Determining the Relationship between Pullback Distance and the Range of a Marble Shot Off a Table

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Introduction: .:. Top

As a child, I ran experiments using a slingshot, testing different angles and drawback distances to determine the best approach to hitting a target using an elastic rubber band. Though I had an understanding that further drawback distances equaled a further distance that an object traveled, I also noticed that further drawback distances required more force. It was not until taking IB Physics that I learned of the fundamental concepts, specifically the topics of energy transfer and conservation, behind this simple relationship and the correlation between the further pullback distances and the distances an object traveled.

The concept of conservation of energy states that in a system, the amount of energy remains constant and energy is neither created nor destroyed. Energy can be converted from one form to another (potential energy can be converted to kinetic energy) but the total energy within the domain remains fixed.

Statement of the Problem: .:. Top

The purpose of this investigation is, therefore, to determine the relationship between the pullback distance of an elastic band and the range of a marble shot off a table.

A conservation of energy equation can be set up for this investigation:

$$E_{Pe} + E_{Gpe} = E_k$$

Within a system using a stretched elastic band and a marble starting at rest on a table (kinetic energy = 0), the equation can be expanded using the equations for elastic potential energy, gravitational potential energy, and kinetic energy:

$$\frac{1}{2}kx^2 + mgh = \frac{1}{2}mv^2$$

The formula for range is as follows:

$$R = \frac{v^2 sin(2\theta)}{g}$$

We can isolate v from the conservation of energy equation and substitute v into the equation for range:

$$\frac{1}{2}kx^2 + mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{k\Delta x^2}{m}}$$

$$R = \frac{v^2 sin(2\theta)}{g} \quad \Rightarrow \quad R = \frac{kx^2}{mg} + 2hsin(2\theta)$$

Therefore, range (R) is directly proportional to the velocity (v) and pullback distance (Δx). An increased pullback distance will result in a greater potential energy stored by the rubber band, which results in a greater kinetic energy transferred to the marble traveling through the air, and ultimately a larger range.

Hypothesis: .:. Top

Using the above formulas and research, it can be hypothesized that as the pullback distance of the elastic rubber band increases, the range of the marble shot off a table also increases, showing a linear correlation between the variables. Range is defined as the horizontal distance the projectile travels before hitting the ground.

Variables: .:. Top

My independent variable is the pullback distance of the elastic rubber band measured using a ruler. My dependent variable is the range of the marble, measured by the horizontal displacement of the marble once it has landed on the ground using a measuring tape. My controls included the elastic rubber band used, the launch height, the launch angle, and the mass of the marble. The elastic rubber band used was kept constant in this experiment to control the spring constant value. The height at which the marble was launched was kept constant at 45 cm with an uncertainty of 0.05 cm, meaning the gravitational acceleration was also constant at an estimated $9.81\ ms^{-2}$. The mass of the marble was kept constant by using the same marble for all trials. Finally, the angle of each launch was constant at 0°.

Materials: .:. Top

To conduct this experiment, I used an elastic band wound between two pieces of wood separated by a distance of 15 ± 0.05 cm. I attached the pieces to a table and a ruler was secured onto the table behind the rubber band using tape to measure the intended pullback distances. A longer measuring tape was placed on the floor to measure the range of the marble. I also set up a camera in front of the setup in order to record the initial landing point of the marble as accurately as possible. Figure 1 depicts a front view of the setup and Figure 2 depicts a side view with the measuring tape extending from the initial launch point.

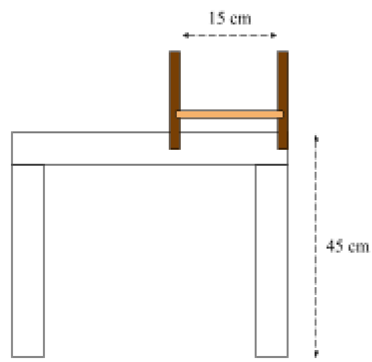


Figure 1

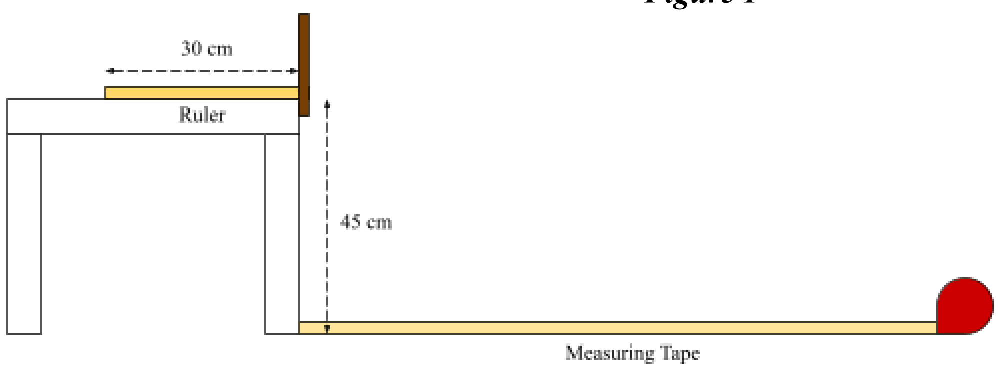


Figure 2

Method: .:. Top

I chose 20 data points, ranging from pullback distances of 0.5 cm to 10 cm increasing by increments of 0.5 cm. These data points were chosen in order to observe significant change in the dependent variable. The increments of 0.5 cm were chosen in part due to practicality to not damage the elastic band with extreme pullback distances. Larger pullback distances also required much more force according to Hooke's Law ($F = kx$) using the same rubber band. The 0.5 cm increments were minimal but sufficient change in distance to notice an effect on the range of the marble. I conducted 3 trials for each data point which allowed me to average the trial results for a more precise result. In order to ensure a constant launch position and angle, the elastic band was pulled straight outwards and the marble was placed in the same spot marked on the table for each trial. I recorded the trials and replayed the videos to get accurate data points as most trials resulted in the marble quickly bouncing away from the initial landing points.

