Investigation of Scaling Laws for Electrosprays

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Existing electrospray thrusters have found successful applications in nano and microsatellites for their high efficiency and specific impulse at compact sizes. Because of these properties, there is also interest in developing large-scale electrospray systems for use as primary thrusters. This paper explores the governing scaling laws for electrospray thrusters (specifically those operating in the droplet or cone jet regime) and endeavors to identify the factors which may limit their implementation on larger spacecraft. The relationship between thrust, voltage, space charge effects, and emitter tip density is investigated, and key scaling relationships are explored. In particular, limitations to achievable thrust are identified. Four key efficiency terms are also presented along with their respective scaling laws, and three are shown to decrease with increasing thrust when power is held constant. The results are discussed in context of how a scaled electrospray might compare to existing forms of electric propulsion.

I. Nomenclature

T	=	Thrust
I_{sp}	=	Specific impulse
$\dot{m}$	=	Mass flow rate
d	=	Distance between grids
q	=	Fundamental charge
U_{ex}	=	Exhaust velocity
A	=	Amps
V_c	=	Extractor voltage
I_B	=	Beam current
I_B	=	Extractor current
m	=	Mass of propellent droplet
ρ	=	Density
P	=	Power
λ	=	Surface Tension
Q	=	Volume Flow Rate
K	=	Conductivity
ε	=	Permittivity
η	=	Efficiency
J	=	Current Density

II. Introduction

Electrospray thrusters are a form of electric propulsion, relying on the extraction and acceleration of propellent particles from a charged liquid by an electrostatic potential. These particles can range from droplets or individual ions. Systems operating in these two regimes are knowns as colloid thrusters and field emission thrusters (FEEP) respectively. The prior typically uses an ionic liquid, while the latter uses liquid metal. A key feature of all electrospray thrusters is the Taylor cone which forms at the extraction site, due to interactions between the surface tension and electric field. Like gridded ion thrusters, electrosprays also require a neutralizing beam downstream of the grid. Fig. 1 shows the basic elements of a typical electrospray thruster [6].

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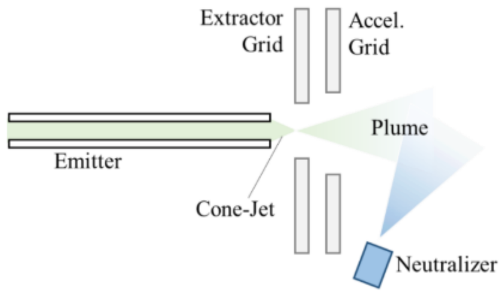

Figure 1: Basic Elements of an Electrospray Thruster. Reprinted from "Disparate Electrospray Systems for Undergraduate and Graduate Education" [6].

The method of fluid transport to the extraction site is still an important area of ongoing research, but to date there are a few established classifications, including: capillary action, externally wetted, and porous emitters [6]. Fig. 2 distinguishes these methods.

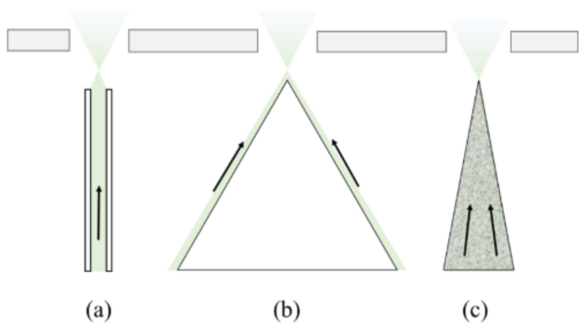

Figure 2: Types of emitter tips - a) Capillary, b) Externally Wetted, c) Porous emitter. Reprinted from "Disparate Electrospray Systems for Undergraduate and Graduate Education" [6].

Colloid thrusters predominately use ionic liquids. Thermochemical and electrochemical properties like surface tension, dielectric constant, electrical conductivity, viscosity, melting and boiling temperature, and electrochemical stability play an important role in their performance as propellants. The best propellants are those that can produce high beam currents and high thrust levels at the highest possible I_{sp} by using as low a voltage potential as possible [2].

