Linear Algebra MAT313 Spring 2023 Professor Sormani

Lesson 2 Solving Linear Systems

Warning: do not start this lesson until you have completed Lesson 1 and submitted all the classwork for that lesson.

Please be sure to mark down the date and time that you start this lesson. Carefully take notes on pencil and paper while watching the lesson videos. Pause the lesson to try classwork before watching the video going over that classwork. If you work with any classmates, be sure to write their names on the problems you completed together.

You will cut and paste the photos of your notes and completed classwork and a selfie taken holding up the first page of your work in a googledoc entitled:

MAT313S23-lesson2-lastname-firstname

and share editing of that document with me <u>sormanic@gmail.com</u>. You can use your Lehman id and hand instead of your face in your selfie. You will also include your homework and any corrections to your homework in this doc.

If you have a question, type **QUESTION** in your googledoc next to the point in your notes that has a question and email me with the subject MAT313 QUESTION. I will answer your question by inserting a photo into your googledoc or making an extra video.

This lesson has two parts and each has its own playlist:

Part 1: Using Row Actions to Solve a Linear System

Part II: How to Solve Any Linear System

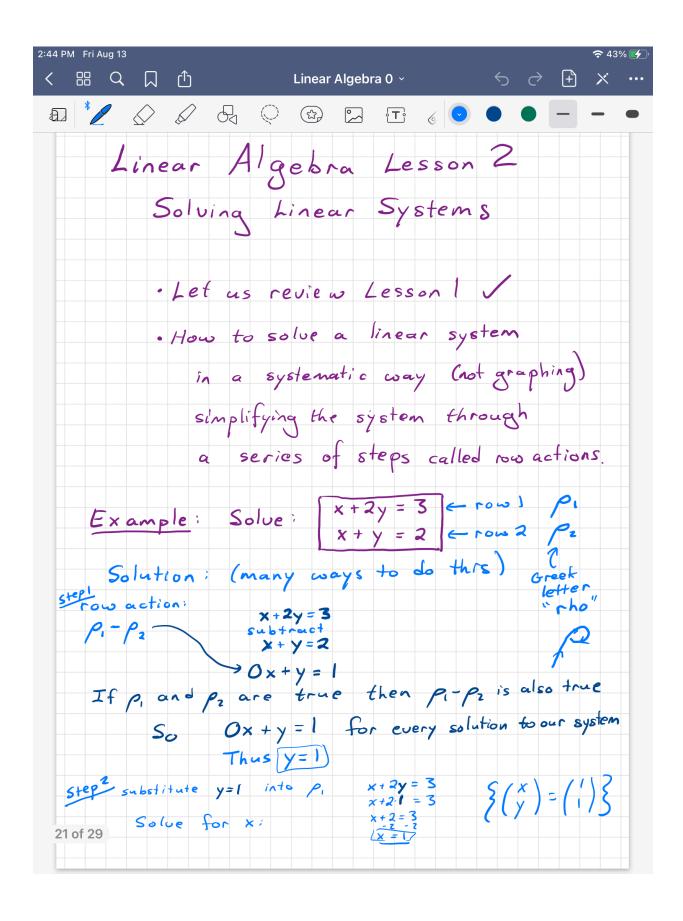
QUESTIONS:

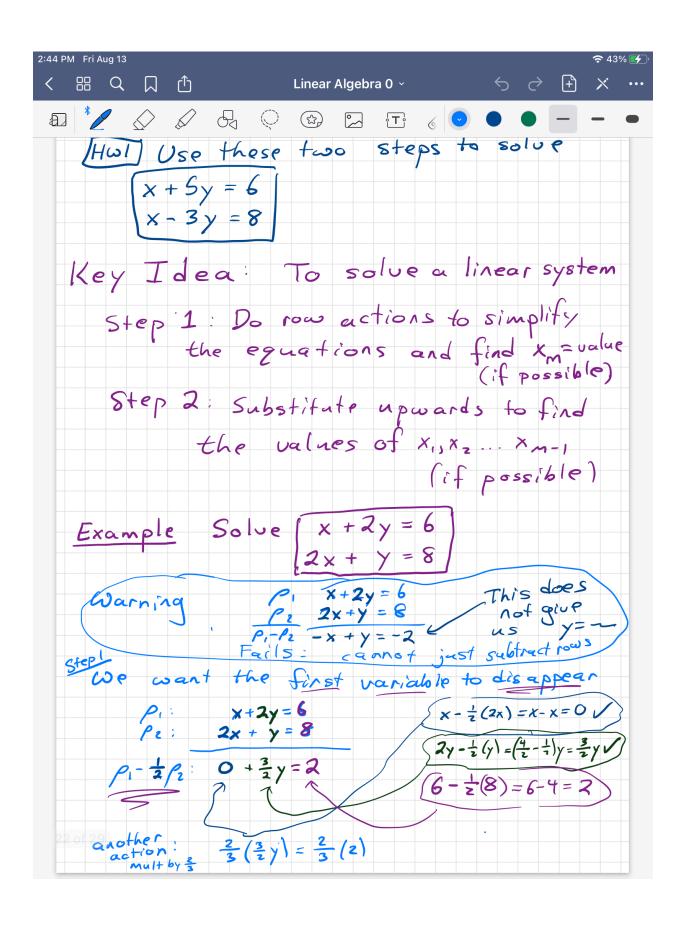
Be sure to learn the methods taught in this lesson even if you already learned to solve a system in another course. I am teaching the method that leads to an algorithm that works with many equations and many unknowns. There are 10 HW problems.

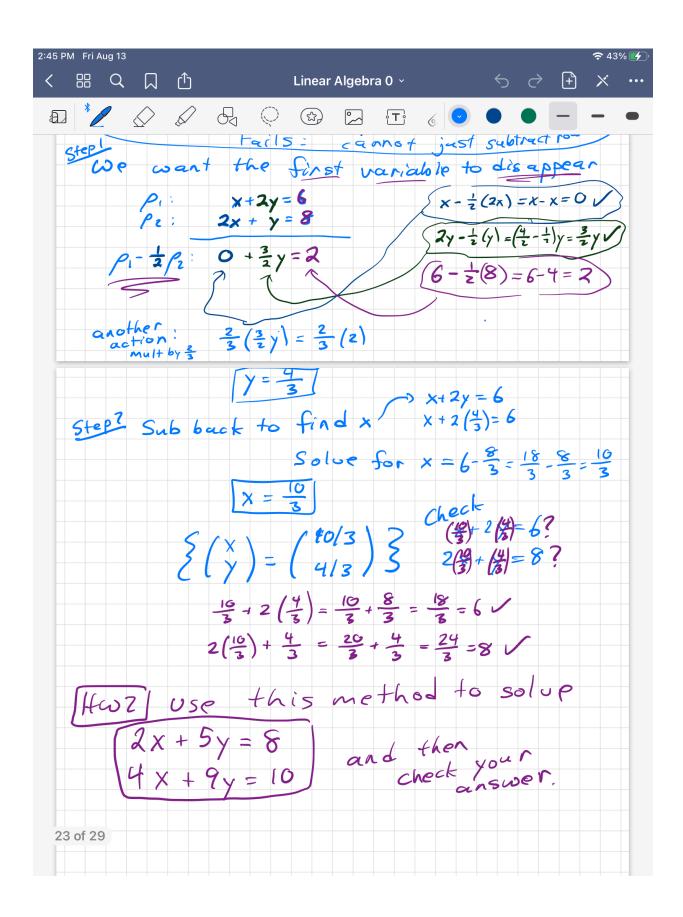
Part 1: Using Row Actions to Solve a Linear System

