

B.Sc. Hons. MATH (Semester – 4th)
DIFFERENTIAL EQUATIONS-II
Subject Code: BMATS1-401
Paper ID: [19131216]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

- a. Obtain a partial differential equation by eliminating the arbitrary constants from $z = c e^{bt} \cos \cos (bx) ..$
- b. Find the complete integral of the partial differential equation $p^2 q^2 (px + qy - z) = 2.$
- c. Find the solution of $(D^2 - D'^2)z = 0.$
- d. Classify the partial differential equation $u_{xx} + 3u_{xy} + u_{yy} = 0.$
- e. Find the solution of $4u_x = u_y$ using variable separable method.
- f. Write the Laplace equation in spherical co-ordinate system.
- g. Write the Bessel's and Legendre's function.
- h. Find the complete integral of the partial differential equation $p^2 y(1 + x^2) = qx^2.$
- i. Find the solution of Cauchy initial value problem $u_t + cu_x = 0, x \in R, t > 0, \text{ with } u(x, 0) = f(x).$
- j. Find the family of characteristic equation of the partial differential equation.

$$x^2 u_{xx} - 2xyu_{xy} + y^2 u_{yy} = e^x.$$

Section – B **(5 marks each)**

Q2. Find the general solution of the partial differential equation $xy^2 p + y^3 q = zxy^2 - 4x^3.$

Q3. Find the general solution of the partial differential equation

$$(2D^2 - D'^2 - DD' + D - D')z = e^{2x+3y}$$

Q4. Find the solution of diffusion equation using variable separable method.

Q5. Show that in cylindrical coordinates system, r, θ, z the Laplace equation, $\nabla^2 u = 0$ takes the form $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = 0$.

Q6. Show that the following partial differential equations are compatible

$$xp - yq = x, \quad x^2p + q = xz.$$

Also, find their solution.

Section – C **(10 marks each)**

Q7. Find the equation of the system of surfaces which cut orthogonally the family of cones $z^2 = c(x^2 + y^2)$. Also obtain the particular surface which passes through the circle $z = 3$, $x^2 + y^2 = 9$.

Q8. Solve the partial differential equation $rq^2 - 2psq + tp^2 = qr - ps$.

Q9. Find the solution of one dimensional wave equation using variable separable method.