Desirable traits for an electrospray propellent include high density, low melting point, low viscosity, high boiling point, high electrochemical stability, high molecular weight, high electrical conductivity, and high surface tension. A high density allows for a larger amount to be able to be stored at a certain volume on a spacecraft. A low viscosity makes easier the transporting of the propellant from the tank to the thrusters. A low melting point allows for the propellant to remain in liquid form by using minimal power to achieve that while high boiling point helps preventing the liquid to change into gaseous form. A high molecular weight allows higher thrust to be achieved since the ions are going to be heavier as well. High surface tension and high conductivity grant higher specific impulse which is inversely proportional to the thrust created but increases the efficiency significantly [1].

Within the trade space for electric propulsion systems, electrosprays have high efficiency, high specific impulse, and lifetimes on the order of 2500 hours [11]. Because they do not have to ionize propellent or interface with plasmas, electrosprays can achieve higher efficiencies in more compact forms than GIT's or HET's. A qualitative trade space is shown below in Fig. 3, reprinted here from a paper by Ma and Ryan. In order to take advantage of their high efficiency at small scales, built systems to date have all existed in the very low thrust and power regime, on the scale of just a few watts [8]. While this is ideal for nanosatellites or fine attitude control, it leaves unanswered the question of whether an electrospray system can be built for high thrust. This paper explores the governing scaling laws for electrosprays (specifically colloids) and demonstrates how a colloid system in the high thrust regime compares to other forms of electric propulsion.

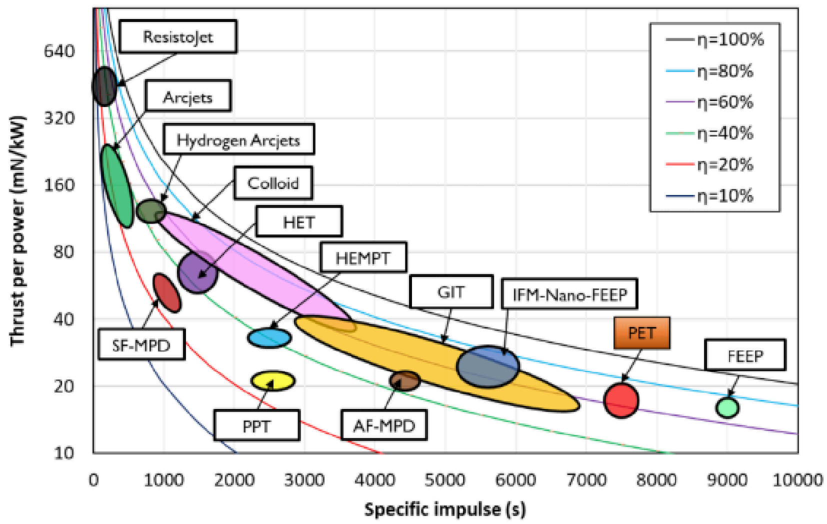

Figure 3: Trade space for electrosprays and other common forms of electric propulsion. Reprinted from "The Design and Characterization of a Porous-emitter Electrospray Thruster (PET-100) for Interplanetary CubeSats" by Ma and Ryan.

III. Historical Background

Electrospray thrusters were first flow in 2015 as a demonstration of the capabilities of the Colloid MicroNewton Thrusters in the LISA Pathfinder mission. A strong characteristic of the CMNTs were being able to produce and control thrust in the order of micronewtons, allowing fine maneuverability of the spacecraft [3]. The thrust noise is also at very low levels, improving precision. However, the lifetime of these thrusters was below the mission target and is in fact an area of ongoing research [12].

Parameter	Value
Maximum Thrust	30 µN
Specific Impulse	150-250s
Operational Lifetime	2500-3000h
Plume Half Angle	<23° (Includes 95% of beam current)
Thrust Noise	≤0.1 µN /√Hz (3-4 Hz)
Total Power	6 W

Table 1: CMNT Performance Specifications [14].