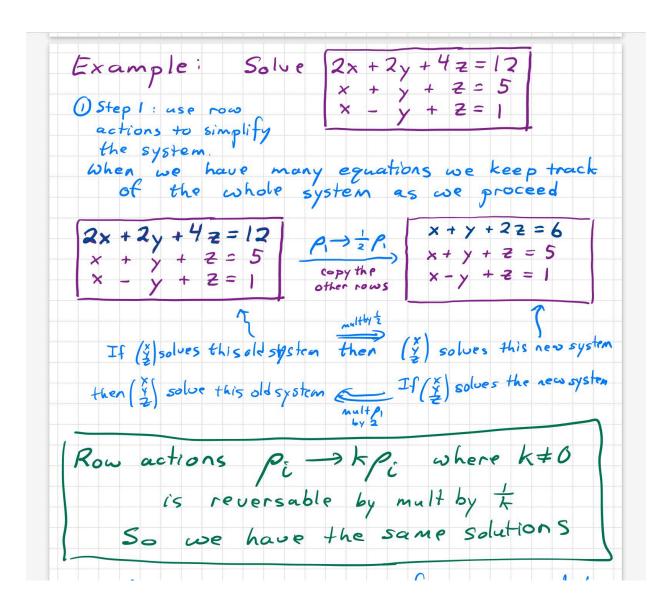
Watch Playlist 313F21-2-1to7

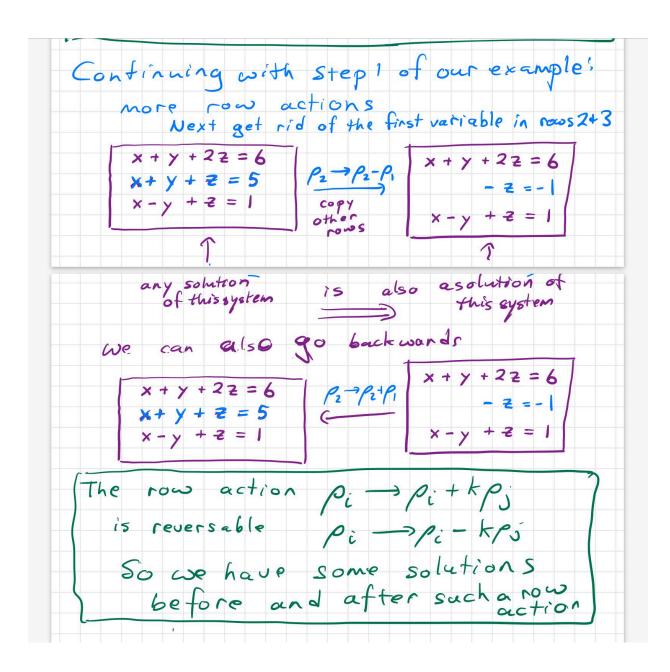
| Here we solve one particular system and explain what row actions are and introduce Echelon form. |
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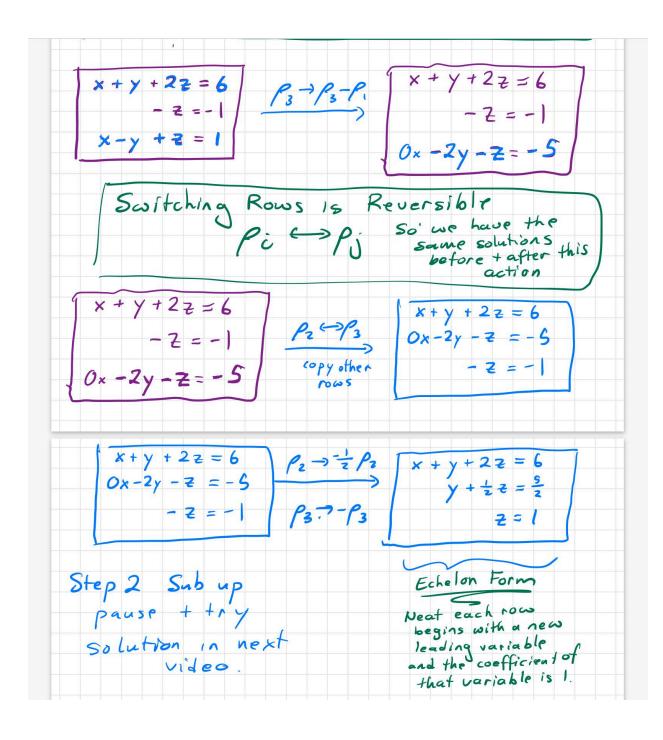


