**XII-MATHEMATICS**

**UNIT-6 - APPLICATIONS OF VECTOR ALGEBRA**

## Important 2 and 3 Marks:

**1). Find the magnitude and the direction cosines of the torque about the point ( 2, 0 , -1) of a force 2 i + j – k , where line of action passes through the origin.**

**2). Find the magnitude and direction cosines of the torque of a force represented by 3 i + 4 j -5 k about the point with vector 2 i – 3 j + 4 k acting through a point whose position vector is 4 i + 2 j – 3 k.**

**3). Find a . (b x c )**

**i) if a = -3 i – j + 5 k , b = i – 2 j + k , c = 4 j – 5 k**

**ii) if a = i – j + 3 k , b = 2 i + j – 2 k , c = 3 i + 2 j + k**

**4). Find the volume of the parallelepiped whose coterminous edges are given by the vectors**

**i) 2 i – 3 j + 4 k , i + 2 j – k and 3 i – j + 2 k**

**ii) – 6 i + 14 j + 10 k , 14 i – 10 j - 6 k , 2 i + 4 j – 2 k**

**5). Show that the vectors i + 2 j – 3 k , 2 i – j + 2 k and 3 i + j - k are coplanar. 6). If 2 i – j + 3 k , 3 i + 2 j + k , i + m j + 4 k are coplanar , find the value of m.**

**7). Determine whether the three vectors 2 i + 3 j + k , i – 2 j + 2 k and 3 i + j + 3 k are coplanar. 8). Show that the four points (6,-7,0 ) , (16,-19,-4) , (0,3,-6 ) , (2,-5,10) lie on a same plane.**

**9). The volume of the parallelepiped whose coterminous edges are 7 i + λ j – 3 k , i + 2 j – k , -3 i + 7j + 5k is 90 cubic units .Find the value of λ .**

**10). If a , b , c are three vectors , prove that [ a + c , a + b , a + b + c] = [ a , b , c ] 11). Prove that [ a x b , b x c , c x a ] = [a , b , c ]$^2$**

**12). For any four vectors a , b, c, d , we have ,**

$$(a \times b) \times (c \times d) = [a, b, d]\, c - [a, b, c]\, d$$

$$= [a, c, d]\, b - [b, c, d]\, a$$

**13). If a = - 2 i + 3 j – 2 k , b = 3 i – j + 3 k , c = 2i – 5 j + k , find ( a x b ) x c and a x ( b x c ). State whether they are equal.**

**14). For any vector $\vec{a}$ , prove that $\vec{i} \times (\vec{a} \times \vec{i}) + \vec{k} \times (\vec{a} \times \vec{k}) = 2\vec{a}$ .**

**15). Prove that $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{d}] = 0$.**

**16). $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$, $\vec{c} = \vec{i} + \vec{j} + \vec{j}$, then find the value of $(\vec{a} \times \vec{b}) . (\vec{a} \times \vec{c})$.**

**17). Find the actue angle between the lines $\vec{r} = (\vec{i} + 2\vec{j} + 4\vec{k}) + t(2\vec{i} + 2\vec{j} + \vec{k})$ and the straight line passing through the points ( 5,1,4) and (9,2,12) .**

**18). Find the actor angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ and state whether they are $\parallel^{el}$ or $^{r}$ .**

**19). Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-3}{-2} = \frac{y-3}{3} = \frac{5-z}{6}$ are $\parallel^{el}$ .**

## 5MARKS

**20). Find the points whether the straight lines passes through (6 , 7, 4 ) ands ( 8, 4, 9) cuts the xz and yz planes.**

**21). Find the actue angle between the following lines:**

**i) $\vec{r} = (4\vec{i} - \vec{j}) + t(-\vec{i} + 2\vec{j} - 2\vec{k})$, $\vec{r} = (\vec{i} - 2\vec{j} + 4\vec{k}) + s(-\vec{i} - 2\vec{j} + 2\vec{k})$ ii)**

**$x + 4 = y - 7 = z + 5$, $\vec{r} = 4\vec{k} + t(2\vec{i} + \vec{j} + \vec{k})$**

**3 4 5**

**iii) $2x = 3y = -z$ and $6x = -y = -4z$ .**

**22). Show that the points (2,3,4) , (-1,4,5) and (8,1,2) are collinear . 23). Determine whether the pair of straight lines**

**$\vec{r} = (2\vec{i} + 6\vec{j} + 3\vec{k}) + t(2\vec{i} + 3\vec{j} + 4\vec{k})$, $\vec{r} = (2\vec{j} - 3\vec{k}) + s(\vec{i} + 2\vec{j} + 3\vec{k})$ are parallel .Find the Shortest distance between them.**