IV. Approach

In arriving at a holistic understanding for scaling of electrosprays, a theoretical, and step by step approach is taken. We begin by exploring the underlying physics of how electrosprays work and establish why the thrust per emitter is so low. This is followed by a consideration of space charge limited current, which is weighed against emitter density limited current. We then outline how the efficiency of an electrospray might scale with thrust. At the end, we will summarize and discuss the key limiting parameters we have identified and compare electrosprays to other forms of electric propulsion. To narrow the scope of this discussion, we will focus on electro sprays that operate in the droplet regime and assume that all droplets have the same charge to mass ratio.

V. Theory

Fundamentally, there are only two strategies for increasing the thrust. Increasing mass flow rate or increasing the exhaust velocity. By applying conservation of energy, thrust can be re-written in terms of the applied voltage, and the charge to mass ratio of accelerated propellent. This yields the following equation [11]:

$$T = \dot{m}\sqrt{\frac{2qV_o}{m}}\#(1)$$

For the case of electrosprays, mass flow rate and exhaust velocity are not independent of one another due to the nature of Taylor cones. At the emission site, a charged fluid when exposed to a suitably high electric field will begin to deform. This minimum electric field is expressed by the following equation [11]:

$$\frac{1}{2}\varepsilon_o E^2 > \frac{2\gamma}{r}\#(2)$$

Meanwhile, the electric field at the apex of the cone is a function of the applied voltage and radius of curvature at the apex [11].

$$E = \frac{-\frac{2V_o}{r}}{\ln ln\left(\frac{4d}{r}\right)}\#(3)$$

The above equations imply that a minimum voltage is required to “start” an electrospray. Eq. 3 also shows that as the radius at the top of the cone decreases (r), the electric field (E) increases. This leads to a positive feedback loop in which (r) tends to zero and the electric field (E) at the apex tends to infinity. In reality, before this point is reached, a jet forms at the apex of the cone, and propellent is emitted at a steady rate. This is known as a cone jet [11].

Figure 4: Taylor Cone with jet [9]

At some distance downstream, the jet breaks up into droplets. These charged propellent droplets are then accelerated by the voltage potential. Many other equations exist which describe the characteristics of the jet and the resultant droplets, but of relevance to this discussion is an expression for the charge to mass ratio of each droplet [5].

$$\frac{q}{m} = \frac{f(\varepsilon)}{\rho}\left(\frac{\gamma K}{\varepsilon Q}\right)^{\frac{1}{2}}\#(4)$$

Where *f(ε)* is an empirical scaling function that is equal to approximately 18 for values of (ε) greater than 40 [9]. The above equation shows that charge to mass ratio increases as volume flow rate decreases. This suggests a direct tradeoff, where decreasing volume flow rate increases the exhaust velocity term but decreases the mass flow rate term. Substituting Eq. (4) back into the thrust expression yields the following:

$$T = \rho Q^{\frac{3}{4}}\sqrt{\frac{f(\varepsilon)}{\rho}\left(\frac{\gamma K}{\varepsilon}\right)^{\frac{1}{2}} 2V_o}\#(5)$$

Eq. 5 shows that the increase to mass flow rate outweighs the penalty to exhaust velocity brought about by increased volume flow. So far this has not yielded any new insight into why the thrust per emitter is so low. To this end, we consider now the relationship between thrust, voltage, and specific impulse. By substituting Eq. (4) into the definition of exhaust velocity, we find [9]:

$$\left(g_o Isp\right)^2 = \frac{f(\varepsilon)}{\rho}\left(\frac{\gamma K}{\varepsilon Q}\right)^{\frac{1}{2}} 2V_o \#(6)$$

The above equation can be solved for the volume flow rate, beam current, and thrust. The resulting expressions are in terms of voltage and specific impulse only.