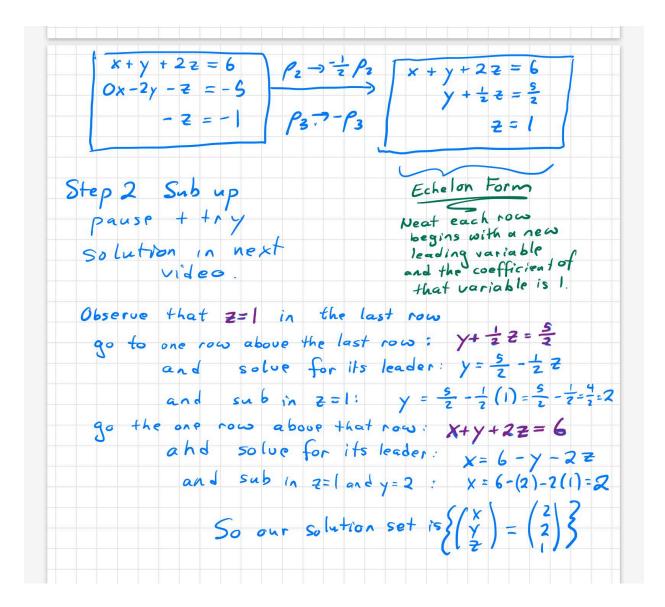












Before continuing to Part 2 you may wish to complete the first five homework problems. Check the solution <u>here</u> to each problem before doing the next.

| Lesson | 2 Homework (answers below) |
|--------|--|
| HWI | 3x+3y-9z=6 What is the first row action you should row action you should do? Do the row action |
| Hwà | (X)+3y+4z=0 What row actions are 2x+7y+9z=6 4x+12y+12z=4 Zeroes under the boxed leader? Do them! |
| НωЗ | $\boxed{X+y+z+4\omega=0}$ What is the 2 nd leader? $0x+0y+5z+5\omega=10$ Move it to the second row $0x+2y+4z+6\omega=12$ using a switch $0x+0y+5z+5\omega=15$ then make its $\omega=ff=1$. |
| μωΨ | X + y + Z = S Ox + Oy + 1Z = 6 Ox + Oy + OZ = O This in Echelon form? If not, complete the row actions to Echelon form |
| Hw 3 | Ox + Oy + 1z = 6 Ox + Oy + 1z = 6 Ox + y + Oz = O This in Echelon form? Ox + y + Oz = O This in Echelon form? The leaders. The leaders. |

Part II: How to Solve Any Linear System

Watch Playlist 313F21-2-8to12

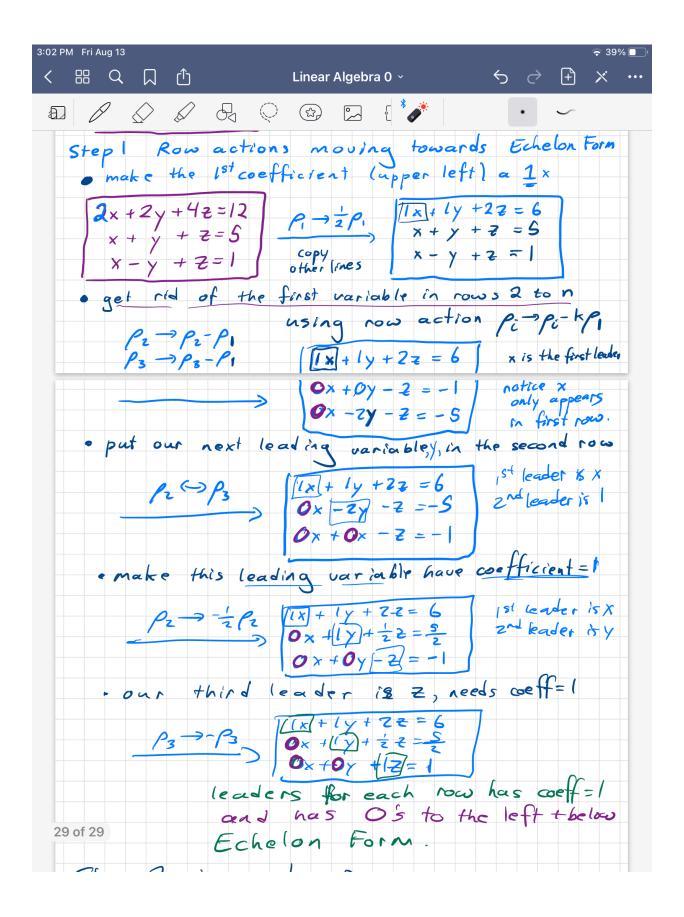
"How to solve a Linear System" is in video 313F21-2-8 and the Example 1 rewritten using this technique is in video 313F21-2-8 and 313F21-2-9:

