BC Calculus Name: _____

Day 5.8 Comparison Tests

1) Consider the infinite series $\sum_{n=1}^{\infty} a_n$

   a) What can we conclude ***about the series above*** if $\lim_{n\to\infty} a_n = 0$? _____

   b) What can we conclude ***about the series above*** if $\lim_{n\to\infty} a_n \neq 0$? _____

2) Which series (circle them) must diverge ***according to the divergence test***? Justify your answers carefully.

   a) $\sum_{n=1}^{\infty} \frac{1}{n}$

   b) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

   c) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

The *comparison test* and the *limit comparison test*

3) Let $\{a_n\}_1^{\infty}$ and $\{b_n\}_1^{\infty}$ be two sequences for which $0 < a_n < b_n$, for $n = 1, 2, 3, \ldots$.

   a) If $\sum_{n=1}^{\infty} a_n$ converges, what can we say about $\sum_{n=1}^{\infty} b_n$? _____

   b) If $\sum_{n=1}^{\infty} a_n$ diverges, what can we say about $\sum_{n=1}^{\infty} b_n$?

   _____

   c) If $\sum_{n=1}^{\infty} b_n$ converges, what can we say about $\sum_{n=1}^{\infty} a_n$? _____

   d) If $\sum_{n=1}^{\infty} b_n$ diverges, what can we say about $\sum_{n=1}^{\infty} a_n$?

   _____

4) ***Use the comparison test*** to determine whether each series converges (C) or diverges (D).

State clearly each of the following: **i)** which series, $\sum_{n=1}^{\infty} b_n$, you are comparing your series to; **ii)** the convergence or divergence of $\sum_{n=1}^{\infty} b_n$ (justify).

a) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}+1}$ C ___ D ___

b) $\sum_{n=1}^{\infty} \frac{n}{n^{5/2}+n}$ C ___ D ___

c) $\sum_{n=1}^{\infty} \frac{n}{n^2-1}$ C ___ D ___

d) $\sum_{n=1}^{\infty} \frac{3^n+n^3}{2^n}$ C ___ D ___

5) ***Use the limit comparison test*** to determine whether the given series converges or diverges.

State clearly each of the following: **i)** which series $\sum_{n=1}^{\infty} b_n$ you are comparing your series to; **ii)** the convergence or divergence of $\sum_{n=1}^{\infty} b_n$ (justify); **iii)** show the steps of the limit test, i.e. evaluate $\lim_{n\to\infty} \frac{a_n}{b_n}$.

a) $\sum_{n=1}^{k} \frac{n+1}{n^{5/2}-n^{3/2}}$ C ___ D ___

b) $\sum_{n=1}^{\infty} \frac{2n-3}{\sqrt{n^3+1}}$