$$Q^{\frac{1}{2}} = \frac{f(\varepsilon)}{\rho\left(g_o Isp\right)^2}\left(\frac{\gamma K}{\varepsilon}\right)^{\frac{1}{2}} 2V_o \#(7)$$

$$I_B = \rho Q \frac{f(\varepsilon)}{\rho}\left(\frac{\gamma K}{\varepsilon Q}\right)^{\frac{1}{2}} \#(8)$$

$$T = \frac{f(\varepsilon)^2}{\rho\left(g_o Isp\right)^3}\left(\frac{\gamma K}{\varepsilon}\right) 4V_o^{\ 2} \#(9)$$

Importantly, we find that that thrust scales with the square of voltage, but inversely with the cube of I_{sp}. Supposing that a 1-Butyl-3-methylimidazolium hexafluorophosphate ([BMI]+[HFP]-) solution is used as propellent, we plot thrust as a function of voltage along lines of constant specific impulse [2]. The properties of ([BMI]+[HFP]-) are shown in table 2 and thrust is plotted for a range of specific impulses in Fig. 6. Notably, we see that voltages can run into the regime of several kV, while thrust remains at a fraction of a micro newton.

Permittivity[N/A]	102
Density [kg/m^3]	1380
Conductivity [Si/m]	0.45
Surface Tension [N/m]	0.05 1

Table 2: Properties of ([BMI]+[HFP]-)

A. Space Charge Limited Thrust Density

In the previous section it was shown that extremely large voltages are required to produce even small amounts of thrust. This practical limit is a result of cone jet formation and the governing charge to mass ratio relationship. A distinct thrust limiting factor we consider now is that of space charge effects. Since electrosprays rely on grids to provide acceleration, the maximum current density is represented by the Child-Langmuir law below:

$$J = \frac{4\varepsilon}{9}\left(\frac{2q}{m}\right)^{\frac{1}{2}} \frac{V_o^{\frac{3}{2}}}{d^2} \#(10)$$

We expect from the above relationship that due to the high voltages found in electrosprays, the space charge limited current density should be very high. Eq. 8 can be written back into the thrust equation to obtain the following relationship.

$$\frac{T}{A} = \left(\frac{4\varepsilon_o}{9}\right)\left(\frac{1}{d^2}\right)\left(\frac{m}{2q}\right)^2\left(g_o Isp\right)^4 \#(11)$$

We find that thrust density scales with the fourth power of specific impulse. Next, we recognize that Eq. 4 provides an expression for the charge to mass ratio that we can substitute into the above expression. Then rewriting the volume flow rate in terms of specific impulse and voltage, we find that the dependence on specific impulse drops away. The space charge imposed thrust density limit reduces simply to:

$$\frac{T}{A} = \frac{4\varepsilon}{9}\left(\frac{2q}{m}\right)^{\frac{1}{2}}\frac{V_o^{\frac{3}{2}}}{d^2}\sqrt{\frac{2mV_o}{q}} = \frac{4\varepsilon}{9}\frac{V_o^{2}}{d^2}\#(12)$$

Using a grid spacing of 1 mm, the above relationship is plotted in Fig. 7.

B. Emitter Tip Density

Since the thrust per emitter is voltage limited, total thrust can be increased instead by increasing the number of emitters. This leads to the following expression for thrust density:

$$\frac{T}{A} = T^{'}\sigma\#(13)$$

Where T' is the thrust per emitter, and σ is the number of emitters per unit area, or an emitter tip density. At constant voltage and specific impulse, thrust density is directly proportional to the emitter tip density.

