{mH}, \mu_H; L)$$

Step 3: Explicit Derivation of IC Conditions

Use the follower's optimal demand

$$Q^*(p_m, \mu) = \frac{1}{2}[a - \delta S + \gamma\mu - bC(p_m)],\ C(p_m) = (1 - \alpha)c_r + \alpha p_m + G.$$

Define

$$A = c_m - c_r - s_b,\ B_\theta = \gamma^2 - 8b\epsilon_\theta$$

Then the first-best emissions under full information are

$$\mu_\theta^* = \frac{\gamma(\alpha bA + Gb + bc_r + \delta S - a)}{B_\theta}.$$

Under asymmetric information, actual μ_θ deviate from μ_θ^*.
High-efficiency IC ($\theta = H$):

$$(p_{mH} - c_H + s_b)\alpha Q^*(p_{mH}, \mu_H) - \epsilon_H \mu_H^2 \geq (p_{mL} - c_H + s_b)\alpha Q^*(p_{mL}, \mu_L) - \epsilon_H \mu_L^2$$

which rearranges to

$$\epsilon_H\left(\mu_L^2 - \mu_H^2\right) \geq \alpha\left[\left(p_{mL} - c_H + s_b\right)Q^*\left(p_{mL}, \mu_L\right) - \left(p_{mH} - c_H + s_b\right)Q^*\left(p_{mH}, \mu_H\right)\right].$$

Low-efficiency IC ($\theta = L$):

$$\left(p_{mL} - c_L + s_b\right)\alpha Q^*\left(p_{mL}, \mu_L\right) - \epsilon_L\mu_L^2 \geq \left(p_{mH} - c_L + s_b\right)\alpha Q^*\left(p_{mH}, \mu_H\right) - \epsilon_L\mu_H^2,$$

yielding

$$\epsilon_L\left(\mu_H^2 - \mu_L^2\right) \geq \alpha\left[\left(p_{mH} - c_L + s_b\right)Q^*\left(p_{mH}, \mu_H\right) - \left(p_{mL} - c_L + s_b\right)Q^*\left(p_{mL}, \mu_L\right)\right]$$

Step 4: Signaling-Induced Inefficiency

To satisfy IC, firms distort their emissions:

$$\Delta\mu_H = \mu_H - \mu_H^* > 0,\ \Delta\mu_L = \mu_L^* - \mu_L > 0$$

The over-signaling cost (high-efficiency) is

$$\epsilon_H\left(\mu_H^2 - \left(\mu_H^*\right)^2\right) > 0$$

and the under-signaling cost (low-efficiency) is

$$\epsilon_L\left(\left(\mu_L^*\right)^2 - \mu_L^2\right) > 0$$

Step 5: Equilibrium Inefficiency Equations

High-efficiency inefficiency:

$$\Delta\pi_{mH}^{ineff} = \pi_m\left(p_{mH}^*, \mu_H^*; H\right) - \pi_m\left(p_{mH}, \mu_H; H\right).$$

Low-efficiency inefficiency:

$$\Delta\pi_{mL}^{ineff} = \pi_m\left(p_{mL}^*, \mu_L^*; L\right) - \pi_m\left(p_{mL}, \mu_L; L\right)$$

Total supply-chain inefficiency:

$$\Delta\pi_{SC}^{ineff} = \rho\Delta\pi_{mH}^{ineff} + (1 - \rho)\Delta\pi_{mL}^{ineff}$$

Appendix D: Centralized decision making Proofs

$$Q = a - bp_r + \gamma\mu - \delta S$$

Profit of Ethanol Company $\left(\pi_m\right)$ *:* The ethanol company's profit function is given by

$$\pi_m = \left(p_m - c_m + s_b\right) \cdot (\alpha.Q) - \epsilon \cdot \mu^2 \#$$

Profit of Petrol Company (π_r *):* The petrol company's profit function is initially defined as

$$\pi_r = \left(p_r - ((1 - \alpha).c_r + \alpha.p_m) - G\right) \cdot Q \#$$

Considering the constraint $p_r - p_m \geq 0$

Incorporating the constraint, the profit function becomes:

$$\pi_{SC} = \left(p_r - ((1 - \alpha).c_r + \alpha.p_m) - G\right)\left(a - b \cdot p_r + \gamma \cdot \mu - \delta \cdot S\right) + \left(p_m - c_m + s_b\right)(\alpha.Q) - \epsilon \cdot \mu^2 - \lambda \cdot \left(p_m - p_r\right)$$

where λ is the Lagrange multiplier associated with the pricing constraint.