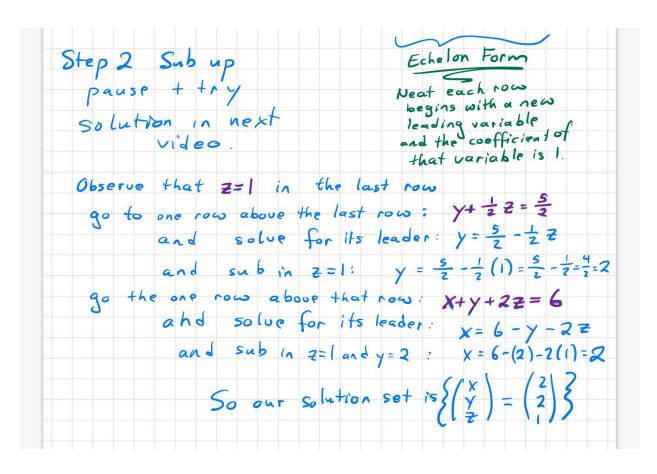
| How to Solve a Linear System: |
|---|
| Step 1: Row Reductions to Echelon Form Row Actions which are reversable |
| Row Actions which are reversable |
| scale · Pi -> kpi where k +0 later we set will set will an |
| skew · Pi -> Pi + kpj will an up thungs |
| switch · Pi => Pi + RPj switch · Pi => Pj algorithmis algorithmis algorithmis |
| Step2: Sub up |
| Start with final row; solve for its leader |
| Start with final row; solve for its leader Next to last row: solve for its leader sub in previous variables |
| West row up: Solve for its leader sub in previous variable |
| subject of solution |
| When all rows are done we have a solution. |