Risk Assessment: .:. Top

Safety and ethical concerns were evaluated for this experiment. Though my investigation involved a moving projectile, the experiment was conducted in an empty room where the marble had a clear launch path so no individuals could be harmed by airborne objects. All materials involved in the experiment were properly secured to ensure that the rubber band would remain in a fixed position and reduce the risk of harm should it snap. There is no environmental harm done as a result of conducting this experiment.

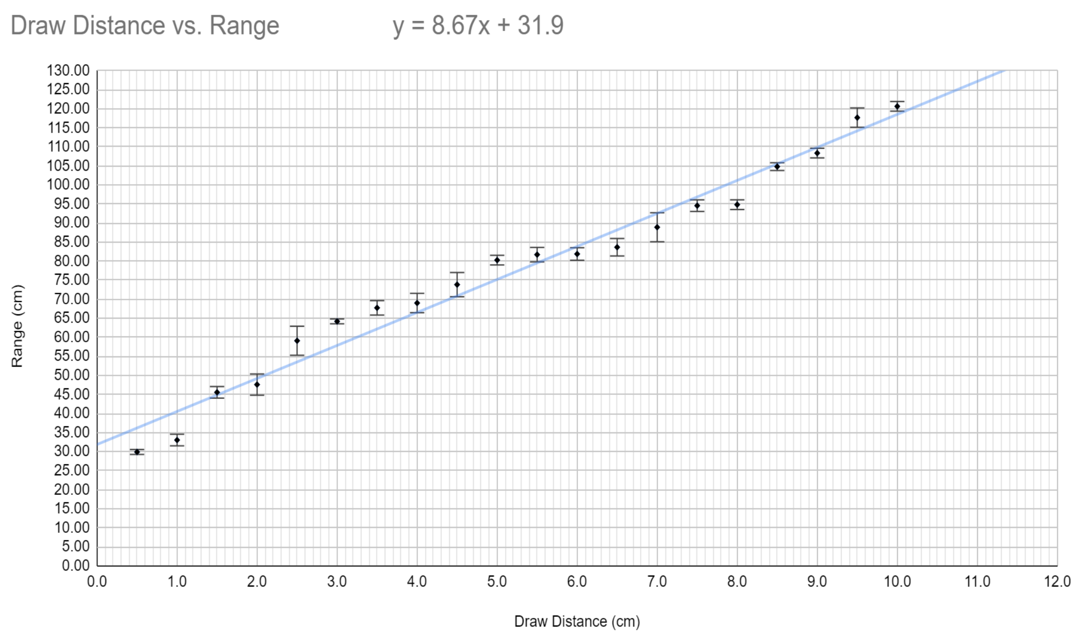
Data and Analysis: .:. **Data File** .:. Top

Data Table 1: Raw data displaying 3 trials for 20 different pullback distance variations

Draw Distance	Range in cm				
+/- 0.05 cm	Range in cm			cm	cm
x / cm	Trial 1	Trial 2	Trial 3	Average	Uncertainty
0.5	29.21	29.97	30.48	29.89	0.64
1.0	34.29	31.24	33.53	33.02	1.52
1.5	43.94	45.72	46.99	45.55	1.52
2.0	45.72	45.72	51.31	47.58	2.79
2.5	54.61	62.23	60.45	59.10	3.81
3.0	64.26	63.50	64.77	64.18	0.63
3.5	69.85	67.31	66.04	67.73	1.90
4.0	66.04	71.12	69.85	69.00	2.54
4.5	72.90	71.12	77.47	73.83	3.18
5.0	78.74	80.77	81.28	80.26	1.27
5.5	80.01	83.82	81.28	81.70	1.90
6.0	80.52	83.82	81.28	81.87	1.65
6.5	83.82	85.85	81.28	83.65	2.29
7.0	86.36	86.36	93.98	88.90	3.81
7.5	92.71	95.76	95.25	94.57	1.52
8.0	93.98	96.52	93.98	94.83	1.27
8.5	104.14	104.14	106.17	104.82	1.02
9.0	109.22	106.68	109.22	108.37	1.27
9.5	115.57	116.84	120.65	117.69	2.54
10.0	120.65	119.38	121.92	120.65	1.27

I used an uncertainty of ± 0.05 cm for measurements made in centimeters because I used a measuring tool with divisions in millimeters. The data points chosen go up to 10 centimeters and increase in half-centimeter increments, so an uncertainty of ± 0.05 is appropriate. The uncertainty calculations take human errors into account.

Data Calculations: .:. Top



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