Solving for λ to yield

$$\lambda = \frac{\alpha bcm-\alpha bcr-\alpha bsb+Gb+bcr-\delta s+\gamma\mu+a+(-\alpha bcm+\alpha bcr+\alpha bsb-Gb-bcr+2pmb+\delta s-\gamma\mu-a)}{2b}$$

To solve the CDM optimization, we take partial derivatives of π_{SC} with respect to p_r and μ, and equate them to zero.

Centralized Decision-Making: Closed-Form Solutions

Step 1: Optimal Retail Price p_r^*

Differentiate π_{sc} with respect to p_r and set to zero. The solution is

$$p_r^* = \frac{\left[-2b\left(c_m-c_r-s_b\right)\alpha+\left(-2G-2c_r\right)b+2\delta S-2a\right]\epsilon+\gamma^2\left[\left(c_m-c_r-s_b\right)\alpha+G+c_r\right]}{-4b\epsilon+\gamma^2}. \#$$

Step 2: Optimal Emission Level μ^*

Substitute p_r^* into $\partial\pi_{sc}/\partial\mu = 0$ and simplify:

$$\mu^* = \frac{\left[\left(\left(c_m-c_r-s_b\right)\alpha+G+c_r\right)b+\delta S-a\right]\gamma}{-4b\epsilon+\gamma^2}. \#$$

Step 3: Total Supply Chain Profit π_{sc}^*

Substitute p_r^* and μ^* back into π_{sc} :

$$\pi_{sc}^* = \epsilon\frac{\left[\left(\left(c_m-c_r-s_b\right)\alpha+G+c_r\right)b+\delta S-a\right]^2}{\left(4b\epsilon-\gamma^2\right)^1}. \#$$

<u>Appendix E1: Proof - Theroem 3</u>

Retail Price under Decentralized Decision-Making (DDM)

$$p_D(r^*) = \frac{[-2bA\alpha+(-2G-2c_r)b+6\delta s-6a]\epsilon+(A\alpha+G+c_r)\gamma^2}{-8b\epsilon+\gamma^2}. \#$$

Retail Price under Centralized Decision-Making (CDM)

$$p_I(r^*) = \frac{[-2bA\alpha+(-2G-2c_r)b+2\delta s-2a]\epsilon+(A\alpha+G+c_r)\gamma^2}{-4b\epsilon+\gamma^2}. \#$$

Difference in Retail Prices

Define

$$\Delta p := p_D(r^*) - p_I(r^*).$$

Substituting the above expressions,

$$\Delta p = \frac{[-2bA\alpha+(-2G-2c_r)b+6\delta s-6a]\epsilon+(A\alpha+G+c_r)\gamma^2}{-8b\epsilon+\gamma^2} - \frac{[-2bA\alpha+(-2G-2c_r)b+2\delta s-2a]\epsilon+(A\alpha+G+c_r)\gamma^2}{-4b\epsilon+\gamma^2} \quad (6)$$

Let

$$M := (A\alpha + G + c_r)b + \delta s - a$$

With algebraic simplification, obtains

$$\Delta p = \frac{4\epsilon(-2b\epsilon+\gamma^2)M}{(-8b\epsilon+\gamma^2)(-4b\epsilon+\gamma^2)} \#$$

Sign of Δp

Under the assumption $\gamma^2 < 2b\epsilon$, we have

$$-2b\epsilon+\gamma^2<0,\ M>0,\ \ -8b\epsilon+\gamma^2<0,\ \ -4b\epsilon+\gamma^2<0$$

Hence

$$\Delta p>0$$

Appendix E2: Proof - Theorem 4

1. Emission Reduction Level under DDM

$$\mu_D^*=\frac{\gamma\left(\alpha bA+Gb+bc_r+\delta s-a\right)}{-8b\epsilon+\gamma^2}\#$$

2. Emission Reduction Level under CDM

$$\mu_I^*=\frac{\gamma\left(b\alpha A+bG+bc_r+\delta s-a\right)}{-4b\epsilon+\gamma^2}\#$$

