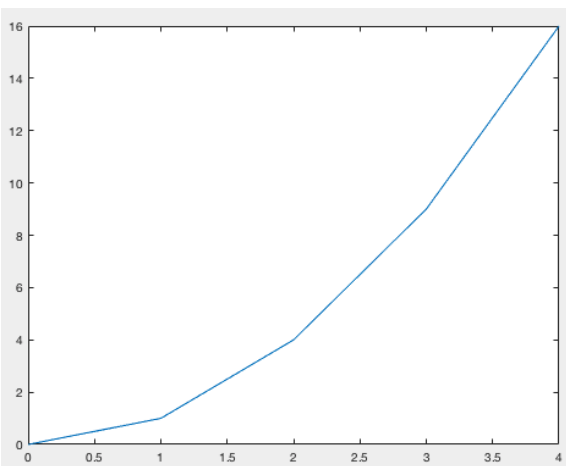
Graphs

- GRAPHING TWO DIMENSIONAL FUNCTIONS -

When plotting functions you must define the independent variable and give its range, like: x=[0:10] or x=[0:2:10]; the former will plot a point every unit on the *x*-axis; the latter will plot a point every 2 units along the *x*-axis. To plot the graph where *x* is the horizontal axis and *y* is the vertical axis use “plot (x, y)”. To flip them, enter “plot (y, x)”.

Here’s an example of plotting $y = x^2$:

```
>> x=[0:4];
y=x.^2;
plot (x,y)
```



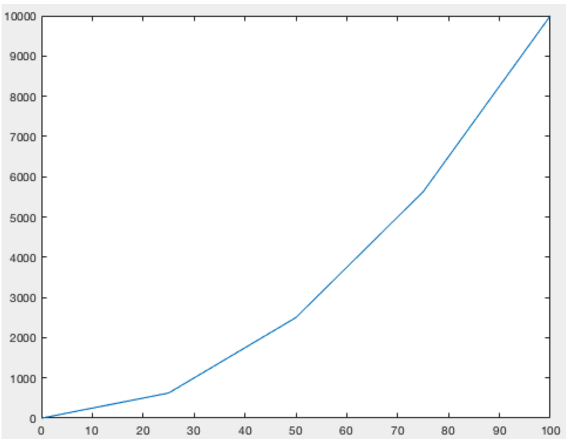
Notice that MATLAB plots a point for every integer x-value and that to type $y = x^2$ into MATLAB, we used “ .^ ” for the exponent. We can also sometimes use ”^”.

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

Here is the same example of plotting $y = x^2$ but by increments of 25.

```
>> x=[0:25:100];
y=x.^2;
plot (x,y)
```



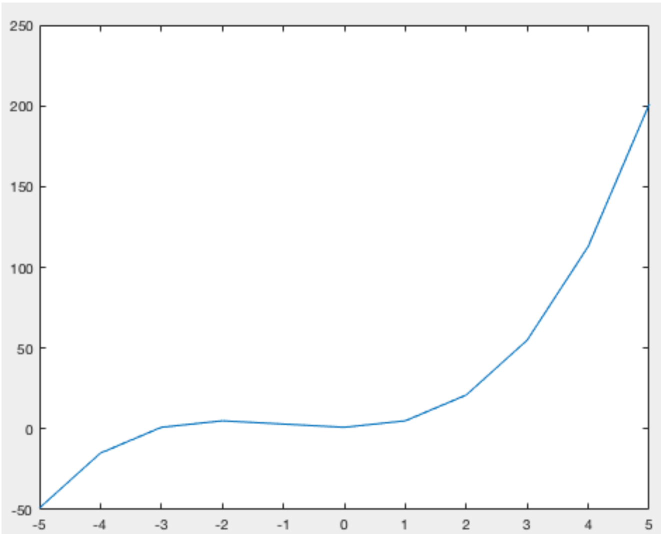
Notice that MATLAB only plotted five points: the x-values 0, 25, 50, 75, and 100

Here's another example, plotting$x^3 + 3x^2 + 1$:

```
>> x=[-5:5]; y=x.^3+3*x.^2+1; plot (x,y)
```

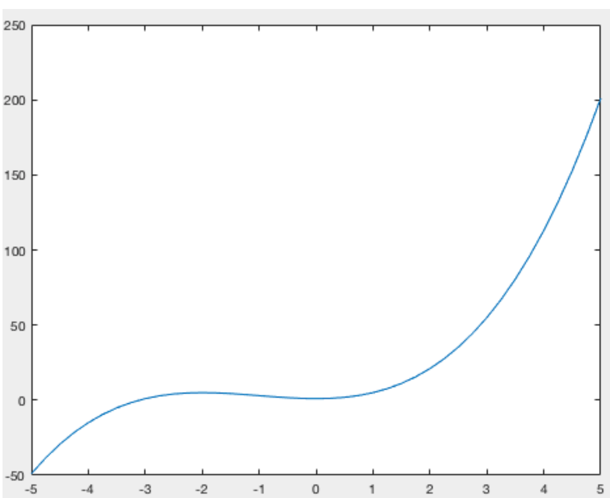
Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set