Example above rewritten (method to use for)
$$2x+2y+4z=12$$

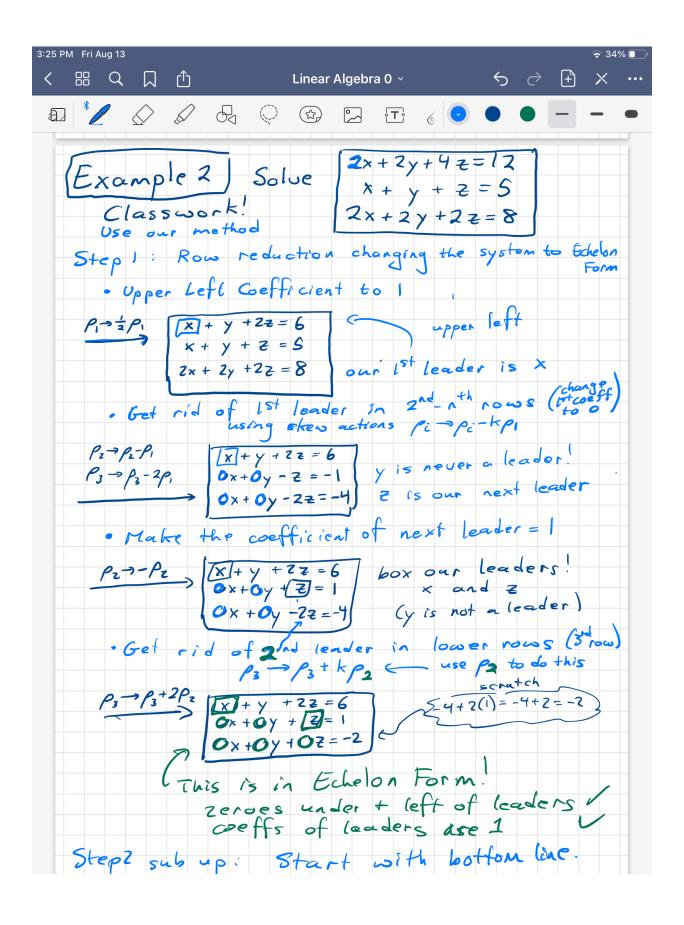
$$x+y+z=5$$

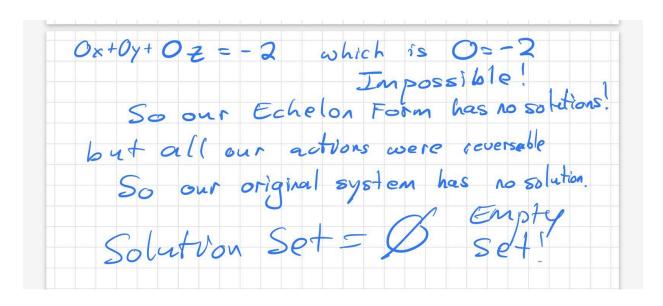
$$x-y+z=1$$



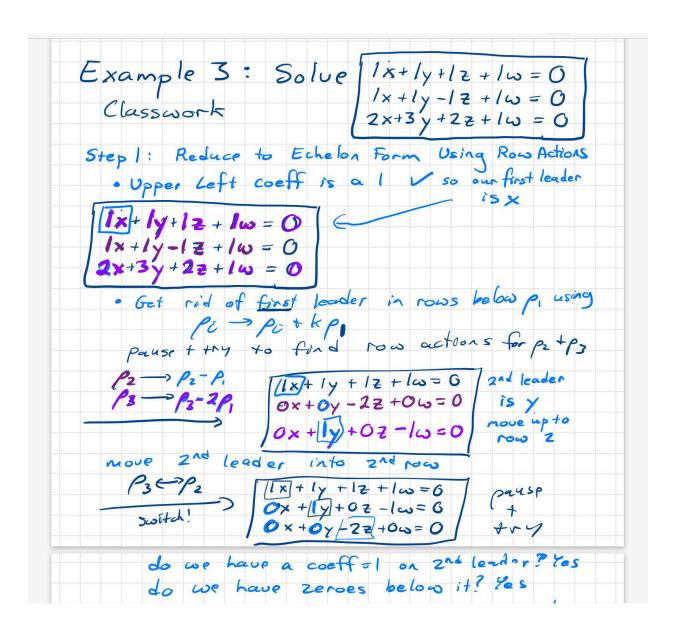


Example 2 is in video 313F21-2-10





Example 3 is in the last two videos 313F21-2-11 and 313F21-2-12 which you should watch pausing and trying as you work:

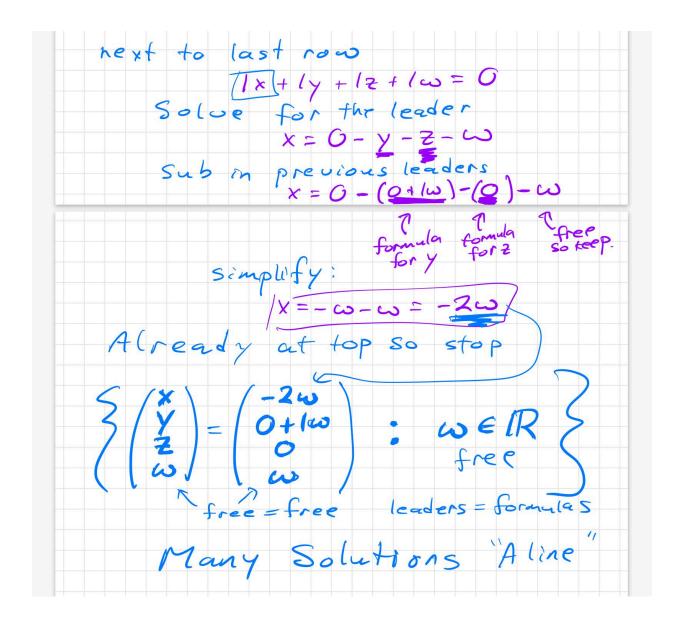


Next look for third leader in next column the next leader is z.

It is already in 3rd row.

No switch needed.