3. Difference in Emission Reduction Levels

Define

$$\Delta\mu:=\mu_D^*-\mu_I^*$$

Substituting the above:

$$\Delta\mu=\gamma\left(\alpha bA+Gb+bc_r+\delta s-a\right)\left(\frac{1}{-8b\epsilon+\gamma^2}-\frac{1}{-4b\epsilon+\gamma^2}\right)\#$$

Let

$$M:=\left(A\alpha+G+c_r\right)b+\delta s-a$$

Then

$$\Delta\mu = \frac{4b\epsilon\gamma M}{\left(-8b\epsilon+\gamma^2\right)\left(-4b\epsilon+\gamma^2\right)} \#$$

Since

$$M = \left(-\left(c_m - c_r - s_b\right)\alpha + G + c_r\right)b + \delta s - a$$

the difference can also be written as

$$\mu_D^* - \mu_I^* = \frac{4b\epsilon\gamma\left[\left(-\left(c_m-c_r-s_b\right)\alpha+G+c_r\right)b+\delta s-a\right]}{\left(-8b\epsilon+\gamma^2\right)\left(-4b\epsilon+\gamma^2\right)}. \#$$

Appendix E3: Proof - Theorem 5

Step 1: Market Demand under DDM

From Theorem 1:

$$Q_D^* = \frac{2\epsilon b\left(\left(\alpha A+G+c_r\right)b+\delta s-a\right)}{-8b\epsilon+\gamma^2} \#$$

Step 2: Market Demand under CDM

From Theorem 2:

$$Q_I^* = \frac{2\epsilon b\left(\left(\alpha A+G+c_r\right)b+\delta s-a\right)}{-4b\epsilon+\gamma^2} \#$$

Step 3: Difference in Market Demand

Let

$$M := \left(\left(\alpha A + G + c_r\right)b + \delta s - a\right).$$

Then

$$\Delta Q := Q_D^* - Q_I^* = 2\epsilon bM\left(\frac{1}{-8b\epsilon+\gamma^2} - \frac{1}{-4b\epsilon+\gamma^2}\right)$$

Compute the difference of fractions:

$$\frac{1}{-8b\epsilon+\gamma^2} - \frac{1}{-4b\epsilon+\gamma^2} = \frac{\left(-4b\epsilon+\gamma^2\right)-\left(-8b\epsilon+\gamma^2\right)}{\left(-8b\epsilon+\gamma^2\right)\left(-4b\epsilon+\gamma^2\right)} = \frac{4b\epsilon}{\left(-8b\epsilon+\gamma^2\right)\left(-4b\epsilon+\gamma^2\right)}.$$

Therefore,

$$\Delta Q = 2\epsilon bM \cdot \frac{4b\epsilon}{\left(-8b\epsilon+\gamma^2\right)\left(-4b\epsilon+\gamma^2\right)} = \frac{8\epsilon^2 b^2 M}{\left(-8b\epsilon+\gamma^2\right)\left(-4b\epsilon+\gamma^2\right)}\#$$

Expanded Final Expression

Since

$$M = \left(-\left(c_m - c_r - s_b\right)\alpha + G + c_r\right)b + \delta s - a$$

we can also write

$$Q_D^* - Q_I^* = \frac{8\epsilon^2 b^2\left[\left(-\left(c_m-c_r-s_b\right)\alpha+G+c_r\right)b+\delta s-a\right]}{\left(-8b\epsilon+\gamma^2\right)\left(-4b\epsilon+\gamma^2\right)}\#$$

<u>Appendix E4: Proof - Theorem 6</u>

Definition

Let

$$M := \left(\left(\alpha A + G + c_r\right)b + \delta s - a\right)$$

Step 1: Supply Chain Profit under DDM

From Theorem 1,

$$\pi_{sc}^{D} = \frac{M^2(12b\epsilon-\gamma^2)\epsilon}{(-8b\epsilon+\gamma^2)^2}\#$$