Notice that MATLAB has plotted a point for each integer value on [-5, 5], thus resulting in a jagged curve. An easy way to fix this issue is to increase the number of plotted points as below, where a point is plotted every .25 units on the *x*-axis:

```
>> x=[-5:.25:5]; y=x.^3+3*x.^2+1; plot (x,y)
```

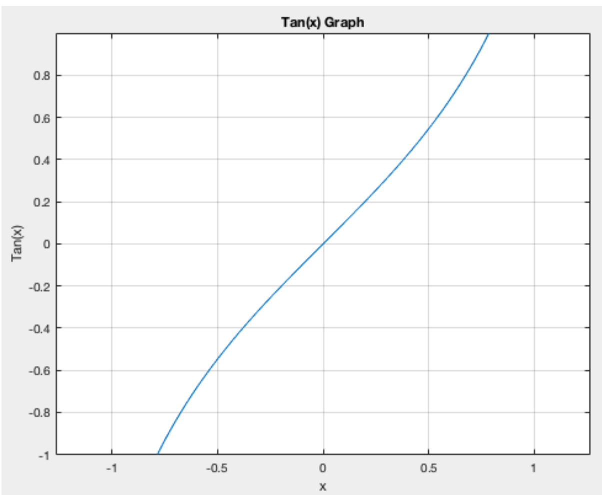


You can also label the graph:

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

```
>> x = [-pi/4:0.01:pi/4];
y = tan(x);
plot(x, y), xlabel('x'), ylabel('Tan(x)'),
title('Tan(x) Graph'),
grid on,
axis equal
```

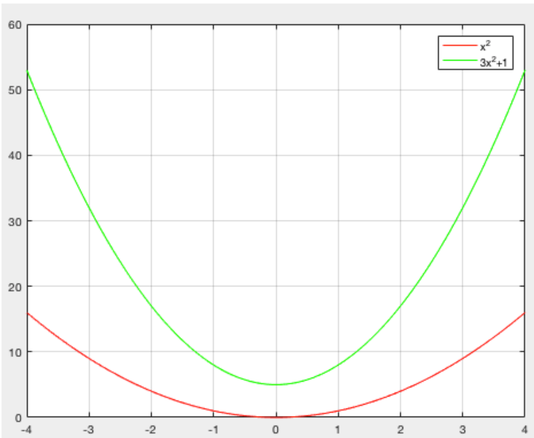


Plot multiple graphs using different colors:

```
>> x = [-4 : 0.01: 4];
y = x.^2;
g = 3*x.^2+5;
plot(x, y, 'r', x, g, 'g' ),
legend('x^2', '3x^2+1'),
grid on
```

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set



$$y = x^2, \ g = 3x^2 + 5$$

Notice the format of the plot command. The ‘r’ and ‘g’ refer to the graphs’ respective colors we can assign.

Here’s the key:

White	**R**ed	**G**reen	**M**agenta
blac**K**	**C**yan	**B**lue	**Y**ellow

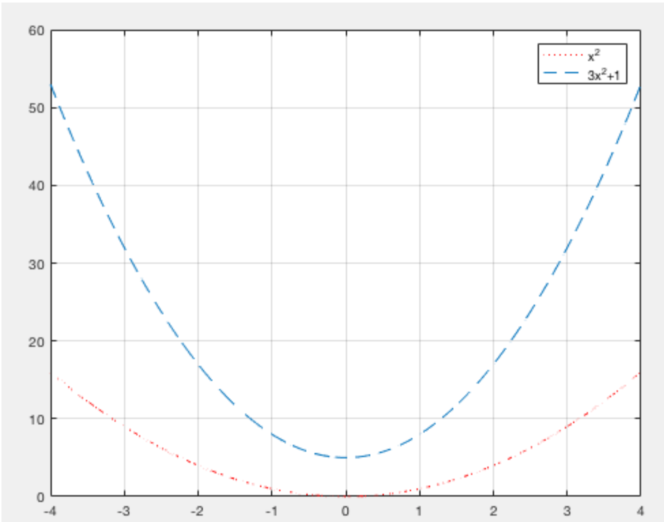
You can modify the line style, too:

. point	. dashdot	* star
o circle	x x-mark	
+ plus	-- dashed	
- solid	: dotted	

```
>> x = [-4 : 0.01: 4]; y = x.^2; g = 3*x.^2+5;
plot(x, y, 'r:', x, g, '--' ), legend('x^2', '3x^2+1'), grid on;
```

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

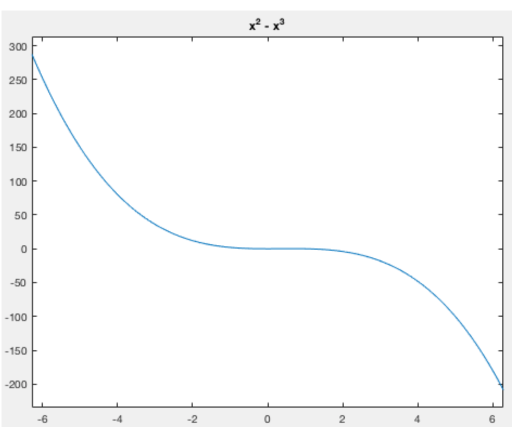


Notice that the ‘r:’ command contains both the red command (r) and the dotted command (:) within the same single quotes. Or you can do them separately as in g, '--'

You may also use “ezplot”:

>> syms x

>> ezplot (x.^2-x.^3)



Challenge Problems.