Must make 3rd leader's coeff = 1 $\begin{array}{c|c} P_3 \rightarrow \frac{1}{2}P^2 & \hline{(1x) + 1y + 12 + 1y = G} \\ \hline{(0x + 1y) + 02 - 1w = G} & Echelon \\ \hline{(0x + 0y + 112) + 0w = O} & Form \end{array}$ last row 0x + 0y + 12 + 0w = 0 Solve for the leader Z=0-00-0x-0y=0 Second to last row So 3=0 0x+/1y/+ 0z-1w=0 Solve for the leader [y= Q+1w]
Sub in leaders (but wis not a lead) is free" not a leader So w can have any value in IR

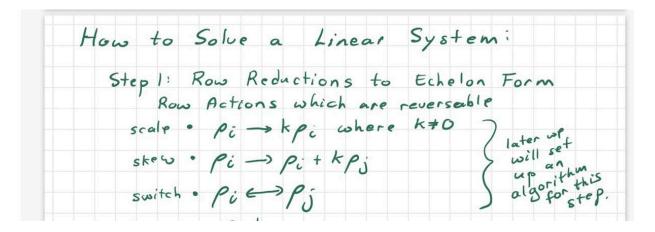


For homework you will solve five systems using today's method.

QUESTIONS:

Email me questions as explained at the top of the doc. Below I include answers to the best questions.

Let us review the key steps again including students questions:



Student Question:

What is Echelon Form?

Here is the precise definition:

Definition: a system is in Echelon Form if

it has zeroes to the left and below all leaders

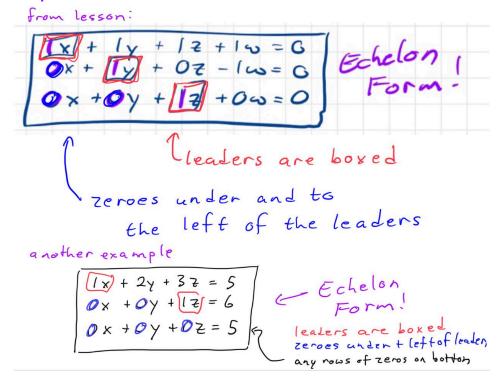
and all the leaders are ones and

all rows with 0+0+0+...+0=something are at the bottom.

Definition: a leading coefficient or leader is

the first nonzero coefficient of a row.

Here are examples of Echelon Form:



Student Question:

How do I do Row Reduction to Echelon Form?

We follow these steps:

Method: Start on the upper left.

Step 1: Find a leader and make it one using row actions.

Step 2: Get zeroes below the leader using row actions.

Go to the next row and column and repeat Step 1 and Step 2.

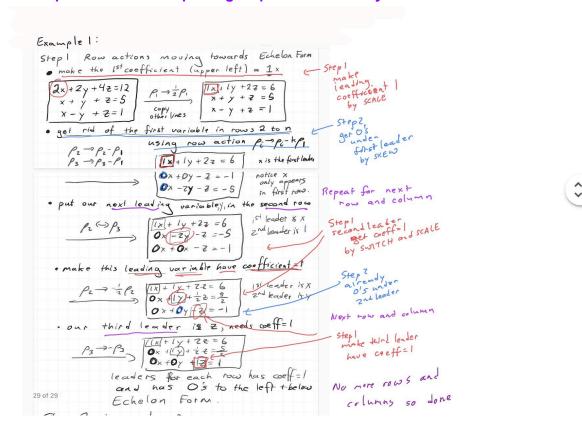
When we run out of rows, the system is in Echelon Form:

it has zeroes to the left and below all leaders

and all the leaders are ones and

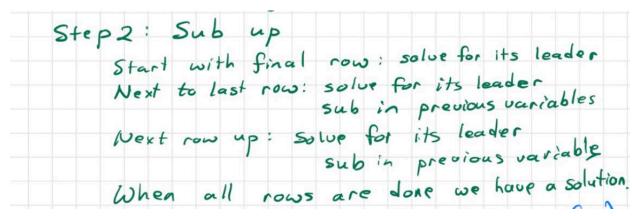
all rows with 0+0+0+...+0=something are at the bottom.

Here is Example 1 with these repeating steps marked clearly:



We will learn a precise way of writing this as an algorithm with a flow chart for any system in the next lesson.