Step 2: Supply Chain Profit under CDM

From Theorem 2,

$$\pi_{sc}^{I} = \frac{M^2\epsilon}{4b\epsilon-\gamma^2}\#$$

Step 3: Difference in Profits

$$\pi_{sc}^{D} - \pi_{sc}^{I} = M^2\epsilon\left(\frac{12b\epsilon-\gamma^2}{(-8b\epsilon+\gamma^2)^2} - \frac{1}{4b\epsilon-\gamma^2}\right)\# \quad = M^2\epsilon\frac{(12b\epsilon-\gamma^2)(4b\epsilon-\gamma^2)-(-8b\epsilon+\gamma^2)^2}{(-8b\epsilon+\gamma^2)^2(4b\epsilon-\gamma^2)}\#$$

Compute the numerator terms:

$$(12b\epsilon - \gamma^2)(4b\epsilon - \gamma^2) \quad = 48b^2\epsilon^2 - 16b\epsilon\gamma^2 + \gamma^4 \, (- 8b\epsilon + \gamma^2)^2 \quad = 64b^2\epsilon^2 - 16b\epsilon\gamma^2 + \gamma^4$$

Thus

$$(12b\epsilon - \gamma^2)(4b\epsilon - \gamma^2) - (- 8b\epsilon + \gamma^2)^2 =- 16b^2\epsilon^2$$

Hence

$$\pi_{sc}^{D} - \pi_{sc}^{I} = M^2\epsilon\frac{-16b^2\epsilon^2}{(-8b\epsilon+\gamma^2)^2(4b\epsilon-\gamma^2)}\#$$

Appendix D: Karush-Kuhn-Tucker (KKT) Proof for the Revenue Sharing Mechanism in the Ethanol-Petrol Supply Chain

The Revenue Sharing (RS) Mechanism is introduced to coordinate the pricing strategies of the ethanol supplier (leader) and the petrol company (follower). This mechanism aligns incentives by requiring the petrol company to share a fraction $\bar{\zeta}$ of its revenue with the ethanol supplier, ensuring supply chain efficiency and environmental benefits.

The goal of the RS mechanism is to determine the optimal wholesale price of ethanol $\left(p_m\right)$ and the optimal retail price of ethanol-blended petrol $\left(p_r\right)$, considering the revenuesharing coefficient $\bar{\zeta}$. The profit functions of both firms are multiplicatively linked under this coordination scheme.

4.1 Optimization Formulation

The decision-making model for the RS mechanism is expressed as:

$$\max_{\bar{\zeta}} \ \Pi_B\left(\bar{\zeta}\right) = \Pi_m\left(\bar{\zeta}\right) \cdot \Pi_r\left(\bar{\zeta}\right)$$

where:

$$\Pi_m\left(\bar{\zeta}\right) = \left(\bar{\zeta} p_r - c_m + p_m + s_b\right)\cdot(\alpha Q) - \epsilon\mu^2\#(33)\ \Pi_r\left(\bar{\zeta}\right) = \left(\left(1 - \bar{\zeta}\right)p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)Q - \lambda\left(p_m - p_r\right)$$

Subject to:

$$\Pi_m\left(\bar{\zeta}\right) \geq \Pi_D^m,\ \Pi_r\left(\bar{\zeta}\right) \geq \Pi_D^r,\ p_r - p_m \geq 0$$

where:

- $\Pi_B\left(\bar{\zeta}\right)$ represents the total supply chain profit.
- $\Pi_m\left(\bar{\zeta}\right)$ is the profit function of the ethanol company.

- $\Pi_r(\bar{\zeta})$is the profit function of the petrol company.

- Π_D^m and Π_D^r are the profits under decentralized decision-making.

Market demand function:

$$Q = a - bp_r + \gamma\mu - \delta S$$

Lagrange multiplier: λ ensures the pricing constraint $p_r \geq p_m$.

The primary goal is to prove that the KKT conditions hold for this revenue-sharing model.

5. Necessary Conditions for KKT Optimality

To verify the KKT conditions, we establish the Lagrangian functions for the ethanol company and the petrol company.

5.1 Lagrangian Formulation

Ethanol Company's Profit Function (Leader)

$$\pi_m = \left(\bar{\zeta} p_r - c_m + p_m + s_b\right)(\alpha Q) - \epsilon\mu^2$$

Petrol Company's Profit Function (Follower)

$$\pi_r = \left(\left(1 - \bar{\zeta}\right)p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)Q - \lambda\left(p_m - p_r\right)$$

6 First-Order Stationarity Conditions

To satisfy the stationarity condition, we take the first derivative of each Lagrangian function.