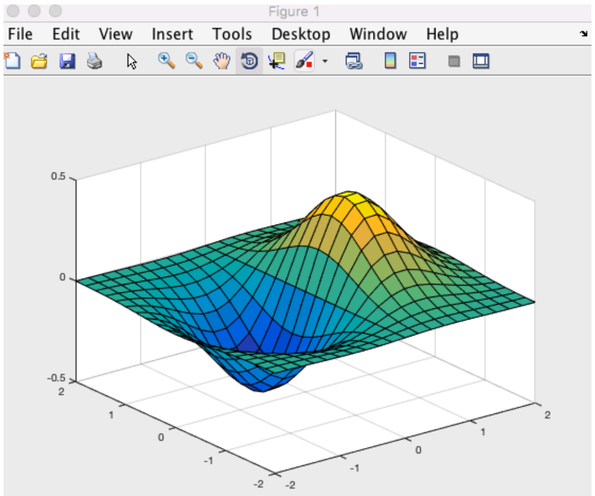
Graph the figures over which the integrands are evaluated on the same set

- GRAPHING THREE DIMENSIONAL FUNCTIONS -

MATLAB uses the commands *meshgrid* and *surf* to plot 3-D functions (functions of two variables). It is essentially the "mesh" of a given surface.

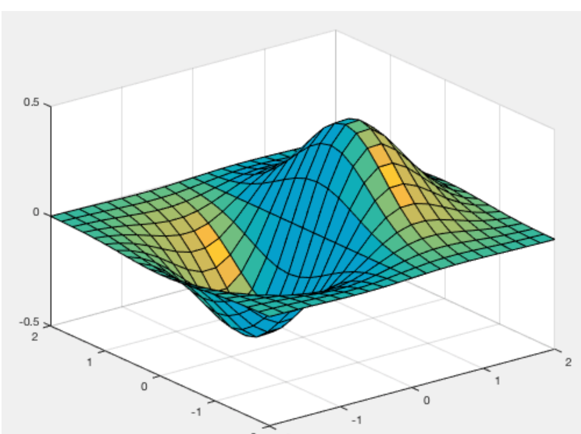
Example: $z = e^{-x^2 - y^2}$

```
>> [x,y] = meshgrid(-2:.2:2); g = x .* exp(-x.^2 - y.^2); surf(x, y, g)
```



You can also use "surfl" for a lighting effect:

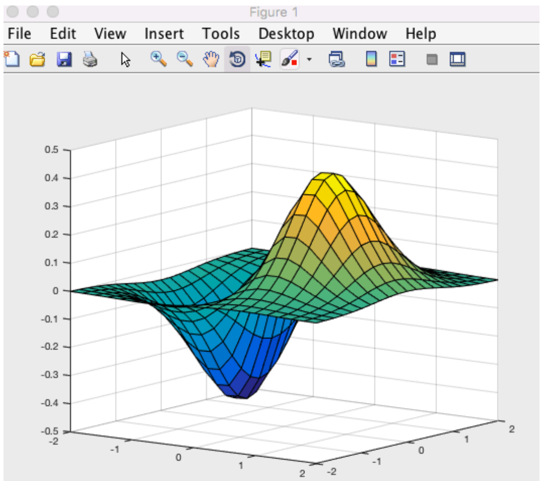
```
>> [x,y] = meshgrid(-2:.2:2); g = x .* exp(-x.^2 - y.^2); surfl(x, y, g)
```



Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

Notice that a point is plotted for every 0.2 units in each direction, which is in the meshgrid command above. You should experiment with the commands at the top of the window. The 3-D rotate button (to the right of the hand) is especially useful.



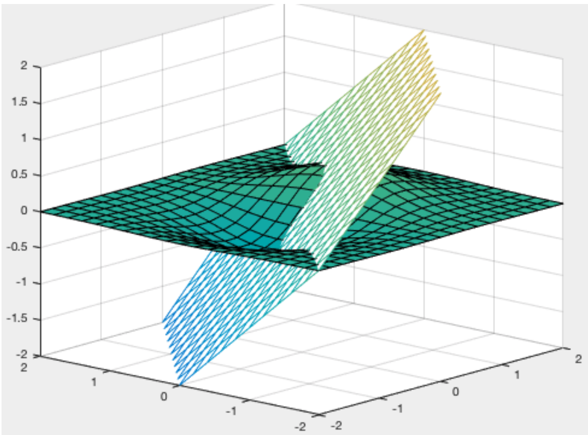
You can also plot two functions on the same set of axes by using the “hold on” command. MATLAB will wait to plot the first function until the second one is plotted. You must use “hold off” afterward to begin a new set of commands.