C. Efficiency

Thus far we have only developed relationships for thrust scaling, but to be practical, electrosprays should be able to maintain high efficiencies in the high thrust regime as well. To that end, we shift now to a discussion of efficiency scaling laws. Four main types of efficiency were identified: extraction efficiency, divergence efficiency, Taylor cone formation or power efficiency, and polydispersity efficiency. We begin by generalizing the following statement for overall efficiency, where P is the total power in.

$$\eta = \frac{T^2}{2P_{in}\dot{m}}\#(14)$$

The first mode of losses we consider is the collision of extracted propellent with the extractor grid. We denote this efficiency as the "extraction" efficiency. In practice this limits the lifetime of electrosprays as material builds up on this surface, but in the context of this paper we are concerned with the resulting reduction in thrust. In their paper, Dahl, Kimber, and Jorns present a theoretical model for extractor current, given by the following equation [4].

$$\frac{I_E}{I_B} = \frac{cos\phi - cos\theta}{1 - \cos cos\,\theta}\#(15)$$

Where θ is the divergence angle of the plume, and ϕ is the line of sight angle to the aperture grid. Importantly, they show that divergence angle is a function of the beam current and extractor voltage [4].

$$\theta \approx \left[\left(R\frac{I_B}{V_o}\right)^{\frac{3}{4}}\right]\#(16)$$

Any propellent which impacts the extractor cannot contribute to the net thrust, and so we adjust the original thrust equation to account for this effect. Combining the above equations yield an extraction efficiency term as a function of beam current and voltage.

$$T = \left(1 - \frac{I_E}{I_B}\right) I_B \frac{m}{q} U_{ex} \#(17)$$

$$\eta_E = \left(1 - \frac{I_E}{I_B}\right)^2 \#(18)$$

However, plugging Eq. 7 into Eq. 8 shows that beam current scales directly with voltage and inversely with the square of specific impulse. This allows us to conclude the following:

- Divergence angle (and by extension, extraction efficiency) must be independent of applied voltage if the thruster is operated at constant I_{sp}.
- Divergence angle decreases and extraction efficiency improves with increasing specific impulse.

The perceptive systems engineer will recognize that the second relationship is in fact undesirable to the goal of increasing thrust, due to the inverse relationship of thrust to power ratio with specific impulse.

Moving on to the divergence efficiency, we suggest here by geometric reasoning that the minimum divergence efficiency is dictated by the line of sight angle to the aperture grid. Portions of the emitted plume traveling at angles greater than ϕ will be intercepted by the grid, and thus will not further decrease the divergence efficiency of the exhaust. If the geometry of the thruster is not changing with thrust, we can say the minimum divergence efficiency should be constant with thrust as well. In the regime where $\theta < \phi$, we expect the divergence efficiency to perform according to the relationships outlined above for extraction efficiency – the divergence angle decreases, and divergence efficiency improves with increasing specific impulse.

The next form of efficiency we consider is the energy cost of forming and sustaining the Taylor cone. This energy loss is manifested as a decreased acceleration voltage. Lozano explores this in his paper and identifies it as the dominant source of inefficiency in colloid thrusters. This efficiency can be expressed as [7]:

$$\eta_P = \frac{V_B}{V_\alpha} \#(19)$$

Where V_B is the beam acceleration voltage and V_α is the applied voltage. Lozano does not go further in identifying the governing scaling laws, but we propose here the following relationship: the voltage cost of forming and sustaining the Taylor cone is directly related to the starting voltage. Since the starting voltage is a function of the geometry and propellent only, the associated voltage cost must be constant. This means that as the applied voltage increases, the efficiency improves.

Lastly, we turn to the polydispersity efficiency, also discussed by Lozano. This efficiency arises due to the presence of both droplets and ions in the emitted propellent. This polydispersity efficiency, η_{poly}, can be represented by the following equations [7]:

$$\eta_{poly} = \frac{\left[1-\left(1-\sqrt{\zeta}\right)f_i\right]^2}{1-(1-\zeta)f_i} \#(20)$$

$$f_i = \frac{I_i}{I_i + I_d} \#(21)$$

$$\zeta = \frac{\left(\frac{q}{m}\right)_d}{\left(\frac{q}{m}\right)_i} \#(22)$$

Where I_i is the fraction of the electric current taken by the ions and I_d is the fraction of the electric current taken by the droplets and ζ is the droplet to ion specific charge ratio [7]. In the figure below, reprinted here from Lozano's paper, we can observe that poly-dispersity efficiency is maximized when the thruster is operated in the ion dominated or droplet dominated regime. However, to increase thrust one would increase the volume flow rate. This in turn pushes the thruster to operate further in the droplet dominated regime [7]. Thus, we conclude that poly-dispersity efficiency remains near unity with increasing thrust.