6.1 First-Order Derivative for the Ethanol Company

$$\frac{\partial \pi_m}{\partial p_m} = \alpha Q$$

Expanding :

$$\frac{\partial \pi_m}{\partial p_m} = \alpha\left(a - bp_r + \gamma\mu - \delta S\right)$$

Similarly, for the revenue-sharing coefficient $\bar{\zeta}$:

$$\frac{\partial \pi_m}{\partial \bar{\zeta}} = \alpha p_r Q$$

6.2 First-Order Derivative for the Petrol Company

$$\frac{\partial \pi_r}{\partial p_r} = \left(1 - \bar{\zeta}\right)Q + \left(\left(1 - \bar{\zeta}\right)p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)(- b) + \lambda$$

Rearranging:

$$\frac{\partial \pi_r}{\partial p_r} = \left(1 - \bar{\zeta}\right)Q - b\left(1 - \bar{\zeta}\right)p_r + b\left((1 - \alpha)c_r + \alpha p_m\right) + bG + \lambda$$

Similarly, differentiating with respect to $\bar{\zeta}$:

$$\frac{\partial \pi_r}{\partial \bar{\zeta}} =- p_r Q$$

7. Primal and Dual Feasibility Conditions

Primal Feasibility:

$$p_r \geq p_m$$

Ensures the retail price is at least the wholesale price.

Dual Feasibility:

$$\lambda \geq 0$$

Ensures the Lagrange multiplier is non-negative.

Complementary Slackness:

$$\lambda\left(p_r - p_m\right) = 0$$

If $p_r > p_m$, then $\lambda = 0$ (constraint inactive).

If $p_r = p_m$, then $\lambda > 0$ (constraint binds).

Conclusion: The necessary KKT conditions hold.

8. Sufficient Conditions: Concavity of Π_m

To confirm that the KKT conditions are also sufficient, we prove that Π_m is concave by constructing the Hessian matrix.

8.1 First-Order Partial Derivatives

With respect to p_m :

$$\frac{\partial \Pi_m}{\partial p_m} = \alpha Q$$

With respect to :

$$\frac{\partial \Pi_m}{\partial \mu} = \alpha\gamma\left(\bar{\zeta} p_r - c_m + p_m + s_b\right) - 2\epsilon\mu$$

8.2 Second-Order Partial Derivatives

Second derivative with respect to p_m :

$$\frac{\partial^2 \Pi_m}{\partial p_m^2} = - 2\alpha b$$

Second derivative with respect to :

$$\frac{\partial^2 \Pi_m}{\partial \mu^2} = - 2\epsilon$$

Mixed derivative:

$$\frac{\partial^2 \Pi_m}{\partial p_m \partial \mu} = \alpha\gamma$$

8.3 Constructing the Hessian Matrix

$$H = [- 2\alpha b \ \alpha\gamma \ \alpha\gamma \ - 2\epsilon]$$

For H to be negative definite:

$$- 2\alpha b < 0, \ 4\alpha b\epsilon > \alpha^2\gamma^2$$

Final Conclusion:

If $4\alpha b\epsilon > \alpha^2\gamma^2$ holds, then Π_m is concave, ensuring global optimality.

The optimal values derivation is as similar detailed in section 13 of Cost Allocation

Appendix E: Karush-Kuhn-Tucker (KKT) Proof for the Cost Allocation (CA) Mechanism in the Ethanol-Petrol Supply Chain

The CA Mechanism is introduced to coordinate the emission reduction efforts between the ethanol supplier (leader) and the petrol company (follower). In this mechanism, the petrol company contributes to the cost of emission reduction in proportion to a cost-sharing coefficient σ, ensuring that both firms have aligned incentives to meet environmental standards.

8.4 Optimization Objective

- Petrol and ethanol companies determine the cost-sharing coefficient σ.
- Ethanol company sets the wholesale price p_m according to σ.
- Petrol company sets the retail price p_r based on p_m and σ.
- Profit functions of both firms are multiplicatively linked under this coordination scheme.