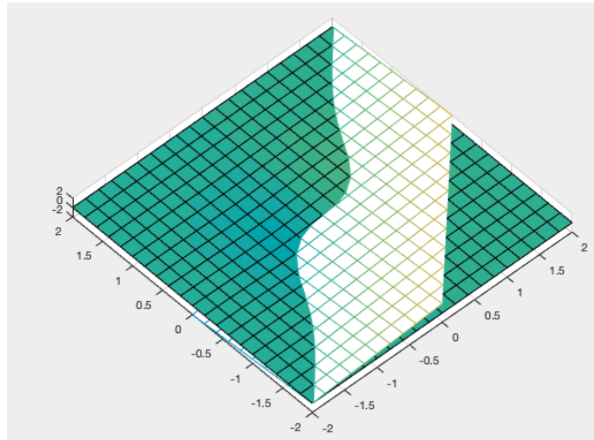
Here is the graph above and the plane $z = x - y$

```
>> [x,y] = meshgrid(-2:.2:2);
g = x .* exp(-x.^2 - y.^2);
surf(x, y, g)
hold on
p= x - y; mesh(x,y,p); axis([-2 2 -2 2 -2 2])
```

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set





The plot is on the left. On the right is a rotation to visualize the intersection. Notice how we've set the range as [-2, 2] for all three *x*, *y*, and *z* axes. (Since the *z*-axis is [-2, 2] instead of [-.5, .5] as in the previous example, the graph of $z = e^{-x^2-y^2}$ appears different.)

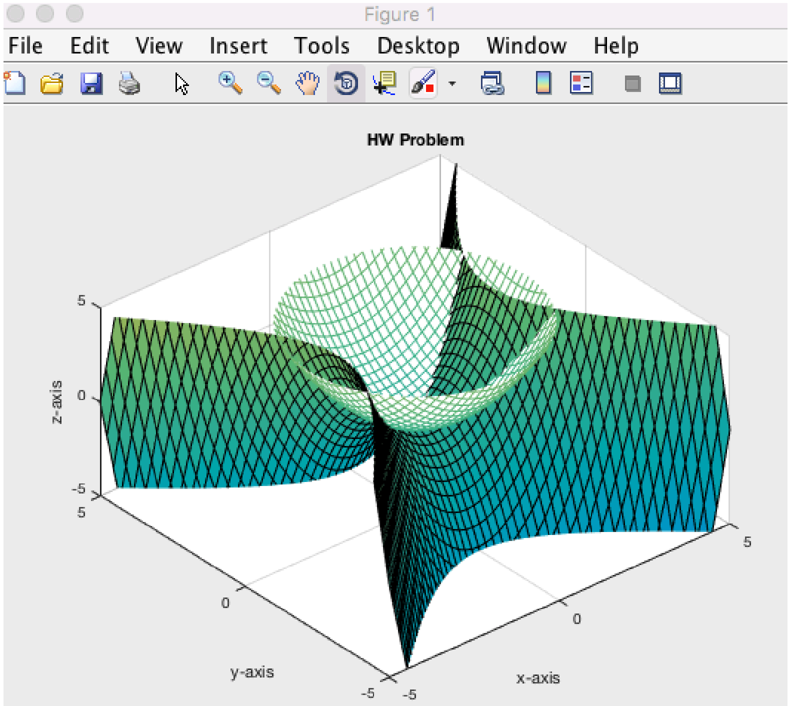
You should experiment with the other options for the window. The labels have been added to the axes and a title added, all by using the Insert tab.

Here's another example:

```
>> [x,y] = meshgrid(-5:.2:5);
g = x.^2 - y.^2;
surf(x, y, g)
hold on
p= .5*(x.^2+y.^2);
mesh(x,y,p);
axis ([-5 5 -5 5 -5 5])
```

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set



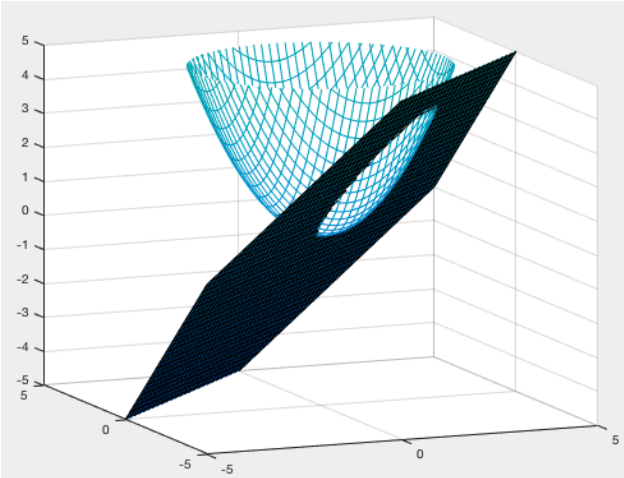
$$z = x^2 - y^2, z = .5(x^2 + y^2)$$

Example:

```
>> [x,y] = meshgrid(-5:.2:5);
g = x-y;
surf(x, y, g)
hold on
p= .5*(x.^2+y.^2);
mesh(x,y,p);
axis ([-5 5 -5 5 -5 5])
```

