Standards for Mathematical Practice: Commentary and Elaborations for 6-8[1]

1. Make sense of problems and persevere in solving them.

Mathematically proficient students set out to understand a problem and then look for entry points to its solution. They analyze problem conditions and goals, translating, for example, verbal descriptions into mathematical expressions, equations, or drawings as part of the process. They consider analogous problems, and try special cases and simpler forms of the original in order to gain insight into its solution. To understand why a 20% discount followed by a 20% markup does not return an item to its original price, they might translate the situation into a tape diagram or a general equation; or they might first consider the result for an item priced at $1.00 or $10.00. While working on a problem, they monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain how alternate representations of problem conditions relate to each other. For example, they can navigate among tables, graphs, and equations representing linear relationships to gain insights into the role played by constant rate of change. Mathematically proficient students check their answers to problems and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and compare different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and relationships in problem situations. For example, they can apply ratio reasoning to convert measurement units and proportional relationships to solve percent problems. They represent problem situations using symbols and then manipulate those symbols in search of a solution. They can, for example, solve problems involving unit rates by representing the situations in equation form. Mathematically proficient students also pause as needed during the manipulation process to double-check or apply referents for the symbols involved. In the process, they can look back at symbol referents and the applicable units of measure to clarify or inform solution steps. Quantitative reasoning also entails knowing and flexibly using different properties of operations and objects. For example, in middle school, students use properties of operations to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use assumptions, definitions, and previously established results in constructing arguments. They make and explore the validity of conjectures. For example, students might conjecture that the diagonals of a parallelogram bisect each other, after having experimented with a representative selection of possible parallelograms. They can recognize and appreciate the use of counterexamples, for example, using numerical counterexamples to identify

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[1] Illustrative Mathematics. (2014, February 12). Standards for Mathematical Practice: Commentary and Elaborations for 6-8. Tucson, Ariz. For discussion of the Elaborations and related topics, see the Tools for the Common Core blog: http//commoncoretools.me.

common errors in algebraic manipulation, such as thinking that 5 - 2x is equivalent to 3x. Mathematically proficient students justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. For example they might argue that the great variability of heights in their class is explained by growth spurts, and that the small variability of ages is explained by school admission policies. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. They can construct formal arguments, progressing from the use of concrete referents such as objects and actions and pictorial referents such as drawings and diagrams to symbolic representations such as expressions and equations. They can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. This might be as simple as translating a verbal description to a mathematical expression. It might also entail applying proportional reasoning to plan a school event or using a set of linear inequalities to analyze a problem in the community. Mathematically proficient students are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. For example, they can roughly fit a line to a scatter plot to make predictions and gather experimental data to approximate a probability. They are able to identify important quantities in a given relationship such as rates of change and represent situations using such tools as diagrams, tables, graphs, flowcharts and formulas. They can analyze their representations mathematically, use the results in the context of the situation, and then reflect on whether the results make sense, possibly improving the model if it has not served its purpose. For example, they can recognize the limitations of linear models in certain situations, such as representing the amounts of stretch in a bungee cord for people of different weights.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a graphing calculator, a spreadsheet, a statistical package, or dynamic geometry software. Proficient students make sound decisions about when each of these tools might be helpful, recognizing both the insights to be gained and their limitations. For example, they use graphs to model functions, algebra tiles to see how properties of operations apply to algebraic expressions, graphing calculators to solve systems of equations, and dynamic geometry software to discover properties of parallelograms. When making mathematical models, they know that technology can enable them to visualize the results of their assumptions, to explore consequences, and to compare predictions with data. For example, they might use a spreadsheet simulation to answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood? Mathematically proficient students are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts, such as a computer applet demonstrating Archimedes' procedures for approximating the value of π.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. For example, they can use the definition of rational numbers to explain why a number is irrational and describe congruence and similarity in terms of transformations in the plane. They state the meaning of the symbols they choose, consistently and appropriately, such as inputs and outputs represented by function notation. They are careful about specifying units of measure, distinguishing, for example, between linear and area measures. They label axes to display the correct correspondence between quantities in a problem, such as the intervals and frequencies on the axes of a histogram. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context, and make explicit use of definitions. For example, they accurately apply scientific notation to large numbers and use measures of center to describe data sets.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. They might use the structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, ascertain the relationship between slopes and solution sets of systems of linear equations, and see that the equation 3x - 2y represents a proportional relationship with a unit rate of 3/2 = 1.5. They might recognize how the Pythagorean theorem is used to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism. They also can step back for an overview and shift perspective, as in finding a representation of consecutive numbers that shows all sums of three consecutive whole numbers are divisible by six. They can see complicated things as single objects, such as seeing two successive reflections across parallel lines as a translation along a line perpendicular to the parallel lines.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Working with tables of equivalent ratios, they might deduce the corresponding multiplicative relationships and connections to unit rates. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity with which interior angle sums increase with the number of sides in a polygon might lead them to the general formula for the interior angle sum of an *n*-gon. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. For example, they learn to see subtraction as addition of the opposite, and use this as a general purpose tool for collecting like terms in linear expressions. They continually evaluate the reasonableness of their intermediate results.