The decision-making model for CA coordination is expressed as:

$$\max_{\sigma} \ \Pi_B(\sigma) = \Pi_m(\sigma)\cdot\Pi_r(\sigma)$$

where:

$$\Pi_m(\sigma) = \left(p_m - c_m + s_b\right)\cdot(\alpha Q) - \xi(1-\sigma)\mu^2 \#(56)\ \Pi_r(\sigma) = \left(p_r - \left((1-\alpha)c_r + \alpha p_m\right) - G\right)Q - \xi\sigma\mu^2 - \lambda\left(p_m - p_r\right)$$

Subject to:

$$\Pi_m\left(\sigma^*\right) \geq \Pi_D^m,\ \Pi_r\left(\sigma^*\right) \geq \Pi_D^r,\ p_r - p_m \geq 0$$

where:

- $\Pi_B(\sigma)$ represents the total supply chain profit.
- $\Pi_m(\sigma)$ is the profit function of the ethanol company.
- $\Pi_r(\sigma)$ is the profit function of the petrol company.
- Π_D^m and Π_D^r are the profits under decentralized decision-making.

Market demand function:

$$Q = a - bp_r + \gamma\mu - \delta S$$

Lagrange multiplier: λ ensures the pricing constraint $p_r \geq p_m$.

The primary goal is to prove that the KKT conditions hold under this cost-sharing model.

9. Necessary Conditions for KKT Optimality

To verify the KKT conditions, we establish the Lagrangian functions for the ethanol company and the petrol company.

9.1 Lagrangian Formulation

Ethanol Company's Profit Function (Leader)

$$\Pi_m = \left(p_m - c_m + s_b\right)(\alpha Q) - \epsilon(1 - \sigma)\mu^2$$

Petrol Company's Profit Function (Follower)

$$\Pi_r = \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)Q - \epsilon\sigma\mu^2 - \lambda\left(p_m - p_r\right)$$

10. First-Order Stationarity Conditions

To satisfy the stationarity condition, we take the first derivative of each Lagrangian function.

10.1 First-Order Derivative for the Ethanol Company

$$\frac{\partial \Pi_m}{\partial p_m} = \alpha Q$$

Expanding :

$$\frac{\partial \Pi_m}{\partial p_m} = \alpha\left(a - bp_r + \gamma\mu - \delta S\right)$$

Similarly, for the cost-sharing coefficient :

$$\frac{\partial \Pi_m}{\partial \sigma} = \epsilon\mu^2$$

10.2 First-Order Derivative for the Petrol Company

$$\frac{\partial \Pi_m}{\partial p_r} = Q + \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)(-b) + \lambda$$

Rearranging:

$$\frac{\partial \Pi_r}{\partial p_r} = Q - bp_r + b\left((1 - \alpha)c_r + \alpha p_m\right) + bG + \lambda$$

Similarly, differentiating with respect to :

$$\frac{\partial \Pi_r}{\partial \sigma} =- \epsilon\mu^2$$

11. Primal and Dual Feasibility Conditions

Primal Feasibility:

$$p_r \geq p_m$$

Ensures the retail price is at least the wholesale price.

Dual Feasibility:

$$\lambda \geq 0$$

Ensures the Lagrange multiplier is non-negative.

Complementary Slackness:

$$\lambda\left(p_r - p_m\right) = 0$$

If $p_r > p_m$, then $\lambda = 0$ (constraint inactive).

If $p_r = p_m$, then $\lambda > 0$ (constraint binds).

Conclusion: The necessary KKT conditions hold.

12. Sufficient Conditions: Concavity of Π_m

To confirm that the KKT conditions are also sufficient, we prove that Π_m is concave by constructing the Hessian matrix.

12.1 First-Order Partial Derivatives

With respect to p_m :

$$\frac{\partial \Pi_m}{\partial p_m} = \alpha Q$$

With respect to :

$$\frac{\partial \Pi_m}{\partial \mu} = \alpha\gamma\left(p_m - c_m + s_b\right) - 2\epsilon(1 - \sigma)\mu$$

12.2 Second-Order Partial Derivatives

Second derivative with respect to p_m :

$$\frac{\partial^2 \Pi_m}{\partial p_m^2} = - 2\alpha b$$

Second derivative with respect to :

$$\frac{\partial^2 \Pi_m}{\partial \mu^2} = - 2\epsilon(1 - \sigma)$$

Mixed derivative:

$$\frac{\partial^2 \Pi_m}{\partial p_m \partial \mu} = \alpha\gamma$$

12.3 Constructing the Hessian Matrix

$$H = [- 2\alpha b\ \alpha\gamma\ \alpha\gamma\ - 2\epsilon(1 - \sigma)\ ]$$

For H to be negative definite:

$$- 2\alpha b < 0,\ 4b\epsilon(1 - \sigma) > \alpha\gamma^2$$

If $4b\xi(1 - \sigma) > \alpha\gamma^2$ holds, then Π_m is concave, ensuring global optimality.