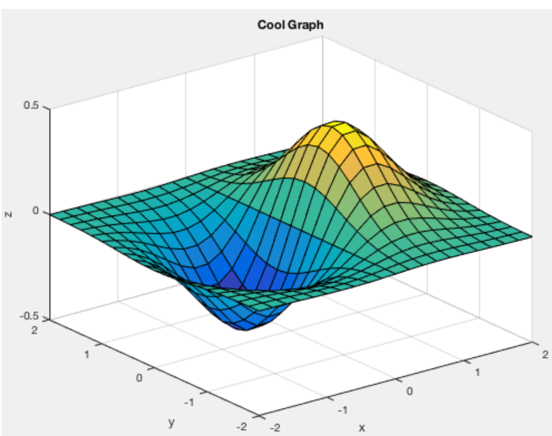
Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set



You can also label within the command structure itself so that the labels are included:

```
>> [x,y] = meshgrid(-2:.2:2);
g = x .* exp(-x.^2 - y.^2);
surf(x, y, g),
xlabel('x'), ylabel('y'), zlabel('z')
title('Cool Graph')
```



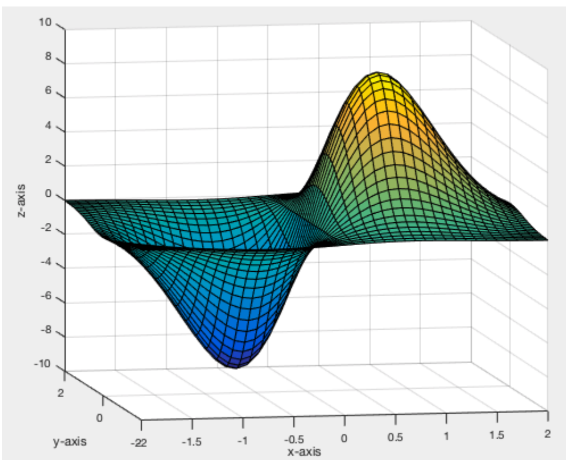
-CONTOUR PLOTS -

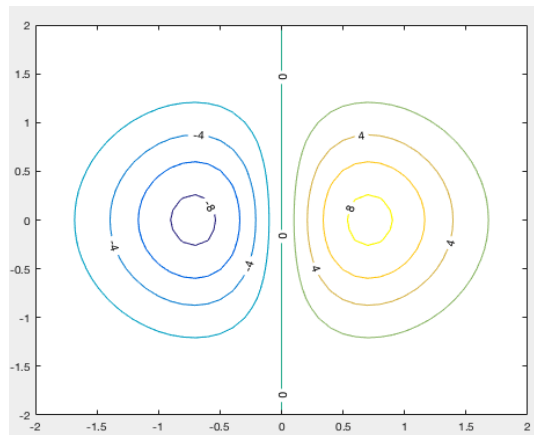
Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

Contour plots represent the graph of a function of two variables set equal to various constants. For example, the contour plot of $z = 20xe^{(-x^2-y^2)}$ is

```
>> [x,y] = meshgrid(-2:0.1:2,-2:0.1:2);
g = 20*x .* exp(-x.^2 - y.^2); [C, h] = contour(x,y,g);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
```





Commands used:

‘ShowText’, ‘on’ -- Contour line labels

TextStep -- Interval between labeled contour lines

LevelStep -- Spacing between contour lines

Here h stands for the contour line functions. We’ve set LevelStep to 2 so the contour lines differ by 2. In other words, the graph represents the functions k=20xe(-x2-y2), k=-8, -6, ... , 6, 8

Challenge Problems.

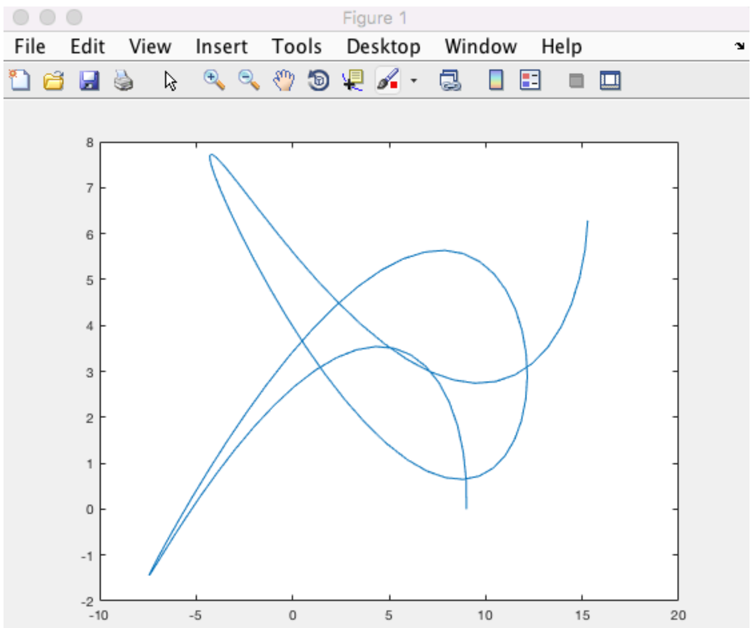
Graph the figures over which the integrands are evaluated on the same set

-PLOTTING PARAMETRIC EQUATIONS -

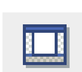
Plotting parametric equations is rather simple. Define the parameter *t* using:

linspace(minvalue, maxvalue)

```
>> t=linspace(0, 6);
x=t+9*cos(2*t);
y=t+3*sin(3*t);
plot(x,y)
```