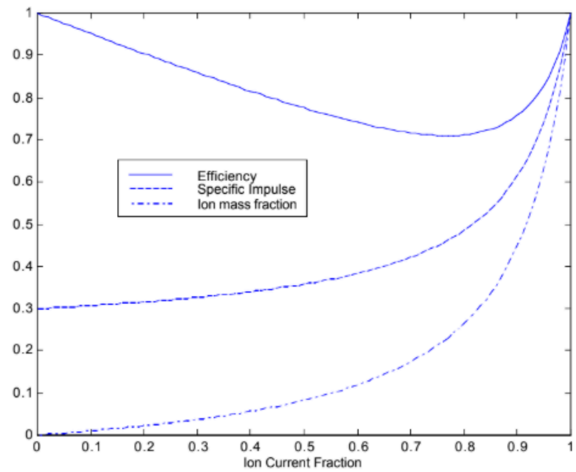

Figure 5: Polydispersity Efficiency as a Function of Current Fraction. Reprinted from "Studies on the Ion-Droplet Regime in Colloid Thrusters" by Lozano [7].

VI. Results

The thrust of an electrospray as a function of specific impulse and voltage as derived in Eq. 7 is plotted below. In particular, we notice that although we are on the order of several thousand volts, the thrust is on the order of a fraction of a micro Newton. We also observe that the thrust drops off significantly with increasing specific impulse (the inverse cube law).

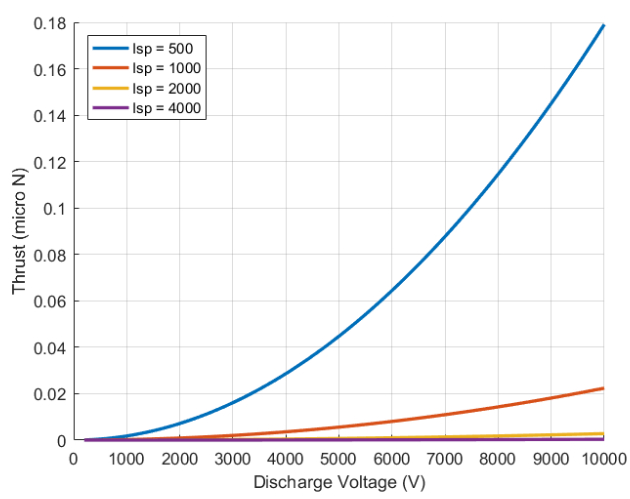

Figure 6: Thrust vs Voltage at constant specific impulse for [BMI]+[HFP]-

Next, we plot the space charge limited thrust density as a function of voltage. A grid spacing of 1 mm is used. As expected, the large voltages result in high allowable thrust densities. Dividing the space charge limited thrust density by the thrust per emitter yields the required emitter tip density at which space charge limits become relevant. The results of this calculation are presented in table 3.

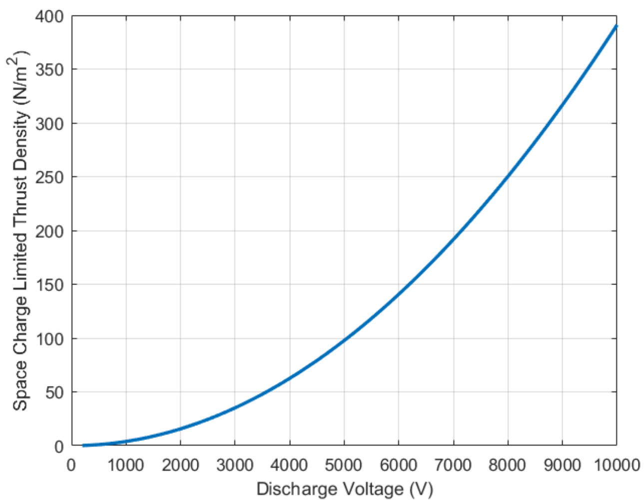

Figure 7: Space charge limited thrust density as a function of applied voltage for an electrospray of grid spacing 1 mm.