The below presents the step-by-step derivation of the optimal values for retail price $\left(p_r^*\right)$, wholesale price $\left(p_m^*\right)$, emission reduction level $\left(\mu^*\right)$, and the total supply chain profit function $\left(\Pi_B\right)$ under the Cost Allocation (CA) mechanism.

The CA mechanism ensures that both the ethanol supplier and the petrol company contribute to emission reduction efforts in proportion to a cost-sharing coefficient σ. The firms optimize their profits while meeting environmental standards.

13 Step-by-Step Calculation of Optimal Values

13.1 Step 1: Deriving the Optimal Retail Price p_r

The petrol company's profit function is:

$$\Pi_r = \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)Q - \epsilon\sigma\mu^2$$

where the demand function Q is:

$$Q = a - bp_r + \gamma\mu - \delta S$$

Differentiate Π_r with Respect to p_r

To find the optimal retail price p_r^*, we take the first-order derivative:

$$\frac{\partial\Pi_r}{\partial p_r} = Q + \left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right)(- b) = 0$$

Expanding :

$$\left(a - bp_r + \gamma\mu - \delta S\right) - b\left(p_r - \left((1 - \alpha)c_r + \alpha p_m\right) - G\right) = 0$$

Solving for p_r :

$$p_r = \frac{-\alpha bc_r + \alpha bp_m + Gb + bc_r - \delta S + \gamma\mu + a}{2b}$$

This is the optimal retail price under the CA mechanism.

13.2 Step 2: Deriving the Optimal Wholesale Price p_m and Emission Reduction Level μ

The ethanol company's profit function is:

$$\Pi_m = \left(p_m - c_m + s_b\right)\alpha Q - \epsilon(1 - \sigma)\mu^2$$

where the demand function remains:

$$Q = a - bp_r + \gamma\mu - \delta S$$

Differentiate Π_m with Respect to μ

$$\frac{\partial \Pi_m}{\partial \mu} = \alpha\gamma\left(p_m - c_m + s_b\right) - 2\epsilon(1 - \sigma)\mu = 0$$

Solving for :

$$\mu = \frac{\left(\left(c_m - c_r - s_b\right)\alpha + G + c_r\right)b + \delta S - a}{8b\epsilon(1-\sigma) + \gamma^2}$$

This is the optimal emission reduction level under cost allocation.

Differentiate Π_m with Respect to p_m

$$\frac{\partial \Pi_m}{\partial p_m} = \alpha Q - 2\epsilon(1 - \sigma)\frac{\partial \mu}{\partial p_m} = 0$$

Solving for p_m :

$$p_m = \frac{-4\left[\left(\left(-c_m - c_r + s_b\right)\alpha + G + c_r\right)b + \delta S - a\right](1-\sigma)\epsilon + \gamma^2\alpha\left(c_m - s_b\right)}{\left(8b\epsilon(1-\sigma) + \gamma^2\right)\alpha}$$

This is the optimal wholesale price under cost allocation.

13.3 Step 3: Compute the Total Supply Chain Profit Π_B

The total profit of the supply chain is given as:

$$\Pi_B = \Pi_m \cdot \Pi_r$$

Substituting the derived values of Π_m and Π_r :

$$\Pi_B = \left[\left(p_m - c_m + s_b\right)\alpha Q - \epsilon(1 - \sigma)\mu^2\right] \cdot \left[\left(p_r - (1 - \alpha)c_r - \alpha p_m - G\right)Q - \epsilon\sigma\mu^2\right]$$

This expression represents the total coordinated supply chain profit under the Cost Allocation (CA) mechanism.