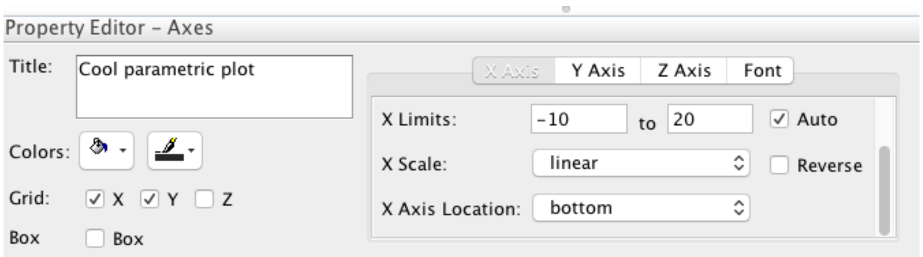
You can experiment with the graph by clicking on the following plot tools icon at the top:



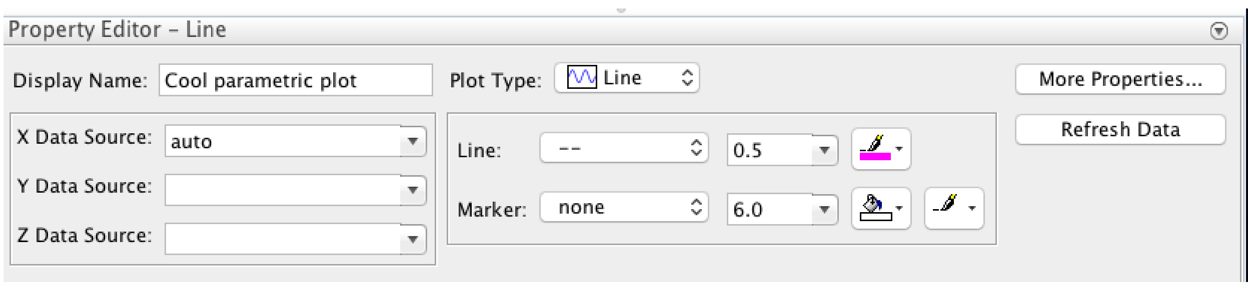
Click on the grid to get the following window:

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set



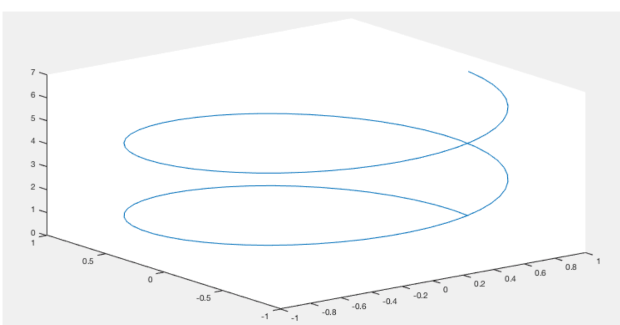
Click on the graph to get this window:



To plot 3-dimensional parametric equations sets, simply insert a *z*-variable and use “plot3” as in this example:

>> t=linspace(0, 6);

>> x=cos(2*t); y=sin(2*t); z=t; plot3(x,y,z)

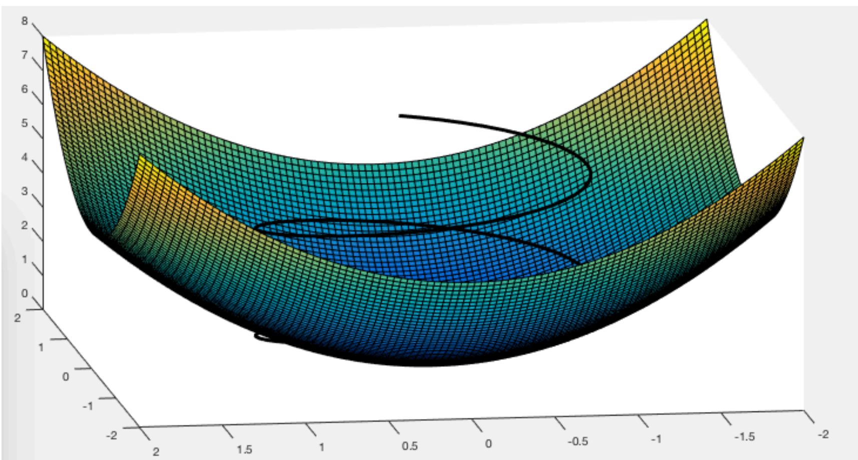


We can superimpose a surface onto the graph above by using the following commands:

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

```
>> t=linspace(0, 6);
>> x=cos(2*t); y=sin(2*t); z=t; plot3(x,y,z); hold on
>> x=linspace(-2,2);
>> y=linspace(-2,2);
>> [x,y]=meshgrid(x,y);
>> z=x.^2+y.^2;
>> surf(x,y,z)
```



(In the figure above we used the plot tools to make the helix more pronounced and we rotated the figure by using the 3-D rotator icon.)