Specific Impulse (s)	Emitter Tip Density (cm^{-2})
500	2.18 e4
1000	1.75 e5
2000	1.40 e6
4000	1.12 e7

Table 3: Emitter tip density required before space charge effects become relevant.

To place the values of table 3 in context, an emitter tip would have to be on the order of at least 1 μm in diameter to meet the required density. This is much smaller than what is practical today, where even the smallest emitter tips are on the order of 10 μm [7].

VII. Discussion

In this paper we have investigated a multitude of factors which affect the scaling of an electrospray system. Due to the dependence of q/m on the volume flow rate, the achieved specific impulse is a function of both voltage and mass flow rate. Importantly, electrosprays operating in the droplet regime achieve charge to mass ratios many times lower than that of gridded ion thrusters or hall effect thrusters. As a result, an electrospray can operate at much higher voltages for a given specific impulse, and thus overcome the space charge limit. Unfortunately, to achieve this low charge to mass ratio, the volume flow rate must be extremely low. This is limited by the largest achievable $\gamma K/\varepsilon$ for existing propellants. The result is a very low amount of thrust per emitter and very high voltages that push up against an engineering limit. Any attempt to decrease the voltage requires an increase in q/m, requiring a decrease in mass flow, resulting in a decrease in thrust. This is the fundamental tradeoff of an electrospray system.

Because of their high operating voltages, electrosprays can achieve much higher thrust densities before running into space charge limits. In practice, the thrust density is limited by how tightly individual emitters can be packed and manufactured. Due to the extremely low thrust per emitter, the emitter density is itself a non-trivial factor for electrospray scaling. If the manufacturable density is too low, then electrosprays can still lose out to the space charge limited thrust density of gridded ion thrusters.

Lastly, we identified four contributions to overall system efficiency and studied how they scale with the thrust per emitter, a) extraction efficiency, b) divergence efficiency, c) power efficiency, and d) polydispersity efficiency. When power is held constant, the only efficiency which increases with thrust is the polydispersity efficiency. This result suggests that there are some efficiency losses associated with trying to increase the thrust per emitter. In arriving at this conclusion, we have assumed uniform plume density, and a mono-dispersive beam. These results should be investigated further with actual testing or more rigorous analysis.

Finally, we note a few limitations to our discussion. First, we've only considered thrusters operating in the droplet regime and have neglected many details regarding ion emission and Taylor cone instability. We've also not discussed the potential effects of high emitter densities, or the manufacturing limitations preventing higher emitter densities. Lastly, the type of propellent plays an important role in determining the overall performance of an electrospray thruster, but here we've only identified some of the key parameters that affect thrust. Future work should aim to fill in these gaps in order to provide a more complete picture of electrospray scaling.

VIII. Conclusion

This paper aimed to review the basic principles of colloid thrusters and identify the scaling laws that concern them. Colloid thrusters have certain traits that grant them an advantage over other types of electric thrusters. In particular, electrosprays can theoretically achieve higher thrust densities than gridded ion thrusters, and higher efficiencies than both GIT's and HET's. From first principles, we derived correlations between thrust, volume flow rate, voltage and specific impulse. In this way, greater insight is gained into how these variables trade off with one another and allow greater control over the system behavior. We find that a critical part of electrospray scaling depends on manufacturing limitations to the emitter density. The dimensions of the tip need to be thousands of times smaller for it to bring satisfactory results. In the future, researchers could focus on surpassing these limitations and optimize electrosprays for improved thrust lifetime. Furthermore, propellants are very important components that are used in electrospray thrusters and depending on their properties, the efficiency and the functionality of the system may vary significantly. Future work should delve further into the nature of different propellants in order to improve system performance & safety. Together, these developments could make colloid thrusters a viable and attractive option for future space missions.

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