**24). Find the shortest distance between the two given straight lines**

**$\vec{r} = (2\vec{i} + 3\vec{j} + 4\vec{k}) + t(-2\vec{i} + \vec{j} - 2\vec{k})$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$**

**25). Show that the lines $\vec{r} = (6\vec{i} + \vec{j} + 2\vec{k}) + s(\vec{i} + 2\vec{j} - 3\vec{k})$ and $\vec{r} = (3\vec{i} + 2\vec{j} - 2\vec{k}) + t(2\vec{i} + 4\vec{j} - 5\vec{k})$ are skew lines and hence find the shortest distance between them.**

**26). Show that the straight lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ are Skew lines and hence find the shortest distance between them.**

**27). Find the foot of the ⊥ drawn from the point (5, 4, 2) to the line $x + \frac{1}{2} = \frac{y - 3}{3} = \frac{z - 1}{-1}$**

**Also find the equation of the perpendicular.**

**28). Show that the lines $x - 3 = \frac{y - 3}{3}, \frac{z - 1}{-1} = 0$ and $\frac{x - 6}{2} = z - 1, y - 2 = 0$ intersect.**

**Also Find the point of intersection.**

**29). Find the vector and Cartesian form of the equations of a plane which is at a distance of 12 units from the origin and ⊥r to $6\,i + 2\,j - 3\,k$.**

**30). Find the direction cosines of the normal to the plane and length of the ⊥r from the origin to the plane $r \cdot (3\,i - 4\,j + 12\,k) = 5$**

**31). Find the vector and Cartesian equations of the plane passing through the point with position vector $4\,i + 2\,j - 3\,k$ and normal to vector $2\,i - j + k$.**

**32). Find the equation of a plane which is at a distance of 7 units from the origin having 3 , -4 , 5 as direction ratio of a normal to it.**

**33). Find the intercepts cut off by the plane $r \cdot (6\,i + 4\,j - 3\,k) = 12$ on the coordinate axes.**

**34). Find the non – parametric form of vector equation , and Cartesian equation of the plane passing through the point ,**

**(i) ( 0, 1 , -5 ) and parallel to the straight lines**

$r = (i + 2\,j - 4k) + s(2\,i + 3\,j + 6\,k)$ **and**

$r = (i - 3\,j + 5\,k) + t(i + j - 6\,k)$

**(ii) ( 2 , 3, 6 ) and parallel to the straight lines $\frac{x - 1}{2} = \frac{y + 1}{3} = \frac{z - 3}{1}$ and $\frac{x + 3}{2} = \frac{y - 3}{-5} = \frac{z + 1}{-3}$**

**35). Find the parametric form of vector equation and Cartesian equations of the plane containing the line $r = (i - j + 3\,k) + t(2\,i - j + 4\,k)$ and perpendicular to plane $r \cdot (i + 2\,j + k) = 8$.**

**36). Find the non parametric form of vector equation and Cartesian equations of the plane $r = (6\,i - j + k) + s(-i + 2\,j + k) + t(-5\,i - 4\,j - 5\,k)$.**

**37). Show that the straight line $\vec{r} = (5\,\vec{i} + 7\,\vec{k} - 3\,\vec{k}) + s(4\,\vec{i} + 4\,\vec{j} - 5\,\vec{k})$ and $\vec{r} = (8\,\vec{i} + 4\,\vec{j} + 5\,\vec{k}) + t(7\,\vec{i} + \vec{j} + 3\,\vec{k})$ are co planar. Find the vector equation of the plane in**

**which they lie.**

**38). Show that the lines $\frac{x-2}{1} = \frac{y-3}{3} = \frac{z-4}{-3}$ and $\frac{x-1}{2} = \frac{y-4}{1} = \frac{z-5}{1}$ are coplanar. Also find The plane containing those lines.**

**39). Find the acute angle between the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} + 2\hat{k}) = 11$ and $4x - 2y + 2z = 15$.**

**40) Find the distance of a point $(2, 5, -3)$ from the plane $\vec{r} \cdot (16\hat{i} - 3\hat{j} + 2\hat{k}) = 5$.**

**41). Find the angle between the straight line $\vec{r} = (2\hat{i} + 3\hat{j} + \hat{k}) + t(\hat{i} - \hat{j} + \hat{k})$ and the plane $2x - y + z = 5$.**

**42). If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are co planar, find the Distinct real values of m .**

**43). If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{a} \times (\vec{b} \times \vec{c}) = l\vec{a} \times m\vec{b} + n\vec{c}$. Find the values of l , m , n .**

**45). Prove by vector method**

**i) $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$**

**ii) $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$**

**iii) $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$**

**iv) $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$**

**46). If G is centroid of a ABC , Prove that**

**( area of GAB ) = (area of GBC ) = ( area of GCA ) = $\frac{1}{3}$ (area of ABC)**

**47). Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.**