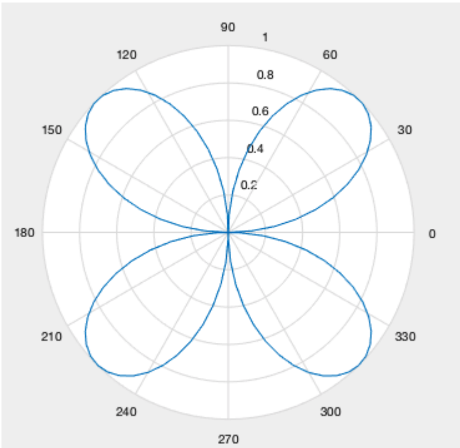
-PLOTTING POLAR EQUATIONS -

Plotting polar equations is rather simple as well. Define the parameter *t* as above.

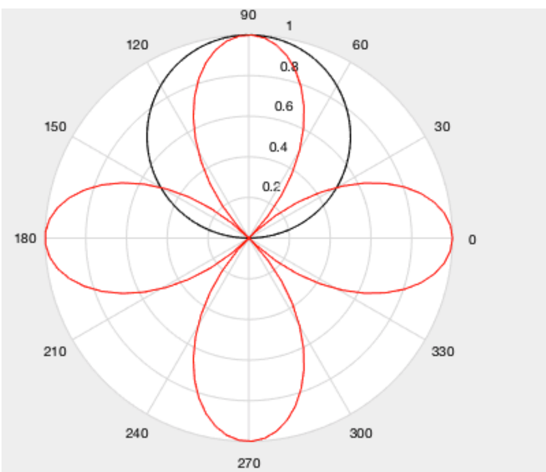
```
>> t=linspace(0, 2*pi);
r=sin(2*t);
polar(t, r)
```

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set




```
>> t=linspace(0, 2*pi);
r=sin(t); s=cos(2*t);
polar(t, r, 'k'); polar (t, s, 'r')
```

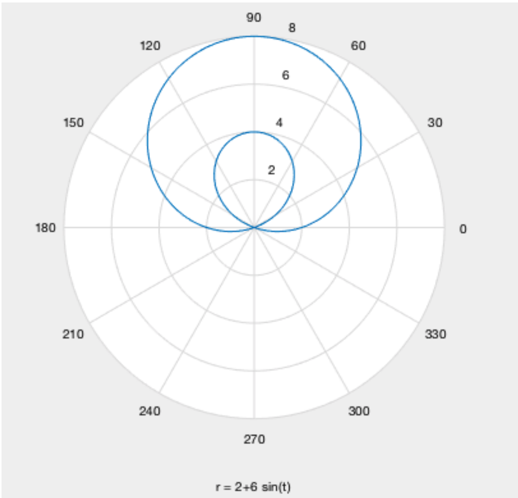


You can also use the simpler **ezpolar** command; Matlab chooses the range for you.

```
>> ezpolar('2+6*sin(t)')
```

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set



EXERCISES

Calculus I

Sketch a graph of each function.

1. $y = x^3 + 3x^2$
2. $y = 2 + 3x^2 - x^3$
3. $y = x^4 - 4x$
4. $y = x^4 - 8x^2 + 8$
5. $y = x(x - 4)^3$
6. $y = x^5 - 5x$
7. $y = \frac{1}{5}x^5 - \frac{8}{3}x^3 + 16x$
8. $y = (4 - x^2)^5$
9. $y = \dfrac{x}{x - 1}$
10. $y = \dfrac{x^2 + 5x}{25 - x^2}$

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

11. $y = \dfrac{x - x^2}{2 - 3x + x^2}$

12. $y = 1 + \dfrac{1}{x} + \dfrac{1}{x^2}$

13. $y = \dfrac{x}{x^2 - 4}$

14. $y = \dfrac{1}{x^2 - 4}$

15. $y = \dfrac{x^2}{x^2 + 3}$

16. $y = \dfrac{(x - 1)^2}{x^2 + 1}$

17. $y = \dfrac{x - 1}{x^2}$

18. $y = \dfrac{x}{x^3 - 1}$

17–22 Use a computer algebra system to graph f and to find f' and f''. Use graphs of these derivatives to estimate the intervals of increase and decrease, extreme values, intervals of concavity, and inflection points of f.

17. $f(x) = \dfrac{x^3 + 5x^2 + 1}{x^4 + x^3 - x^2 + 2}$

18. $f(x) = \dfrac{x^{2/3}}{1 + x + x^4}$

19. $f(x) = \sqrt{x + 5 \sin x}, \quad x \leqslant 20$

20. $f(x) = x - \tan^{-1}(x^2)$

21. $f(x) = \dfrac{1 - e^{1/x}}{1 + e^{1/x}}$

22. $f(x) = \dfrac{3}{3 + 2 \sin x}$

Calculus II

Sketch each parametric curve.

1. $x = t^2 + 4t, \quad y = 2 - t, \quad -4 \leqslant t \leqslant 1$
2. $x = 1 + e^{2t}, \quad y = e^t$
3. $x = \cos\theta, \quad y = \sec\theta, \quad 0 \leqslant \theta < \pi/2$
4. $x = 2\cos\theta, \quad y = 1 + \sin\theta$

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

Sketch each polar graph.