The CA Mechanism and Optimal Values

The CA mechanism ensures that both firms contribute to emission reduction costs in proportion to a cost-sharing coefficient σ. By maximizing the total supply chain profit:

$$\Pi_B = \Pi_m \cdot \Pi_r$$

where:

- Π_m represents the ethanol supplier's profit.

- Π_r represents the petrol company's profit.

The firms coordinate through the cost-sharing coefficient σ, which influences the optimal wholesale price p_m^*, retail price p_r^*, and emission reduction level μ^*.

The following sections derive the optimal expressions for these variables.

14 Optimal Cost-Sharing Coefficient σ^*

By differentiating Π_B with respect to σ and solving for the optimal value, we obtain:

$$\sigma^* = \frac{\gamma^2+12b\epsilon+\sqrt{64b^2\epsilon^2+4b\gamma^2\epsilon+\gamma^4}}{20b\epsilon}$$

This represents the optimal proportion of emission reduction cost borne by the petrol company.

Wholesale Price of Ethanol (p_m)

$$p_m = \frac{-4\left[\left(\left(-c_m-c_r+s_b\right)\alpha+G+c_r\right)b+\delta s-a\right]\left(-1+\frac{\gamma^2+12b\epsilon+\sqrt{64b^2\epsilon^2+4b\gamma^2\epsilon+\gamma^4}}{20b\xi}\right)\xi+\gamma^2\alpha\left(c_m-s_b\right)}{\left(8b\epsilon\left(-1+\frac{\gamma^2+12b\epsilon+\sqrt{64b^2\epsilon^2+4b\gamma^2\epsilon+\gamma^4}}{20b\epsilon}\right)+\gamma^2\right)\alpha}$$

Retail Price of EBP (p_r)

$$p_r = \frac{\left[\left(\left(c_m-c_r-s_b\right)\alpha+G+c_r\right)b-3\delta s+3a\right]\sqrt{64b^2\epsilon^2+4b\gamma^2\epsilon+\gamma^4}-8\left[\left(c_m-c_r-s_b\right)\alpha+G+c_r\right]\epsilon b^2+\left(11\gamma^2\left(c_m-c_r-s_b\right)\alpha+\left(11G+11c_r\right)\gamma^2+24\epsilon(\delta s-a)\right)b-3\gamma^2(\delta s-a)}{14b\gamma^2-32b^2\epsilon+4b\sqrt{64b^2\epsilon^2+4b\gamma^2\epsilon+\gamma^4}}$$

Emission Reduction Level (μ):

$$\mu = \frac{5\left[\left(\left(c_m - c_r - s_b\right)\alpha + G + c_r\right)b + \delta s - a\right]\gamma}{7\gamma^2 - 16b\epsilon + 2\sqrt{64b^2\epsilon^2 + 4b\gamma^2\epsilon + \gamma^4}}$$

The profit functions for the ethanol and petrol companies under the optimal strategies are:

Profit of Ethanol Company $\left(\pi_m\right)$:

$$\pi_m = \frac{9\left(\left(\gamma^2 - \frac{32b\epsilon}{9}\right)\sqrt{64b^2\epsilon^2 + 4b\gamma^2\epsilon + \gamma^4} + \gamma^4 - \frac{64b\gamma^2\epsilon}{9} + \frac{256b^2\epsilon^2}{9}\right)\left[\left(\left(c_m - c_r - s_b\right)\alpha + G + c_r\right)b + \delta s - a\right]^2}{4b\left(7\gamma^2 - 16b\epsilon + 2\sqrt{64b^2\epsilon^2 + 4b\gamma^2\epsilon + \gamma^4}\right)^2}$$

Profit of Petrol Company (π_r):

$$\pi_r = -\frac{3\left(\left(\gamma^2 + \frac{16b\epsilon}{3}\right)\sqrt{64b^2\epsilon^2 + 4b\gamma^2\epsilon + \gamma^4} + \gamma^4 + 24b\gamma^2\epsilon - \frac{128b^2\epsilon^2}{3}\right)\left[\left(\left(c_m - c_r - s_b\right)\alpha + G + c_r\right)b + \delta s - a\right]^2}{4b\left(7\gamma^2 - 16b\epsilon + 2\sqrt{64b^2\epsilon^2 + 4b\gamma^2\epsilon + \gamma^4}\right)^2}$$