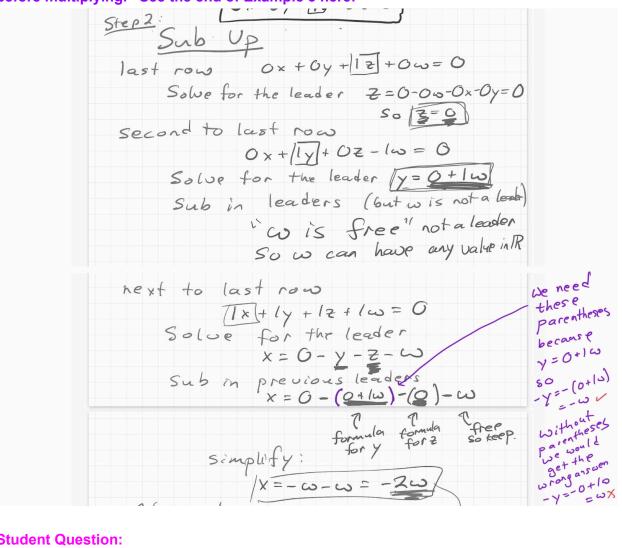
Continuing forward:



Student Question:

Why do you use parentheses when you sub up?

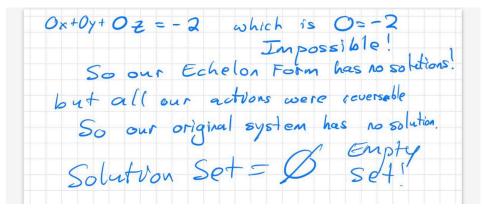
Parentheses are used because they tell us to compute the value inside the parentheses before multiplying. See the end of Example 3 here:



Student Question:

What if there is a row of zeroes adding up to something that is not zero?

If so, then there is no solution! Write the empty set. This happened in Example 2 above:



Be careful not to write brackets around the empty set symbol.

Student Question:

What if there is a row of zeroes adding up to zero?

If you get something like 0x+0y+0z=0 then it still has a solution. Sub up to find the solution starting with the last row that has a leader.

You finally complete the problem by writing the solution set.

Student Question:

How do I know which variables are free?

All variables which are not boxed as leaders are free.

Student Question:

In some places I see the solution sets written without brackets and free variables. Linear Algebra is an ancient subject dating back a thousand years in China and has many notations. In this course we will use only the notation I teach to keep things simple. Do not use other notation in your work. All mathematicians understand the notation I am teaching you. In your own fields: chemistry, computer science, economics, etc other notations may be used that you will learn in those courses.

Homework:

Glance over the lecture notes above to be sure you watched all the videos and read over the students' questions. Then do the following ten problems. Check the solution here to each problem before doing the next.

| Lesson | 2 Homework | (answers below) |
|--------|---|---|
| HWD | 3x + 3y - 9z = 6 What $x - y + z = 0$ row $x + 4y + 2z = 5$ | t is the first ow action of the row action |
| | [x] + 3y + 4z = 0 Who $2x + 7y + 9z = 6$ | at row actions are exceed here to get croes under the boxed cader? Do them! |
| Нω3 | (x)+y+z+4w=0 W 0x+0y+5z+5w=10 0x+2y+4z+6w=11 | hat is the 2 nd leader? oup it to the second pow using a switch hen make its coeff = 1. |
| | X+y+z=5 $DX+DX+1Z=($ Z | s the leaders. This in Echelon form? I not, complete the row actions to Echelonform |
| Hw s | (x + y + z = 5) $0x + 0y + 1z = 6$ $0x + y + 0z = 0$ If | this in Echelon form? not, complete the row actions to Echelon form |
| Hwg S | polue the system in | (HWS) by subing up |
| (HW7) | Solve the system in Solve the system | EN HWY) |
| Hwa |) Solve the syste. | n in (HWZ) |
| (Hw10) |) Solve the sys | tem in (HW3) |

Note your solution is only correct if you do the same row actions in the same order exactly as in the solutions <u>here</u>. If you do something different, and do not know why it is wrong, send a question.

It is very important to email me if you do not understand why your problems are incorrect. See how to email questions at the top of this document.

You can use your Lehman id and hand instead of your face in your selfie. This can be helpful if you are not dressed well or are shy or have difficulty taking a selfie.

Submit your classwork and homework following the directions at the top of this document.