9. $r = 1 + \sin\theta$

10. $r = \sin 4\theta$

11. $r = \cos 3\theta$

12. $r = 3 + \cos 3\theta$

13. $r = 1 + \cos 2\theta$

14. $r = 2\cos(\theta/2)$

15. $r = \dfrac{3}{1 + 2\sin\theta}$

16. $r = \dfrac{3}{2 - 2\cos\theta}$

19. The curve with polar equation $r = (\sin\theta)/\theta$ is called a **cochleoid**. Use a graph of r as a function of θ in Cartesian coordinates to sketch the cochleoid by hand. Then graph it with a machine to check your sketch.

20. Graph the ellipse $r = 2/(4 - 3\cos\theta)$ and its directrix. Also graph the ellipse obtained by rotation about the origin through an angle $2\pi/3$.

Calculus III

Quadric Surfaces

31–38 Reduce the equation to one of the standard forms, classify the surface, and sketch it.

31. $y^2 = x^2 + \frac{1}{9}z^2$

32. $4x^2 - y + 2z^2 = 0$

33. $x^2 + 2y - 2z^2 = 0$

34. $y^2 = x^2 + 4z^2 + 4$

35. $x^2 + y^2 - 2x - 6y - z + 10 = 0$

36. $x^2 - y^2 - z^2 - 4x - 2z + 3 = 0$

37. $x^2 - y^2 + z^2 - 4x - 2z = 0$

38. $4x^2 + y^2 + z^2 - 24x - 8y + 4z + 55 = 0$

39–42 Use a computer with three-dimensional graphing software to graph the surface. Experiment with viewpoints and with domains for the variables until you get a good view of the surface.

39. $-4x^2 - y^2 + z^2 = 1$

40. $x^2 - y^2 - z = 0$

41. $-4x^2 - y^2 + z^2 = 0$

42. $x^2 - 6x + 4y^2 - z = 0$

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

Domains and Level Curves

1–2 Find and sketch the domain of the function.

1. $f(x, y) = \ln(x + y + 1)$

2. $f(x, y) = \sqrt{4 - x^2 - y^2} + \sqrt{1 - x^2}$

3–4 Sketch the graph of the function.

3. $f(x, y) = 1 - y^2$

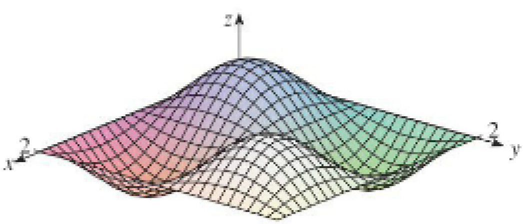
4. $f(x, y) = x^2 + (y - 2)^2$

5–6 Sketch several level curves of the function.

5. $f(x, y) = \sqrt{4x^2 + y^2}$

6. $f(x, y) = e^x + y$

7. Make a rough sketch of a contour map for the function whose graph is shown.



Tangent Planes to Surfaces

7–8 Graph the surface and the tangent plane at the given point. (Choose the domain and viewpoint so that you get a good view of both the surface and the tangent plane.) Then zoom in until the surface and the tangent plane become indistinguishable.

7. $z = x^2 + xy + 3y^2$, $(1, 1, 5)$

8. $z = \sqrt{9 + x^2y^2}$, $(2, 2, 5)$

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set

47–48 Use a computer to graph the surface, the tangent plane, and the normal line on the same screen. Choose the domain carefully so that you avoid extraneous vertical planes. Choose the viewpoint so that you get a good view of all three objects.

47. $xy + yz + zx = 3$, $(1, 1, 1)$ **48.** $xyz = 6$, $(1, 2, 3)$

68. (a) Show that the function $f(x, y) = \sqrt[3]{xy}$ is continuous and the partial derivatives f_x and f_y exist at the origin but the directional derivatives in all other directions do not exist.

(b) Graph f near the origin and comment on how the graph confirms part (a).

Graphs of Limits of Integration (Multiple Integrals)

Graph the functions on which the integrand is evaluated on the same set of 3D coordinates.

9. $\iiint_E y\,dV$, where
$E = \{(x, y, z) \mid 0 \leqslant x \leqslant 3, 0 \leqslant y \leqslant x, x - y \leqslant z \leqslant x + y\}$

10. $\iiint_E e^{z/y}\,dV$, where
$E = \{(x, y, z) \mid 0 \leqslant y \leqslant 1, y \leqslant x \leqslant 1, 0 \leqslant z \leqslant xy\}$

11. $\iiint_E \frac{z}{x^2 + z^2}\,dV$, where
$E = \{(x, y, z) \mid 1 \leqslant y \leqslant 4, y \leqslant z \leqslant 4, 0 \leqslant x \leqslant z\}$

12. $\iiint_E \sin y\,dV$, where E lies below the plane $z = x$ and above the triangular region with vertices $(0, 0, 0)$, $(\pi, 0, 0)$, and $(0, \pi, 0)$

13. $\iiint_E 6xy\,dV$, where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$

14. $\iiint_E (x - y)\,dV$, where E is enclosed by the surfaces $z = x^2 - 1$, $z = 1 - x^2$, $y = 0$, and $y = 2$

15. $\iiint_T y^2\,dV$, where T is the solid tetrahedron with vertices $(0, 0, 0)$, $(2, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$

Challenge Problems.

Graph the figures over which the integrands are evaluated on the same set